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XXVIII. *On the Physical Structure of the Earth.* By HENRY HENNESSY, F.R.S., Professor of Applied Mathematics in the Royal College of Science for Ireland.*

THE structure of the Earth as a mechanical and physical question is closely connected with the origin and formation of its satellite, and of the planets and satellites belonging to the same solar system. The brilliant results obtained during the present and preceding century by the aid of mathematical analysis, whereby the motions of these bodies have been brought within the grasp of dynamical laws, may have led to the notion that by similar methods many obscure problems relating to the planet we inhabit might be accurately solved. But, although the general configuration of the Earth and planets has been treated mathematically with results which leave little to be desired, when applications of analytical methods are attempted to questions of detail in terrestrial structure, the complication of the conditions is so great as to impose the necessity on some investigators of so altering these conditions as to make their results perfectly inapplicable to the real state of the Earth. Physical Geology presents problems the solution of which undoubtedly calls for mechanical and physical considerations; but these may in general, under the complex nature of the phenomena, be often better reasoned out without the employment of the symbolical methods of analysis. In most cases the conditions are totally unlike those above alluded to, which admit of precise nume-

* Communicated by the Author.

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rical computations. The heterogeneous character of the rocks composing the Earth's crust, and the probably varied nature of the matter composing its interior, render mathematical applications rarely possible, and sometimes misleading. Such views seem to be gradually gaining strength among geologists who pay attention to questions of a general nature, and no one has better expressed them in recent times than Professor M. E. Wadsworth*.

The principle upon which I have ventured to found all my researches on terrestrial physics is this: to reason on the matter composing the globe from our knowledge of the physical and mechanical properties of its materials which come under our notice. Of these properties the most important are density, compressibility, contraction or dilatation from changes of temperature. Newton and other philosophers have already adopted the same principle to a limited extent, when assuming for the mass of fluid composing the Earth in its primitive condition those specific properties which have been assigned to all kinds of fluids observed at the surface. It is impossible to frame any statement more erroneous and misleading than that I have endeavoured to render the question more hypothetical than it was. On the contrary, I have discarded the invariable assumption of mathematicians who treated the question, namely, the hypothesis of the invariability of positions of the particles composing the solidifying earth. The speculations of all rational inquirers upon the Earth's internal structure must necessarily start from the same general principle as above. Some investigators have disregarded that principle, and made the problem thereby a purely mathematical exercise.

In order to reason upon the Earth's figure, we must assume that the laws of fluid equilibrium apply to the inner portions of the fluid as well as the outer. There is nothing hypothetical in reasonings as to the formation of the solid shell and the law of increase of ellipticity of its inner surface as a result of the transition of the formerly fluid matter to the state of solidity. On the contrary, the assumptions of Mr. Hopkins and other mathematicians, that this transition created no change in the law of density of the matter composing the Earth, and in the ellipticity of the strata of equal pressure, are not merely hypothetical; they are directly opposed to well established physical and mechanical laws.

On the other hand, those who have concluded that nothing can be known of the form of the fluid nucleus, seem to deny

* "Lithological Studies." *Memoirs of Harvard College Museum*, vol. i. p. 3; and 'American Naturalist,' June 1884, p. 587.

that the recognized laws of matter apply to the internal condition of the Earth. The shape of the nucleus and the figures of its strata of equal density follow from physical and mechanical laws, just as the forms of the isothermal surfaces within the spheroid follow from the known laws of conduction of heat. Some of the mechanical reasonings regarding the strata of the nucleus and the structure of the solid shell can be presented without employing mathematical symbols, and in what follows I have as far as possible avoided the use of such symbols.

This course, moreover, possesses the advantage of making many parts of the reasonings more clear to geologists and observers of the stratigraphical features of the Earth, who are in reality the ultimate judges of the matter, and not mathematicians. The necessity under which the latter are constrained when dealing with problems, of throwing the preliminary propositions into simple well-defined shapes, admitting of definite deductions, obliges them to overlook the most essential conditions of the very questions at issue, and they thus arrive at results which may be precise, but which are totally inconclusive with reference to the Earth's structure.

*The Mechanical and Physical Properties of the Matter
composing the Earth.*

(1) The materials of the Earth must manifestly influence its general structure, and no inquiries with this structure can be usefully made if the physical properties of these materials are not kept in view. If the interior of the Earth is in a fluid state it is reasonable to believe that the fluid is not the ideal substance called by mathematicians a perfect liquid, namely a substance not only endowed with perfect mobility among its particles but also absolutely incompressible. It is more reasonable to believe that the fluid in question resembles the liquid outpourings of volcanoes, or at least some real and tangible liquid whose properties have been experimentally studied. I have already shown that by overlooking this simple principle certain untenable conclusions, which assert the exclusively solid character of the Earth, have been deduced. Here I propose to develop some additional arguments relative to one of the properties of liquids which has an essential bearing upon the internal structure of the Earth.

