XII.—Seismic Radiations. By Professor C. G. Knott, D.Sc.

(Read December 2, 1907 ; and January 20, 1908.)

WHEN a large earthquake occurs in any part of the earth, its tremors can be recorded on suitable instruments all over the surface of the globe. These instruments are of various types, the most familiar being the horizontal pendulum, the evolution of which we owe to the labours of a small band of enthusiasts who were engaged by the Japanese Government in the late seventies and early eighties to teach the students of Japan the scientific methods of the West. The most conspicuous of these is undoubtedly Professor John Milne, who in his seismological laboratory established in the Isle of Wight continues to study the mysterious movements of the earth. Prompted by him, the British Association has installed some fifty instruments in various parts of the world; and from the accumulating records furnished by these instruments, Milne pursues his seismological studies. In addition to these fifty British Association stations, there have grown up in recent years many seismological laboratories in Europe, Asia, and America; and the data supplied from all these sources place us in a much better position than ever before to draw sure conclusions from the character of the records.

In 1898 I drew a rough sketch of the probable form of wave-fronts and rays of seismic disturbance, basing my calculations upon the early results obtained by Milne as to the times of transit of the different types of motion which characterise the distant earthquake record. This was published as part of the Seismological Committee's Report to the British Association in 1899. A little earlier, Rudzki * and V. Kövesligethy † had independently worked out some brachistochronic problems suggested by the earthquake phenomena; but their mathematical assumptions were made so as to get a soluble case, and had no dynamical basis in harmony with known seismological facts.

In a second paper published in 1905, V. Kövesligethy † works out his original theory more fully, and shows how it may be harmonised with certain of these facts. But his fundamental assumption that the speed of propagation of the seismic waves through the earth diminishes as the depth

* *Beiträge zur Geophysik*, iii., 1898.† *Mathem. u. Naturw. Berichte aus Ungarn*, xiii., 1897 ; xxiii., 1905.

increases, is not one which can be readily admitted in these days. The evidence seems to be all the other way.

In 1905 and 1906, Benndorf * published two important papers, in the second of which he works out a law connecting speed of propagation with depth, starting from the angles of emergence as measured by Schlüter.

Before I knew of these papers of Benndorf, I had myself returned to the problem of nine years ago, and after some trials had obtained a form of solution which seemed to satisfy all the known phenomena. It is this which forms the subject of the present paper.

It is recognised by all seismologists that the records obtained at stations outside the destructive area of a large earthquake consist of two well-marked portions—called by Milne the “preliminary tremors” and the “large waves.” In all well-developed seismograms two distinct phases of the preliminary tremors can be distinguished, a fact first clearly discussed by Oldham; and Omori, the Professor of Seismology in Tokyo, further divides the large waves into five distinct phases.

Here I consider only the two phases of the preliminary tremors, regarding the nature of which there has been a good deal of controversy. The most obvious view regarding these is that they are elastic waves propagated from the earthquake source through the body of the earth. This is the view I have always held along with Milne and Oldham; but it is not the view favoured by the Japanese seismologists. Benndorf's second paper, already referred to, strongly supports the view now to be presented.

When an earthquake occurs it sends forth in all directions seismic disturbances apparently of three distinct kinds. Two of these, the phases, namely, of the preliminary tremors, are transmitted by brachistochronic paths through the earth with speeds of propagation which are determined by the elastic constants and the density of the earth. The large waves, on the other hand, pass round the circumference as surface waves, taking fully two hours to reach the antipodal point after the epoch of the earthquake. This is proved by the fact that the time of transit from the epicentre to any station is proportional to the arcual distance of the station. But the time of transit of the preliminary tremors does not follow this simple law. The times of transit for various distances are given below when the results of the calculation are compared with the results of observation.

In the following investigation I assume—

1. That the source of disturbance is close to the surface of the globe.

* *Mitt. d. Erdbeben-Kommission* (Kaiserl. Akad. der Wissens., Vienna), Nos. xxix., xxxi.

