MINOR CONTRIBUTIONS.

THE APPLICATION OF THE INTERFEROMETER TO THE MEASUREMENT OF SMALL ANGULAR DEFLECTIONS OF A SUSPENDED SYSTEM.

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IN an interesting paper in the last number of the PHYSICAL REVIEW, "On the development of the Idiostatic Electrometer," Professor Barus has described an application of the Michelson "interferometer" to the measurement of the small *linear* movements of the electrometer disk, and has suggested the use of the same instrument in the measurement of the *angular deflections* of an ordinary suspended system. If such an application is practicable, it is at once evident what a field of usefulness is open to the interferometer in the measurement of minute deflections, which our present means of reading do not enable us to measure with any degree of accuracy. It may therefore not be uninteresting in this connection to state that this application has already been made, and its practicability under proper experimental conditions demonstrated. The difficulties encountered, however, particularly by those who should attempt to use the instrument for the first time, are even greater than those indicated by Professor Barus. Indeed I have found that many find difficulty at first in finding and adjusting the fringes at all, even when working with the most stable form of instrument with all the mirrors fixed. These difficulties are rendered greater than they need be by the fact that, although the theory of the instrument has been fully discussed and explained by both Professor Michelson and Professor Morley,¹ and the general types and special forms for both linear, angular, and spectroscopic measurements described by Michelson in a number of papers,² there has not been pub-

¹ See paper, Interference Phenomena in a new form of Refractometer, A. A. Michelson, Phil. Mag., Vol. 46, p. 395, 1882; Théorie de L'Appareil D'Interférence, A. A. Michelson, Trav. et Mem. du Bureau internat. des Poids et Mesures, Annexe II., Tome XI., p. 115; On the Measurement of the Expansion of Metals by the Interferential Method, Part I., Optical Principles on which the Method depends, E. W. Morley, PHYSICAL REVIEW, Vol. IV., p. 3. Also joint papers by Michelson and Morley, On the Feasibility of making the Light Wave the Ultimate Standard of Length, Amer. Jour. Science, Vol. 38, Sept., 1889, and Journal of Association of Eng. Soc., May, 1888.

² Measurements by Light Waves, Amer. Jour. Science, Vol. 39, p. 115; Application of Interference Measurements to Astronomical Measurements, Phil. Mag., Vol. 30,

lished, so far as I am aware, any description of those details of construction and manipulation which seem comparatively unimportant to those rendered thoroughly familiar with the instrument by long use, but which nevertheless are likely to occasion difficulties to those taking up the construction or the use of one of these instruments for the first time, particularly when, as in this case, they are not commercially obtainable in standard forms. For this reason I take this occasion to go over the general construction and details of adjustment common to all forms of interferometer before describing the special forms of instrument with which the experiments on the measurement of angular deflections of a suspended system were made.

In his paper "Measurements by Light Waves" in the *American Journal*, Michelson divides interferometers into five general groups, the distinction between which in some cases does not seem very clear. It has always seemed to the writer that a division into two general classes, according to the character of the source of light with which the instrument is to be used, would be better and more comprehensive. On this basis the first class would include all those forms of interference apparatus in which the source is necessarily of very small angular magnitude; types of these being Fresnel's and Lloyd's mirror systems, Billet's bi-lenses, and Michelson's "telescope refractometers."

The second, and by far the more important class, comprises those instruments which may be used with a source of any angular magnitude, *i.e.* an extended luminous surface. In this class, of which the earliest example was Jamin's refractometer,¹ there is necessarily a plane of separation, generally an optical surface of some transparent material, at which the incident light is divided into two beams, one reflected, the other transmitted; which pass away from the surface in different directions, and are afterwards brought together again by any desired combination of reflecting surfaces or lenses. If the transmitted ray is allowed to emerge from a second surface parallel to the first (surface of separation), it is evident that we may, by simply changing the inclination of the latter to the incident light, obtain any angular divergence of the two separated beams from 0° to 180° . Figs. 1, 2, 3, and 4 show four forms in which the separation β of the two rays is respectively 45° , 90° , 135° , and 180° .

In these figures the single heavy arrow indicates the beam of incident light, the two thin arrows the reunited rays which produce interference. The shaded plates are mirrors (silvered on front surface), the unshaded

p. 1; Application of Interference Measurements to Spectroscopic Measurements, Part II., *Phil. Mag.*, Vol. 34, p. 280; and *Smithsonian Contributions to Knowledge*, Application of Light Waves to Metrology, *Nature*, Nov. 16, 1893.