(2) In a former paper, on the limits of hypotheses regarding the properties of matter composing the Earth's interior*, I find that, having referred to published statements where the

* Philosophical Magazine for October 1878, p. 265.

facts were not clearly put forward, I underrated the compressibility of liquids as compared with solids. The influence of the imperfect experiments of the Academia del Cimento has long injuriously operated in defining liquid and solid matter, and has produced a remarkable conflict of opinions.

On taking the results of the best experimental investigations, it appears that, although liquids are but slightly compressible as compared with gases, they are highly compressible as compared with solids. In many treatises on Physics and Mechanics which have a high reputation matter is divided into solids, elastic fluids or gases, and incompressible fluids or liquids. Hence the erroneous inference seems to have arisen that liquids are incompressible, not only in comparison with gases, but also in comparison with solid bodies. I was surprised to find this remarkably misleading proposition formally stated long after the decisive experiments of Oersted, Colladon and Sturm, Regnault, Wertheim, and Grassi, in such a work as Pouillet's *Éléments de Physique*, and also in the German translation by Müller. The greater compressibility of liquids as compared with solids is seldom affirmed as a distinct general proposition in books on Physics. It occurs, however, in Deschanel's treatise, both in the original and in the English edition. Daguin states, in vol. i. of his *Traité de Physique*, 2nd edition, p. 40, that the compressibility of liquids was long considered doubtful, but nevertheless they are more compressible than solids.

Lamé also pointed out the great compressibility of liquids as compared to solids. I have before now referred to the statement of the same proposition in the comprehensive work of the late Professor C. F. Naumann, the *Lehrbuch der Geognosie*, vol. i. p. 269, 2nd edition*.

Although in many physical questions the compressibility of liquids may be neglected as well as the compressibility of solids, we are not entitled to assume at any time that the latter are relatively more compressible than the former. In questions where the pressure of columns of liquid of great magnitude comes under consideration, we can no longer treat the liquid as incompressible. In the problem of oceanic tides the incompressibility of the water has been assumed, but if a planet were covered with water to a depth of one hundred miles it would be scarcely correct to make such an assumption. The compressibility is negligible in a small mass of water, but it cannot be neglected in a large mass. Such an assumption is equally unwarrantable with regard to properties

* "Flüssige Körper sind aber mit einer weit stärkeren Compressibilität begabt, als starre Körper."

of matter which, though negligible in some problems, are not in others. Thus in the common hydraulic questions, liquids are assumed to be incompressible; it would be more correct to say the compressibility is neglected. In small problems connected with limited portions of the atmosphere, the compressibility of air may be also neglected, but we could not neglect it for a high column of the atmosphere. If, as before remarked, the Earth were surrounded with an ocean 100 miles deep, the compressibility of the water could not be well overlooked in tidal questions; then, *à fortiori*, compressibility cannot be neglected in such a problem as the tides of a liquid spheroid having a radius nearly equal to that of the Earth. This is immediately made manifest by expressing the compressibilities of liquids, not in terms of the amount due to a single atmosphere of pressure, as is done in most tabulated groups of results, but by some very much greater standard, such as one or two thousand atmospheres. In the experiments of Perkins* the highest pressure employed was two thousand atmospheres, and with this he reduced a column of water by nearly $\frac{1}{12}$ of its volume. The results of experiments with great pressures such as this are highly illustrative of the force by which a fluid may be compressed in the Earth's interior. The actual coefficients of cubical compressibility, on which calculations could be based, may be partly obtained from the more exact researches of Regnault, Grassi, and other recent experiments, or from special investigations on fluid matter conducted with precautions such as these observers have employed. By then comparing the moduli of compressibilities calculated from pressures of 1000 or 10,000 atmospheres, there could be no possibility of overlooking the consequences as to the relations of liquid and solid bodies in any case where they would be subjected to pressures of abnormal magnitude.

(3) The propagation of sound in solids and liquids gives further proof of the greater compressibility of liquids.

The rate v of transmission of sound in solids and liquids is a function of their compressibilities. In solids,

$$v = \sqrt{\frac{gE}{\rho}},$$

where E is the modulus of elasticity and ρ the density. In liquids,

$$v_1 = \sqrt{\frac{gHa}{\mu\rho_1}},$$

* Phil. Trans. 1826, p. 541.

where μ is the coefficient of cubic compressibility, H the pressure of the atmosphere, and a the density of mercury. But, as in solids, the modulus of elasticity is inversely as the compressibility k , we have

$$\frac{v}{v_1} = \sqrt{\frac{\mu \rho_1}{k p H a}}.$$

Both in solids and liquids the velocity of sound is inversely as the square roots of the densities and compressibilities. Although such solids as metals and rocks are denser than most liquids, the limits of their elastic compressibility are so much less, that sound is propagated far more quickly through such solids than through liquids. In steel and metals generally this has been long since established. In rocks the velocity of sound has been computed from direct experiment by Mallet, and has been found to be greater in continuous homogeneous rock than the velocities observed in liquids*.