2. That the speed of propagation depends only on the distance from the centre.

Let v be the speed at distance r and T the time along any path. Then by Hamilton's general method applied to brachistochronic problems we have

$$\left(\frac{\partial T}{\partial r}\right)^2 + \left(\frac{\partial T}{r\partial\theta}\right)^2 = \frac{1}{v^2} \quad (1)$$

where r, θ are the polar co-ordinates of a ray passing in any chosen diametral plane including the origin of the disturbance.

Put $r = xR$, where R ($= 6370$ kilometres) is the radius of the globe, and $T = SR$. Equation (1) then becomes

$$x^2\left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial\theta}\right)^2 = \frac{x^2}{v^2} \quad (1')$$

We get a solution of this by putting $\partial S/\partial\theta = \alpha$, a quantity independent of x and θ , and then integrating the equation

$$x\frac{\partial S}{\partial x} = \sqrt{\frac{x^2}{v^2} - \alpha^2}$$

between appropriate limits.

All rays are supposed to begin at the source $x = 1, \theta = 0$. Hence

$$\frac{T}{R} = S = \alpha\theta + \int_1^x \frac{dx}{x} \sqrt{\frac{x^2}{v^2} - \alpha^2} \quad (2)$$

an equation which gives the time of transit from the origin to any point x, θ .

The equation of the path is obtained by equating to an arbitrary constant the result of differentiating S with respect to the constant α . Hence with the same limits any ray is represented by

$$0 = \theta - \alpha \int_1^x \frac{dx}{x \sqrt{\frac{x^2}{v^2} - \alpha^2}} \quad (3)$$

each particular ray corresponding to a definite value of the parameter α . If ψ is the angle at which the radius meets the ray at any point, we have

$$\cotan \psi = \frac{\partial x}{x\partial\theta} = \sqrt{\frac{x^2}{\alpha^2 v^2} - 1} \quad (4)$$

At the surface where $x = 1$, this quantity becomes the tangent of the angle of emergence of the ray. Representing the angle of emergence by e , and writing U for the value of v at the surface, we get

$$\tan e = \sqrt{\frac{1}{\alpha^2 U^2} - 1} \quad \text{or} \quad \cos e = \alpha U \quad (4')$$

When a wave-front impinges on the earth's surface, the speed of pro-

pagation of the surface trace of the disturbance is obviously greater than the true speed of the wave in the ratio of unity to the cosine of the angle of emergence. Hence α is the reciprocal of the fictive speed of propagation of the surface trace. This relation, which can be established almost intuitively by consideration of the manner in which an impinging wave runs along a boundary, must be as old as Fresnel, and is made use of explicitly by Green and many later writers on the laws of reflexion and refraction of light and elastic waves.¹ It gives, indeed, the most lucid explanation of the phenomenon of total reflexion. Nevertheless, both V. Kövesligethy and Benndorf think it necessary to prove the relation analytically, and to enunciate it as something altogether new in the theory of wave motion.

It is clear that a knowledge of the angle of emergence can give no information regarding the speeds of propagation in the deeper parts of the earth. Benndorf has, however, by certain plausible assumptions as to the limiting values of the quantities involved, obtained a solution which satisfies the values of the angles of emergence determined experimentally by Schlüter. The experimental determination of the angle of emergence depends on the comparison of the vertical displacement of the ground with the simultaneous maximum horizontal displacement, and involves a mathematical reduction. The displacements were measured on appropriate forms of seismometer. They were very small and subject to large errors. Taken as a whole, Schlüter's values for the angles of emergence at one locality due to earthquake tremors coming from sources at different distances are in accordance with what would be expected from the nature of the problem; but the data are too meagre to establish any peculiarity of detail such as seems to be indicated.

I purpose to work out a definite case, assuming for the speed v the expression

$$v^2 = V^2 - \mu^2 x^2 = \mu^2 (\alpha^2 - x^2).$$

To determine V and μ we have the two conditions, (1) the value of v at the surface, (2) the time of transit across a diameter.