(4) If we had not the results of direct experiment on the compressibilities of liquids and solids to assure us that these properties in liquids are in excess of those obtained for solids, we might fairly infer this conclusion from the relative dilatability of such substances under differences of temperature†. The construction of our common thermometers is based on the greatly superior dilatability of the liquids enclosed in the thermometer-tube over the material of the tube itself. The dynamical theory of heat clearly establishes that the expansion of solids and liquids is a mechanical action as much as their compression under the action of force, and the substances which contract least by cooling are precisely those which contract least under pressure. Gases which contract most by pressure are also the most dilatable by heat. Liquids occupy

* See 'Philosophical Transactions' for 1861 and 1862.

† Expansions of Metals and Glass for 1° Centigrade, according to Dulong and Petit, at different Temperatures T.

SOLIDS.							LIQUID.	
Platinum.	T.	Iron.	T.	Copper.	T.	Glass.	T.	Mercury.
$\frac{1}{37,700}$	100	$\frac{1}{28,200}$	100	$\frac{1}{19,400}$	100	$\frac{1}{38,700}$	160	$\frac{1}{5550}$
						$\frac{1}{36,800}$	213	$\frac{1}{5425}$
$\frac{1}{38,800}$	311	$\frac{1}{22,700}$	372	$\frac{1}{17,700}$	328	$\frac{1}{32,000}$	353	$\frac{1}{5300}$

an intermediate place between solids and gases, in relation both to the dynamical effect of pressure and the action of loss of heat. If, instead of the experiment of the Academia del Cimento, with globes of porous metals, an experiment with equally strong but impervious vessels had been made, the deformation of each globe would have been unaccompanied by the exudation of the liquid, and the totally false statement that solids are more compressible than liquids would not have so long injuriously influenced physical science.

*The Rotation of the Earth considered as partly Fluid
and partly Solid.*

(1) The problem of the precessional motion of the Earth, considered as a solid shell filled with liquid devoid of viscosity and friction, has been elaborately investigated by Mr. Hopkins in his 'Researches of Physical Geology' in the 'Philosophical Transactions' for 1839, 1840, and 1842, and the result obtained by him has been often quoted as extremely remarkable. Before treating the same question, it may be necessary to state that on the continent of Europe the application made by Mr. Hopkins of his result to Geology is not generally admitted, and views such as I have always firmly upheld seem to be more generally adopted; but some confusion appears to exist as to Mr. Hopkins's results and those to which I have been led. Thus in a recent treatise on Systematic Geology, the author says, with reference to the thickness of the solid crust of the Earth, there are plainly only four possibilities to be thought of:—

1. The Earth is through and through solid.
2. The Earth is through and through fluid, with a solid crust.
3. The Earth has a solid nucleus and a solid crust, with a fluid stratum lying between.
4. The Earth is solid but furnished with cavities which are filled with fluid.

The first and last of those possibilities are not admissible, according to astronomical observations. According to the investigations of Hopkins the action exercised by the Sun and Moon on the position of the Earth's axis in space, by which Precession and Nutation are produced, would be different according to the structure we attribute to the Earth. *The values established by observation compel us to regard the Earth as for the most part in a fluid state, in order that the results may harmonize with calculation* (Pfaff, *Grundriss der Geologie*). This is the reverse of what Hopkins has concluded, and is precisely what I have long since enunciated, which I have

always continued to maintain, and which forms the cumulative result of the investigations in the text of this paper. In a Report to the Royal Irish Academy on "Experiments on the Influence of the Molecular Influence of Fluids on their Motion when in Rotation," p. 57*, I referred to a proof obtained by me of the result alluded to, and I now may be allowed to submit this proof to those interested in the question.

(2) Let us suppose the earth to consist of a solid spheroidal shell, composed of nearly similar spheroidal strata of equal density, and having the ellipticities of the inner and outer surfaces small and nearly equal. The shell is supposed to be full of liquid and to rotate around its polar axis. Under these conditions the attraction of an exterior body would tend to produce pressure between the fluid nucleus and the inner surface of the shell. Whatever may be the direction of this pressure, it can be resolved into a force normal to the shell's surface and into forces in its tangent plane. The normal force might be effective in causing a deformation of the shell, or, if the latter were rigid, it would be destroyed by the shell's resistance. If friction existed between the materials of the shell and the fluid of the nucleus, the resolved forces in the tangent plane would tend to change the motion of the shell from the motion it would have if empty. But if no friction and no adhesion existed between the particles of the liquid and the shell's nearly spherical surface, and if the particles of the liquid are free from viscosity and internal friction among themselves, this purely tangential component could exercise no influence on the motions of the shell. If the solid envelope containing fluid was bounded by planes such as a prismatic vessel or box, it is manifest that unequal normal pressures on the faces of such prism would tend to produce couples, and thus possibly rotations. Such a case has been considered by Professor Stokes, and he has shown that a rectangular prism filled with fluid will have the same motion as if the fluid was replaced by a solid having the same mass, centre of gravity, and principal axes, but with much smaller moments of inertia corresponding to these axes. But in a continuously curved and nearly spherical vessel, the normal pressure arising from disturbance of the liquid could not produce the same results. The tangential components of the forces acting at the surface of the liquid could, in this case, be alone effective, and if no friction or viscosity existed at this surface such tangential action would totally disappear. The conclusion of Mr. Hopkins's first memoir is, that if the ellipticity of the inner and outer surfaces of the solid shell were