After some trials I chose the following expression as satisfying these two conditions fairly well, namely,

$$v = \mu \sqrt{\alpha^2 - x^2} = 13.6 \sqrt{1.2 - x^2}.$$

Putting $\phi = \mu \alpha$, we get equations (2) and (3) in the somewhat simpler forms

¹ See, for example, my paper of 1888 on Earthquakes and Earthquake Sounds, etc., *Trans. Seism. Soc. of Japan*, vol. xii., p. 123; also *Phil. Mag.*, July 1899, p. 71.

$$\frac{\mu T}{R} = \phi \theta + \int_1^x \frac{dx}{x \sqrt{\frac{x^2}{a^2} - \phi^2}} \quad (2)$$

$$0 = \theta - \phi \int_1^x \frac{dx}{x \sqrt{\frac{x^2}{a^2} - \phi^2}} \quad (3)$$

The integrals of these are

$$\begin{aligned} \frac{\mu T}{R} = \phi \theta + \sqrt{1 + \phi^2} \left\{ \sin^{-1} \sqrt{\frac{x^2}{a^2}(1 + \phi^2) - \phi^2} - \sin^{-1} \sqrt{\frac{1}{a^2}(1 + \phi^2) - \phi^2} \right\} \\ + \phi \left\{ \sin^{-1} \phi \sqrt{\frac{a^2}{x^2} - 1} - \sin^{-1} \phi \sqrt{a^2 - 1} \right\} \end{aligned} \quad (4)$$

$$\begin{aligned} 0 = \theta + \frac{\phi}{\sqrt{1 + \phi^2}} \left\{ \sin^{-1} \sqrt{\frac{x^2}{a^2}(1 + \phi^2) - \phi^2} - \sin^{-1} \sqrt{\frac{1}{a^2}(1 + \phi^2) - \phi^2} \right\} \\ + \left\{ \sin^{-1} \phi \sqrt{\frac{a^2}{x^2} - 1} - \sin^{-1} \phi \sqrt{a^2 - 1} \right\} \end{aligned} \quad (5)$$

Multiply (5) by ϕ and subtract from (4) and we get a very simple expression for the time, namely,

$$\frac{\mu T}{R} = \frac{1}{\sqrt{1 + \phi^2}} \left\{ \sin^{-1} \sqrt{\frac{x^2}{a^2}(1 + \phi^2) - \phi^2} - \sin^{-1} \sqrt{\frac{1}{a^2}(1 + \phi^2) - \phi^2} \right\} \quad (6)$$

For each possible value of ϕ equation (5) gives a ray, and the least value of x for this ray is determined by the condition

$$x^2 = \frac{a^2 \phi^2}{1 + \phi^2} = \frac{1 \cdot 2 \phi^2}{1 + \phi^2} \quad (7)$$

Also, since $a^2 \phi^2$ cannot exceed $1 + \phi^2$, we find for the greatest possible value of ϕ^2 the expression

$$\phi^2 = \frac{1}{a^2 - 1}.$$

In the case to be discussed, $a^2 = 1 \cdot 2$; hence ϕ^2 cannot exceed 5, and ϕ cannot exceed 2.236.

When x^2 has the limiting value just given, equation (6) becomes

$$\frac{T}{574} = \frac{\mu T}{R} = \frac{1}{\sqrt{1 + \phi^2}} \left\{ \sin^{-1} 0 - \sin^{-1} \sqrt{\frac{5 - \phi^2}{6}} \right\} \quad (8)$$

and gives the time of describing half the ray.

Again, to find half the arc subtended at the centre by a complete ray, we put $x^2(1 + \phi^2) = a^2 \phi^2$ in equation (5) and obtain

$$\begin{aligned} 0 = \theta + \frac{\phi}{\sqrt{1 + \phi^2}} \left(\sin^{-1} 0 - \sin^{-1} \sqrt{\frac{5 - \phi^2}{6}} \right) \\ + \sin^{-1} 1 - \sin^{-1} \phi \sqrt{0 \cdot 2} \end{aligned} \quad (9)$$

Finally, we have by equation (4') for the angle of emergence

$$e = \cos^{-1} \frac{\phi U}{\mu} = \cos^{-1} \phi \sqrt{1.2} = \cos^{-1} (1.095 \phi) \quad . \quad . \quad (10)$$

From these four equations (7), (8), (9), and (10) we can quickly calculate for each assumed value of ϕ between zero and 2.236 the important characteristics of various rays. The results are given in the following table:—

CASE I. $v = 13.6 \sqrt{1.2 - x^2}$ throughout the globe.