* Proceedings of R. I. A., 2nd series, vol. iii. Science.

the same, the precession would be unaffected by the fluid, and any small inequality of nutation would be totally inappreciable to observation, p. 423, Phil. Trans. 1839. This may be rendered more manifest by recalling the general equations for the surface of a fluid obtained by Poisson, Navier, Meyer, and other mathematicians, when the internal friction of the fluid is taken into account. If α, β, γ be the angles made by the normal to the curved surface of the fluid, X, Y, Z the components parallel to the rectangular axes of x, y , and z , it appears that we shall have at the fluid surface, when nearly spherical,

$$X = hk^2 \left[2 \frac{du}{dx} \cos \alpha + \left(\frac{du}{dy} + \frac{dv}{dx} \right) \cos \beta + \left(\frac{du}{dz} + \frac{dv}{dx} \right) \cos \gamma \right],$$

$$Y = hk^2 \left[\left(\frac{dv}{dx} + \frac{du}{dy} \right) \cos \alpha + 2 \frac{dv}{dy} \cos \beta + \left(\frac{dv}{dz} + \frac{dw}{dy} \right) \cos \gamma \right],$$

$$Z = hk^2 \left[\left(\frac{dw}{dx} + \frac{du}{dz} \right) \cos \alpha + \left(\frac{dw}{dy} + \frac{dv}{dz} \right) \cos \beta + 2 \frac{dw}{dz} \cos \gamma \right],$$

where u, v, w are components of velocity parallel to the coordinate axes, and where k is a coefficient depending on friction and viscosity. If no viscosity and no friction exists we must have $k=0$, and hence also

$$X=0, Y=0, Z=0.$$

Now as X, Y , and Z are the effective components with which the nearly spherical mass of fluid acts at its surface when each of them is separately equal to zero, it follows that the fluid can do no work at the surface, and the motions of the shell would take place quite independently of the contained mass of fluid when the latter is totally devoid of friction and viscosity.

(3) It has long since been clearly shown that the motion of the axis of the earth, considered as a solid body, may be determined by the differential equations

$$\frac{d\psi}{dt} = - \frac{1}{Cn \sin \theta} \frac{dV}{d\theta},$$

$$\frac{d\theta}{dt} = \frac{1}{Cn \sin \theta} \frac{dV}{d\psi}.$$

V is the potential of the rotating solid, C its maximum moment of inertia, θ and ψ direction-angles of the axis of rotation. In the case of the Earth θ has a particular value when it becomes the obliquity of the ecliptic, and ψ the longitude of the first point of Aries. It follows that the determination of ψ and θ at any time depends upon C and V .

By analytical transformations, which are fully given by Poisson in his memoir, *Sur la rotation de la Terre autour de son centre de Gravité*, and other writers, it finally appears that the variations of θ and ψ depend on equations in which a factor enters of the form

$$\frac{2C-A-B}{C},$$

where A, B, C are the three principal moments of inertia of the earth. In a spheroid of revolution $A=B$, and the factor becomes $\frac{2(C-A)}{C}$. As precession depends essentially on the variation of the angle ψ , it follows that the complete expression of the factor $\frac{C-A}{C}$ is of primary importance.

(4) Mathematicians, during the past two centuries, have devoted much attention to the question of the figure of a rotating mass of fluid, with especial reference to the explanation of the spheroidal figures of the Earth and her sister planets. Solutions of this problem have been presented, especially by Clairaut, Legendre, Laplace, Gauss, Ivory, Jacobi, and Airy; and it is not a little remarkable that in applying these solutions to the case of the Earth every one of these investigators has not only supposed the Earth to have been originally in a fluid state, but that the particles of the mass retained the same positions after solidification had taken place. This tacit or openly expressed assumption of the unchangeable position of the particles of the original fluid mass on their passage to a complete or partial state of solidity lies at the root of the whole question of the Earth's structure. For the first time in the treatment of the physico-mathematical problem, I distinctly discarded this assumption, and I affirmed that the position of the particles of matter, on passing from the state of fluidity to solidity, must assume positions in conformity with mechanical and physical laws. In this way the hypothesis of the Earth's primitive fluidity became more simple and much more rational; for it was as manifestly absurd to assume that the particles of the fluid mass, on passing into a solid state of consistence, retained their original positions, as it would be to assume that if the whole Earth became liquified the positions of its particles would be unchanged. The corrected and simplified hypothesis is also fruitful in important results; but it is singular that, as far as I am aware, no mathematician seems to have understood or appreciated its bearing on the physical structure of the Earth, except M. Plana, by a remark in a memoir published by him towards the close of his career.

(5) Before presenting my conclusions on the shape of the inner surface of the solidified shell and Plana's remark relative to the same subject, it is necessary to recall some results established by Clairaut, and frequently put forward by mathematical investigators of the Earth's figure. It seems to be universally admitted that if a mass of heterogeneous fluid composed of strata of equal density, each increasing in density from the surface of the mass to its centre, is set in rotation, the several strata will be spheroidal, but their ellipticities will not be equal. The ellipticities will decrease from the outer surface towards the centre. This law of decrease of ellipticity towards the centre is not a hypothetical result, but a necessary deduction from the properties of fluids. As all known fluids are compressible, such an arrangement of strata of equal density as that referred to must follow from the supposition of the existence of any mass of fluid of such magnitude as the whole Earth. The increase of the Earth's density from its surface to its centre is, moreover, a fact clearly revealed by the mean density of the Earth being double that of the materials composing the outside of its solid shell.

If the increase of density, in going from the surface to the centre of a large mass of fluid, is due to compression exercised by the outer upon the inner strata, it follows that the greater the total quantity of fluid the greater will be the difference between the density at its surface and its centre, and the less the quantity of fluid the less will be this difference. With a small spheroid of compressible fluid, the variation of density might be neglected, and the mass regarded as homogeneous. Suppose such a small mass of fluid to be set in rotation, its surface will become spheroidal, and it will have the well-known ellipticity $\frac{5}{4} m$, where m is the ratio of centrifugal force to gravity at the equator of the spheroid. If, now, this original spheroid be supposed to be overlaid with masses of the fluid, one after another, the inner portions will be sensibly compressed, and the whole mass will begin to vary in density in going from centre to surface. The outer surface will now present an ellipticity less than $\frac{5}{4} m$. If fresh layers of fluid are continually applied to the outer surface, the variation of density will continue, and the difference between the density at the centre and surface will increase. The ellipticity of the outer stratum of fluid will at the same time diminish to a value corresponding to the law of density. Let us now reverse this operation, and suppose a great mass of liquid in rotation, its outer stratum will be less dense than those beneath, and its greatest density must be at the centre.

Let the outer strata of equal density be successively removed, so as to leave a succession of free fluid surfaces, until a spheroid is reached in which the difference of density is insensible. It is manifest that, with each successive removal of the upper stratum of liquid, the compression in the remaining strata becomes reduced, and also the variation in density from surface to centre, until this variation becomes altogether extinguished. With the same velocity of rotation, the ellipticities of the surfaces of liquid thus successively exposed would increase up to the limiting value, $\frac{5}{4} m$.

If at any time of the Earth's solidification we suppose a nucleus of fluid to be enclosed within the solid shell, the successive increasing of thickness of the shell, from the congelation of the fluid matter of the nucleus, must be accompanied by the removal of successive outer strata from the nucleus. From what has been seen already, the nucleus will tend to acquire an increase of ellipticity, and therefore to mould the semifluid pasty matter about to pass into a solid state into a shape different from what it would have if no change whatever in the positions of the particles had taken place. As the nucleus is supposed to be in a state of fusion from heat, the successive additions to the inner surface of the shell from the matter of the nucleus must proceed at a very slow rate. The congelation of the surface-stratum of the nucleus must be a process of the same order of slowness as the flow of heat through the shell; and the mathematical theory of conduction established by Fourier shows that this cannot proceed otherwise than slowly. The changes in shape of the surface of the nucleus would be correspondingly slow and gradual. When once a comparatively rigid outer crust had been formed, the process of moulding additional strata of solidified matter against the inner surface of the crust from the nucleus would proceed in a slow and gradual order, so that the resulting solid strata would conform to the shape impressed upon them by the moulding forces. A remarkable illustration of the way in which fused matter ejected from the Earth's interior may, while turning on its centre and at the same time cooling, mould itself against a solid crust formed upon it has been adduced by Charles Darwin, and has been already quoted by me on a former occasion. From these considerations I have been led to conclude, *that the ellipticity of the shell's inner surface may exceed, but cannot be less than, the ellipticity of its outer surface**; and, referring to the same question, Plana used the words, "*La loi des ellipticités a subi dans le passage de l'état*

* See the subjoined representation of a section of the shell and nucleus.

liquide à l'état solide une alteration sensible par laquelle toutes les couches se sont constituées de manière à avoir *un même* applatissement et plus grand que le précédent." M. Plana has further stated his views in the same volume of the *Astronomische Nachrichten* for 1852, thus:—"Il est permis de penser que ces couches (de la fluide intérieure) en se consolidant, ont subi des modifications à la vérité fort petites, mais assez grandes pour nous empêcher de pouvoir dériver, avec tout l'exactitude que l'on pourrait souhaiter, l'état de la Terre solide de son état antérieure de fluidité."