Parameter ϕ .	Arc 2 θ° .	Transit time 2T min.	Minimum radius x_1 .	Emergence angle e° .	Energy distribution over surface defined by arc.
2.2	5.03	1.6	0.998	10.3	3.26
2	10.1	2.9	.98	26.6	20.04
1.8	15.6	4.3	.958	36.4	35.26
1.5	24.9	6.5	.913	47.9	54.99
1.2	37.7	8.8	.841	57.6	71.24
1	49.6	10.6	.775	63.4	80.03
0.8	65	12.4	.684	69.0	87.22
.5	98.7	14.9	.492	73.4	95.02
.2	144.2	17.5	.214	84.9	99.21
.1	161.7	17.8	.109	87.4	99.82
.05	170.9	17.9	.055	88.7	99.97
.01	178.2	18	.011	89.8	99.99
0	180	18	0	90	100

As we shall see immediately, the above table gives for arcs smaller than 100° values of times of transit distinctly too high; but for arcs greater than 100° the values agree fairly well. This shows that the speed of propagation must begin to increase more rapidly with depth than is indicated by the assumed formula for v , attaining an almost constant value through a large proportion of the inner parts of the earth.

Let us assume, then, a second case in which the formula

$$v^2 = V^2 - \mu^2 x^2$$

applies from the surface to a depth of only one-tenth of the radius. For greater depths the speed is the same throughout.

After a few trials I chose the expression

$$v = 24.45 \sqrt{1.06 - x^2}$$

as holding true from $x = 1$ to $x = 0.9$. At the surface the speed is 6 kilometres per second; and at all points in the globe up to value $x = 0.9$ the speed is 12.23 kilometres per second.

The ray will be wholly curved when it does not penetrate deeper than $x = 0.9$; but when it penetrates deeper than this limit, the middle portion of the ray will be straight.

The position of the straight part and the angle which it subtends at the centre are obtained by consideration of the angle with which the outer curved part of the ray meets the spherical surface of radius 0.9. This angle might be called the *angle of immergence* into the nucleus of constant speed of propagation. For the rays which penetrate this nucleus we use the formulæ already given between the limits $x=1$ and $x=0.9$; and then for the straight parts we use the simple formula for constant velocity.

The important data are given in sufficient detail in the following table, which differs from the former table only because of the necessity of discriminating between the curved and straight parts of the rays when these exist. The immergence angle has the meaning already defined, and quantities which have to do with the curved and straight parts are marked with suffixes 1 and 2 respectively.

CASE II. $v=24.45\sqrt{1.06-x^2}$ through the outer shell of one-tenth the radius.

Parameter ϕ .	Arc.		Transit time.		Minimum radius x_1 .	Emergence angle e .	Immer- gence angle i .	Energy dis- tribution over surface defined by arc.
	$2\theta_1^\circ$.	$2\theta_2^\circ$.	$2T_1$.	$2T_2$.				
4	1.7	...	0.4	...	0.999	11.5	...	8.03
3	6	...	2.0	...	.977	42.4	...	45.96
2	17.9	...	3.9	...	.922	60.7	...	76.10
1.8	21.6	...	4.5	...	.9	63.8	...	80.10
1.7	14.2	38.5	3.5	5.1	.85	65.4	19.2	82.65
1.5	10.5	67.1	3	8.6	.75	68.4	33.6	86.49
1	6.7	111.4	2.5	12.9	.5	75.8	68.6	94
0.5	2.6	146.2	2.4	15.3	.25	83.0	73.9	98.5
0	...	...	2.3	15.6	...	90	90	100

The whole arc in each case is $2\theta_1+2\theta_2$, and the time of transit $2T_1+2T_2$.