This paragraph gives a distinct adhesion to the improved form of the hypothesis of the original fluidity of the Earth; and this concurrence on the part of M. Plana is the more important, as it is possible that he had formed his conclusions independently. He refers to a letter written by him on the subject to Humboldt; and it is remarkable that, in the fifth and last volume of 'Cosmos,' published not long before the author's death, some adjacent notes allude to Plana's views, and contain references to the investigations of Mr. Hopkins and to my early researches. At this period Humboldt could scarcely have had time to examine the mechanical and physical reasonings, and he merely quoted the papers in the 'Philosophical Transactions' as if he had seen them for the first time. I am not aware of any evidence as to whether Plana had known their contents; and it is possible that his conclusions as to the forms of the strata of the shell and nucleus had been formed independently, though published a short time after my investigations.

The annexed figure may assist in making clear the results of the preceding paragraphs. The outer ellipse represents the outline of the exterior surface of the Earth's crust, which is shaded and bounded inwardly by a surface slightly more elliptical. The fluid nucleus included within the shell is represented with strata decreasing in ellipticity towards the centre. This arrangement is necessarily followed by a mass of fluid under such conditions

as the nucleus, or under the conditions of the entirely fluid Earth. If the matter composing the Earth underwent no change in passing from the fluid to the solid state, instead of the arrangement here represented, the inner surface of the

shell would have a smaller ellipticity than its outer surface, and the strata of the shell, as well as those of the nucleus, would be less oblate in going from the outer surface.

(6) It is important to distinctly bear in mind that the constitution of the shell and nucleus indicated by the foregoing reasonings is not based on any hypothesis of a specific law of density of the interior strata of the Earth. It is a deduction from the established properties of fluids quite as rigorous as the conclusions regarding the spheroidal shape of a mass of rotating liquid. On the other hand, the supposition tacitly or openly made by Mr. Hopkins and his followers, that the ellipticity of the inner stratum of the solid shell is precisely the same as that which this stratum had when fluid, is not merely a hypothesis—it is an assumption which is directly contradicted by the recognized physical properties of all known liquids, and even contradicted by the fundamental principles of hydrodynamics. Upon this assumption was based the calculation of the ratios of the inner and outer ellipticities of the shell which would correspond to the observed value of the precession of the Earth's axis, and hence the limiting value of the thickness of the shell. But when the fundamental assumption on which this ratio is calculated is shown to be in contradiction to physical and mechanical laws, the whole of the conclusions drawn from such a calculation must fall to the ground.

In the *Mécanique Celeste*, Laplace, following Clairaut, proved that if the density in a fluid spheroid decreases from the centre to the surface, the ellipticity of the strata of equal density must decrease from the surface towards the centre. This result forms the groundwork of some of the arguments employed in the present inquiry. Legendre and Laplace also deduced a law of density from the properties of compressible fluids, and from this law the latter unfolded a law of ellipticity of the strata of equal density. The results arrived at in my present inquiry are manifestly totally independent of the law of density $\rho = \frac{A \sin qa}{a}$, deduced by Legendre and Laplace.

In order to apply this law to the strata of the solidified shell, the assumption must necessarily be made that the particles of the fluid underwent no change in position on passing to the solid state. This was assumed by Mr. Hopkins and Archdeacon Pratt; and, as we have seen, such an assumption is not only unwarranted, but is absolutely contradicted by the established laws of hydrodynamics. My conclusions are not only in harmony with those laws, but necessarily require them to be kept constantly in view throughout the whole investigation.

(7) The result obtained in section (3) allows of an immediate and easy application to the inquiry before us, if we admit that the strata of equal density in the shell have all equal ellipticities—an admission which has been already shown to be a particular case of a rigorous and exact deduction from hydrodynamical principles. In this case let us consider the ratio of the difference of the moments of inertia of any spheroidal stratum to its greatest moment of inertia. It will readily appear that the difference of the greatest and least moments of inertia, of all the strata divided by the sum of the greatest moments of inertia will be the same as that for a homogeneous shell whose inner and outer ellipticities are equal.

If ρ be the density of any spheroidal stratum of equal density, then for that stratum

$$\frac{C_1 - A_1}{C_1} = \frac{\int \rho(x^2 + y^2) dx dy dz - \int \rho(x^2 + y^2) dx dy dz}{\int \rho(x^2 + y^2) dx dy dz},$$

and as ρ may be placed outside the sign of integration, it disappears both from numerator and denominator. As we shall presently see,

$$\frac{C_1 - A_1}{C_1} = \frac{1}{2} \left(1 - \frac{b_1^2}{a_1^2} \right),$$

where b_1 and a_1 are the semi-axes of the stratum; and for all other strata of equal density we would have

$$\frac{C_2 - A_2}{C_2} = \frac{1}{2} \left(1 - \frac{b_2^2}{a_2^2} \right),$$

$$\frac{C_3 - A_3}{C_3} = \frac{1}{2} \left(1 - \frac{b_3^2}{a_3^2} \right), \dots \frac{C_n - A_n}{C_n} = \frac{1}{2} \left(1 - \frac{b_n^2}{a_n^2} \right).$$

Now if these strata are all similar, and have equal ellipticities,

$$\frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \dots = \frac{b_n}{a_n};$$

and hence

$$\frac{C_1 - A_1}{C_1} = \frac{C_2 - A_2}{C_2} = \frac{C_3 - A_3}{C_3} = \dots = \frac{C_n - A_n}{C_n} = \frac{1}{2} \left(1 - \frac{b^2}{a^2} \right),$$

where b and a are the outer semi-axes of the shell composed of all the strata of equal density. But

$$\frac{1}{2} \left(1 - \frac{b^2}{a^2} \right) = \frac{C - A}{C} = \frac{C_1 + C_2 + \dots + C_n - (A_1 + A_2 + \dots + A_n)}{C_1 + C_2 + \dots + C_n}.$$

This is the symbolical form of the proposition just stated.

In a homogeneous solid of revolution the general expression for the moment of inertia is

$$\pi \int y^2 x dx;$$

and from the ordinary treatises on Mechanics it readily appears that for a spheroid,

$$C = \frac{8}{15}\pi a^2 b, \quad A = B = \frac{4}{15}\pi a^2 b(a^2 + b^2),$$

where b is the semipolar and a the semiequatorial axis. Hence we have

$$\begin{aligned} \frac{C-A}{C} &= \frac{2a^4b - a^4b - a^2b^3}{2a^4b} = \frac{a^4b - a^2b^3}{2a^4b} = \frac{(a^2 - b^2)}{2a^4b} a^2b \\ &= \frac{a^2 - b^2}{2a^2} = \frac{1}{2} \left(1 - \frac{b^2}{a^2} \right), \end{aligned}$$

and

$$\frac{2(C-A)}{C} = \left(1 - \frac{b^2}{a^2} \right).$$

In a spheroidal shell for whose inner surface the semiaxes are b_1 and a_1 , we have the moments of inertia with respect to the axes by taking the moments for the inner spheroid bounded by b_1 and a_1 from those of the outer spheroid.

Calling the former C_1 and A_1 , we have, as before,

$$C_1 = \frac{8}{15}\pi a_1^4 b_1, \quad A_1 = \frac{4}{15}\pi a_1^2 b_1(a_1^2 + b_1^2).$$

Calling C_1 and A_1 the moments of inertia of the shell, we have, therefore,

$$C_1 = \frac{8}{15}\pi(a^4b - a_1^4b_1), \quad A_1 = \frac{4}{15}\pi[a^2b(a^2 + b^2) - a_1^2b_1(a_1^2 + b_1^2)];$$

and hence

$$\begin{aligned} \frac{C_1 - A_1}{C_1} &= \frac{a^2b(a^2 - b^2) - a_1^2b_1(a_1^2 - b_1^2)}{2(a^4b - a_1^4b_1)} \\ &= \frac{a^4b \left(1 - \frac{b^2}{a^2} \right) - a_1^4b_1 \left(1 - \frac{b_1^2}{a_1^2} \right)}{2(a^4b - a_1^4b_1)}. \end{aligned}$$

If e and e_1 be the outer and inner ellipticities of the shell,

$$e = 1 - \frac{b}{a}, \quad e_1 = 1 - \frac{b_1}{a_1},$$

and if $e = e_1$,

$$\frac{b}{a} = \frac{b_1}{a_1}.$$

In this case

$$\frac{C_1 - A_1}{C_1} = \frac{(a^4b - a_1^4b_1) \left(1 - \frac{b^2}{a^2} \right)}{2(a^4b - a_1^4b_1)} = \frac{1}{2} \left(1 - \frac{b^2}{a^2} \right),$$

or

$$\frac{C_1 - A_1}{C_1} = \frac{C - A}{C}.$$

Consequently the precessional motion of such a shell would be the same as that of a homogeneous spheroid of the same ellipticity. If $e = \frac{1}{300}$, it appears that the value of precession for such a spheroid would be $57''$, while its observed value is $50''.1^*$. Now, as it is impossible to admit such a difference where the result of observation is so well established, we must conclude that the solid shell of the Earth, composed of nearly equielliptic strata, cannot extend to its centre—in other words, that the Earth cannot be altogether a solid from its surface to its centre. On the other hand, the fluid nucleus contained within the shell cannot be devoid of friction and viscosity, but must possess these properties in common with all fluids that have ever been observed on the Earth's surface. These properties of the liquid may, as I have long since announced, cause the shell and fluid nucleus to rotate together as one solid mass. The same conclusion has been afterwards put forward by M. Delauney; and experiments made under his direction, and afterwards, at the instance of the Royal Irish Academy, by me, show that, in rotating glass vessels filled with water, the amount of friction and viscosity is such as to render any difference of slow motion between the liquid and its containing vessel insensible. With liquids so viscid that water is in comparison limpid, such as pitch, honey, and especially volcanic lava in a fused state, the results would be absolutely decisive. To this class of liquids the fluid matter of the Earth's interior, so far as it has come under observation, undoubtedly belongs; and hence the overwhelming certainty of our general conclusions as to the connexion between the Earth's structure and its rotation.