Drawing the time-graphs for these two cases and picking out the values for every 30° of arc, we get the following abridged table, in which Milne's corrected values are added for the sake of comparison:—

TRANSIT TIMES.

Arc.	Case I.	Case II.	Milne.
30°	7.5	5.7	5.2
60°	11.9	9.5	9.8
90°	14.5	13	13.1
120°	16.4	15.6	15.3
150°	17.6	17.6	17
180°	18	17.9	18

The values for Case II. agree very well with Milne's corrected values. That is to say :—

The observed facts of seismic radiation can be co-ordinated on the assumption that throughout all but a comparatively thin crust of the earth the elastic waves of highest speed are transmitted with a speed of 12·23 kilometres per second, and that within this crust, of thickness equal to one-tenth the radius, the speed diminishes from value 12·23 kilometres per second at the inner surface to 6 kilometres per second at the outer surface.

If this wave of highest speed be a compressible wave with longitudinal vibrations, the *cosine* of the angle of emergence of the ray will give the ratio of the magnitude of the horizontal motion to the whole amplitude. Now, most of the observations have been obtained with instruments recording horizontal motion only, and comparatively few with instruments recording vertical motion. But at great arcual distances from the epicentre the angle of emergence increases towards 90°, with corresponding diminution in the value of the *cosine*, and the horizontal component of the displacement will be very small, ultimately vanishing at the antipodal point. At such great distances there will consequently be a tendency for the preliminary tremors, assumed to be mainly compressional, to be retarded in their arrival. As a matter of fact, it is extremely difficult at times to determine the exact moment at which the tremors begin on records which have been obtained at stations further distant than 100° from the epicentre.

According to the theory here developed, we may obtain the times for the second phase of the preliminary tremors by increasing the times for the first phase in the ratio of 31·3 to 18, assuming for the purpose Benndorf's corrected values. The results are as follows :—

TIMES OF TRANSIT OF SECOND PRELIMINARY TREMORS.

Arc.	Case I.	Case II.	Benndorff.
30°	13	9·9	11·9
60	20·7	16·5	17·5
90	25·2	22·6	23·8
120	28·5	27·1	27·5
150	30·6	30·6	30·5
180	31·3	31·1	31·3

The agreement here is not quite so good as in the case of the first phase. It could be improved by slightly increasing the depth of the shell through

which there is variation of speed of propagation. Viewed as a problem in elasticity, this means that the modulus on which the speed of the second phase depends varies with depth according to a law not quite the same as that which holds for the modulus determining the propagation of the first phase,—a highly probable truth.

If the second phase be supposed to be due to the arrival of the first tremors of the distortional wave with displacements perpendicular to the direction of propagation, then the horizontal component of the displacement will be equal to the maximum displacement multiplied by the *sine* of the angle of emergence. Consequently, at great arcual distances the second phase, as recorded by a horizontal pendulum, should tend to become proportionately in greater evidence than the first phase. There are, indeed, cases in which the second phase has been mistaken for the first, the latter having had too small a horizontal component to produce a record.

It seems to me that the theory here sketched is amply sufficient to co-ordinate all the known phenomena. The first phase of the preliminary tremors is thus identified with the compressional waves passing through the body of the earth. No doubt, especially in the more heterogeneous crust, surfaces of discontinuity will start waves of distortional type along with the incident waves of compressional type.* But across the practically homogeneous nucleus the compressional waves will run ahead of their associates of other type, so that what emerges at the surface, although modified in detail, must be referable to these compressional waves. Similarly, when somewhat later the distortional waves flow in in quantity, there will be mingled with them waves of the compressional type. Nevertheless, the second phase will be largely composed of disturbances which have passed through the homogeneous nucleus as distortional waves, but have emerged modified in detail by refraction across discontinuous surfaces.

The distortional waves need not necessarily be more energetic than the compressional; but their generally greater amplitude, as measured on instruments recording horizontal motion, is at once explained in terms of the angle of emergence. Looking back to pages 222, 223, we see that for arcual distances greater than 60° the sine of the angle of emergence is greater than the cosine in ratios exceeding the value 2, rapidly increasing for greater arcual distances.

The longer periods which observation proves to be associated with the

* See my paper on "Reflexion and Refraction of Elastic Waves," *Phil. Mag.*, July 1899, and abstract of address on Earthquakes, *R.S.E. Proceedings*, vol. xxii., 1899.

second phase do not seem to find an immediate explanation along the lines of this theory. But may these not be explained as due to the intermingling of the quicker distortional vibrations with compressional vibrations of longer wave-length which, because of their longer period, have travelled slower than the compressional waves of shorter wave-length? True, the mathematical theory of elasticity does not recognise any relation between speed of propagation and length of wave; but this theory is only a first approximation to reality, and proves nothing, either one way or the other, as to what may occur in seismic vibrations. The fact that the first phase, when well developed, always begins with comparatively rapid oscillations, seems indeed to establish the truth that the shorter waves of a seismic disturbance do travel faster than the longer waves. If we take four seconds to be the shortest period, we find that the disturbance travelling with a speed of 12·23 kilometres per second will have a wave-length of nearly 49 kilometres.

It may be of some interest to compare the elastic constants of the material of the nucleus of the earth on the assumption that we are dealing with compressional and distortional waves. The ratio of the speeds of the two types is 31·3 to 16, or 1·74 to unity. The ratio of the wave-moduli will be as the square of this, or almost exactly 3 to 1. Hence in the notation of Thomson and Tait we have

$$k + 4n/3 = 3n$$

where k is the incompressibility and n the rigidity. This gives

$$3k = 5n$$

a noteworthy result, showing that the inner parts of the earth almost accurately fulfil the conditions of isotropy possessed by the ideal elastic solid of Navier and Poisson. This conclusion seems to me to be an additional argument in favour of the view now being presented.

Here in the heart of the earth is a material at a high temperature and under great pressure, brought into a physical state suggesting homogeneity, though not necessarily implying it. As shown long ago by Tait, this globe is held together mainly by gravitational attraction. The cohesion between the molecules is, however, the force which is involved in the propagation of the elastic disturbances which radiate from a seismic centre. The view of the French elasticians was that true homogeneity required a definite relation between incompressibility and rigidity. This definite relation is not realised in the case of materials tested by ordinary combinations of stress and strain. This fact, however, was not admitted by de St Venant as disproving the

uniconstant theory developed by Navier and Poisson; for as soon as an æolotropic stress is applied to our rods and wires, the material ceases to be truly isotropic. It would appear from the calculation just made that the interior of the earth is in a condition which at each point might be described as isotropic; and the relation required by the uniconstant theory is accurately satisfied by the constants calculated from the propagation of the two phases of the preliminary tremors when these are assumed to be respectively the compressional and distortional vibrations.

We shall now consider the significance of the last column of figures in the tables on pages 222 and 223, the columns headed “energy distribution.” The meaning of the figures is best explained by consideration of particular cases in connection with the accompanying diagram.*

The diagram represents a section of the globe, and some of the particulars corresponding to each of the cases are figured on the one half. The full lines show the paths of the seismic disturbances as they radiate out from the origin O. Each ray corresponds to one of the particular set of values calculated and tabulated on pages 222, 223. The following short table

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indicates the value of the parameter which corresponds to each ray, the rays being represented by the terminal letter on the diagram :—

Ray.	A.	B.	C.	D.	E.
Case I., parameter ϕ . . .	1.5	1	0.8	0.5	0.2
Case II., parameter ϕ . . .	1.8	1.7	1.5	1	0.5

To each full line OA, OB, OC, etc., there corresponds a dotted line Oa, Ob, Oc, etc., which starts tangential to the curved ray and is therefore the direction in which the disturbance begins to radiate outwards from the origin. Considerations of symmetry show at once that the angle which each dotted line makes with the surface at the origin is the same as the angle with which the ray emerges at the other end. In other words, this angle is equal to the emergence angle as tabulated above.