(8) If the tendency of the solid crust is to become more elliptical at its inner surface as it increases in thickness, some interesting consequences appear to follow. If the shell were unaccompanied by the nucleus, or if no friction existed at their surfaces, the changes in the relations of the principal moments of inertia of the shell might be supposed to cause

* A revision of the numerical data from recent astronomical results leads me to conclude that the precession for the solid spheroid would be a little less and about $55''$ instead of $57''$. This I propose to prove in a short paper, entitled "Note on the Annual Precession calculated on the hypothesis of the Earth's Solidity." This note leaves the general conclusions of the present paper unaltered.

its rotation to become unstable, so as to bring about conditions which might result in a change of the axis of rotation. It is easy to show, on the most favourable suppositions, that this could not occur. The increasing ellipticity of the inner surface of the shell would be due to the increasing oblateness of the surface of the fluid nucleus, and this would be at its maximum if the nucleus approached a state of homogeneity. But the fluid cannot approach this state unless the radius of the nucleus is so small that the variation in density due to pressure becomes insensible, whence all its strata would possess the same density. This condition, with a certain thickness of the solid shell, may bring about equality in the two principal moments of inertia of the shell. The most favourable case would be for a homogeneous shell; hence we have only to solve the very simple problem:—Given the thickness of a homogeneous spheroidal shell at its pole, required its thickness at the equator so as to make its principal moments of inertia equal. We have from the expressions for C_1 and A_1 in (7),

$$a^2b(a^2-b^2)=a_1^2b_1(a_1^2-b_1^2),$$

or

$$a_1^4-a_1^2b_1^2=\frac{a^2b(a^2-b^2)}{b_1},$$

which gives

$$a_1=\frac{1}{\sqrt{2}}\sqrt{b_1^2+\sqrt{\frac{4a^2b(a^2-b^2)}{b_1}}}+b_1^4.$$

This may be written

$$\frac{a_1}{b_1}=\frac{1}{\sqrt{2}}\sqrt{1+\sqrt{\frac{4a^2b(a^2-b^2)}{b_1^5}}}+1.$$

If we take $e=\frac{1}{293}$ for the outer ellipticity of the shell, and

$e_1=\frac{1}{230}$ for its maximum inner ellipticity, we can easily find

the values of $\frac{a}{b}$ and $\frac{a_1}{b_1}$; from whence it appears that in order to have equal moments of inertia the thickness of the shell should be .047 of its equatorial semiaxis, and the mean radius of the nucleus would thus be reduced from the original value when the whole mass was fluid by a fraction less than one twentieth. Under these conditions the ellipticity of $\frac{1}{230}$, corresponding to homogeneity, could not exist; and hence it may be concluded that, whether the shell is thin, or whether the

Earth has become almost altogether solid, the moment of inertia of the shell with respect to its polar axis must be always greater than the moment of inertia for its equatorial axis.

The tendency of the fluid nucleus to increase in ellipticity might produce a result worthy of examination by volcanologists, namely, a possible increase in the development of volcanic phenomena in equatorial as compared to polar regions with the progressive solidification of the Earth up to a certain point. Until the thickness of the shell has become very great, recent periods should exhibit a greater development of volcanic energy towards the equator than towards the poles, as compared to remote epochs.

XXIX. *On the Magnetic Torsion of Iron and Nickel Wires.*

By SHELFORD BIDWELL, *M.A., F.R.S.**

IN a paper published in the *Phil. Mag.* for July 1886, p. 50, Prof. G. Wiedemann refers to his well-known experimental researches into the relations between the torsion and magnetization of iron wires. Let a longitudinally magnetized wire NS, be fixed at the south end S, the other end N being free, and let a battery current be passed through it from S to N, then the free end will be observed to twist in the direction of the hands of a clock as seen from the fixed end.

Maxwell † explains the phenomenon thus. The wire being magnetized both circularly and longitudinally, the resultant magnetization is in the direction of a right-handed screw round the wire. Now Joule found that an iron bar when longitudinally magnetized was increased in length, its transverse dimensions being at the same time contracted. We should expect therefore that a spirally magnetized wire would expand in the direction of the magnetization and contract in directions at right angles to the magnetization. And thus the twisting would be produced.

This explanation assumes that the torsional effect is simply secondary to and dependent upon the phenomena observed by Joule, and Prof. Wiedemann, for reasons given in his paper, appears not to be satisfied with it.

Some additional light may perhaps be thrown on the subject by a few experiments which I made at the beginning of the present year. They were intended to be merely of a

* Communicated by the Author.

† 'Electricity,' ii. § 448.