In the diagram the left-hand semicircle shows the rays for the first case, in which the variation of speed is assumed to take place throughout the whole globe; and the right-hand diagram the second case, in which the variation takes place only through the upper layer of thickness, equal to one-tenth of the radius. In the latter case the first ray OA lies wholly within the layer of varying speed of propagation, and is curved throughout its whole length. All the other rays represented pass partly through the interior of constant speed of propagation and are straight throughout a part of their course. Thus the rays OD, OE to distant points are very approximately coincident with chords, but for shorter rays such as OC and OB the chordal coincidence is not so close. We shall discuss this case in some detail.

The dotted line Oa in Case II. gives the direction in which a ray would have passed if the speed of propagation had been absolutely constant throughout the whole globe. This condition would have given rise to what we may call the spherical distribution of energy over the surface of the globe, half the energy being in fact distributed over the hemisphere of which the origin is the pole. But in the case represented in the right-hand semicircle the ray starting originally along the dotted line Oa becomes bent round by refraction so as to assume the position OA. The energy, of course, passes with it. Hence the energy, which in the simple case of spherical distribution would have been distributed over the part of the surface defined by the arc Oa with O as pole, is concentrated within the much smaller part of the surface defined by the arc OA. We are to imagine Oa to be one of a cone of rays which divides the spherical surface into two parts, defined

respectively by the arcs into which Oa divides the semicircle. The semi-vertical angle of this cone is equal to $90^\circ - e$, where e is the corresponding angle of emergence belonging to the ray OA ; and the area on the spherical surface defined by the arc Oa is proportional to

$$1 + \cos (180^\circ - 2e) = 1 - \cos 2e.$$

This number, divided by 2, the value when $e = 90^\circ$, represents the fraction of the energy which is finally distributed over the surface defined by the arc OA . Thus in the particular case which has been the subject of discussion, 80.1 per cent. of the whole energy is found distributed over the comparatively small fraction of the surface whose boundary lies $21^\circ 6'$ from the epicentre. In spherical distribution only 3.5 per cent. of the whole energy would have appeared over this surface. Glancing back to the table for Case II., we see that 50 per cent. of the whole energy is distributed over the small surface whose boundary lies about 7° from the epicentre, and that 75 per cent. is distributed over the surface which does not extend to 18° from the epicentre. In spherical distribution these surfaces would have received respectively only $\frac{1}{3}$ and $2\frac{1}{2}$ per cent. of the whole energy.

It is interesting to compare the two cases figured side by side on the diagram, and to notice how much more concentrated the energy is in the neighbourhood of the epicentre in Case II., which is characterised by a rapid variation of speed of propagation within the upper layers of the earth.

In these calculations I have, for simplicity, assumed the origin to be at the surface. This is never quite the case in large, world-shaking earthquakes. These originate at depths which may vary from 10 to 50 miles. Nevertheless, because of the curving of the seismic rays the energy will be distributed unequally in a manner similar to what is here indicated. The deeper the origin the less unequal will the final distribution be; but so long as the origin lies within the layer of changing velocity, there must be the curving round of the seismic rays, carrying their energy with them.

Let there be two earthquakes of equal intensity but with their origins at different depths. The one with the shallow focus will have its energy strongly concentrated towards the surface regions immediately in the vicinity of the epicentre; while the energy associated with the deeper focus will be less unequally distributed over the whole surface. The latter will be registered all the world over as a world-shaking earthquake, while the former may appear much more limited in its sphere of action, simply because of the small intensity of the tremors which pass to distant regions.

It would be possible, though somewhat laborious, to work out the surface distribution of energy for several depths of origin along the lines indicated above. If this were done, and if instruments could be constructed to give an accurate measure of the energy associated with seismic movements at the surface, we should be in possession of a method for determining the depths of origins—a problem which has hitherto baffled all endeavours.

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