¹ *Ann. de Chem. et de Phys.*, 3d Ser., Vol. 52, p. 163, 1858.

plates are plane parallel glasses, the heavy shaded side of the one marked A being the side at which the separation of the two rays takes place. In Figs. 1 and 2, the two rays are reunited at this same surface; in Fig. 3, at a second surface, A' .¹ In the former case a compensator C is required, in the latter none is needed.²

Fig. 1.

Fig. 2.

It is evident that an infinite number of individual forms may be produced by changing the angle β and the subsequent arrangement of the mirrors which bring about reunion. None of these, however, differs in any essential particular either as regards the method of mounting and adjusting the mirrors, or the character of the interference phenomena produced. As

Fig. 3.

Fig. 4.

Fig. 5.

¹ In this case the function of the surfaces A and A' may of course be interchanged; *i.e.* A' may be the surface of separation, A that of reunion.

² The exact correspondence between this case and any of the others shown may be easily seen by projecting the path of the reflected ray on the path of the transmitted one by cutting the paper along the dotted lines (the prolongation of the intersection of the separating surface A with the plane in which the angle β is measured) and folding the parts to the right over upon the parts to the left. This process also makes clear the part played by the compensator C in making the two paths optically the same.

will be most readily seen from Figs. 4 and 5, this is simply that produced by reflection when the light falls upon a thin air film whose thickness at the center is a, b , and whose sides are inclined to each other at an angle θ . When the angle θ is 0, as in Fig. 4, the interference fringes are circles whose diameters are determined by the distance a, b (the thickness of the film) ; when θ is not zero, as in Fig. 5, the fringes are hyperbolas or parabolas which are symmetrically arranged on each side of a single straight fringe lying at the line of intersection p of the two sides of the film.¹

In all the instruments which have so far been used for either angular or linear measurements, the general arrangement shown in Fig. 2, for which $\beta = 90^\circ$, has been adopted either in the exact form there shown or with some simple modification for securing (as in the case of the Paris interferometer), greater economy of construction and greater convenience of manipulation ; or (as in the case of the form adopted for the angular measurements presently to be described) a more symmetrical arrangement of parts about the axis of rotation. This form may therefore be taken as the general type of construction for all instruments of this class.

In the descriptions of the details of setting up and adjusting the interferometer which follow, I shall assume that the reader has made himself familiar with the general theory and method of operation of the instrument from the papers already referred to. For convenience of description I will indicate the four principal optical parts by the following letters (see Fig. 2) :

- A indicates the separating surface.
- C indicates the compensating glass.
- B indicates the fixed mirror of the interferometer. Sometimes called the reference plane.²
- D indicates the movable mirror of the interferometer.

The difference in path between the two rays is in every case twice the thickness of the equivalent air film ab . This will be indicated by the symbol Δ .

In regard to the general design of the instrument the following points must be considered :

1. The bed of the whole instrument should be very heavy and rigid in order to afford an unusually steady and stiff support for the whole mirror system. It should be H or U shaped in section, of a width of at least twice the aperture of the mirrors B and D , of a height at least one and one

¹ For fuller discussion of these phenomena see papers already referred to, particularly that of Michelson's in the *Phil. Mag.* for 1882, and that of Morley's in this journal: Vol. IV. p. 3.

² In Fig. 4, A performs also the function of B .

half times the width, and of a length depending on the range of movement allowed for the movable mirror. The extreme difference of path Δ at which the interference fringes have yet been observed is about 45 cm. (with the light from the green mercury line), and since, as just stated, the difference of path introduced by the movement of D is twice the movement itself, it is not generally necessary to provide for a greater range of motion than 20 cm. The carriage upon which D is carried should be long (three to five times its width), in order to minimize the effect of error in the ways. The best form for the latter is that shown in Fig. 6,¹ which is a cross section through the bed, movable carriage, and mirror. These ways should be as true and smooth as possible, a good way of securing the desired result being to grind them by the method described in a recent paper in the *Journal of the Franklin Institute*.²

The screw used to move the carriage does not usually need to be an especially accurate one, but it should work very smoothly in the nut.

Fig. 6.

Fig. 7.

The nut may for the same reason be a simple square solid nut, as shown, with its lower side sliding against the surface of the cross bar of the bed, and with a pin projecting from its upper surface and entering a slot in the carriage. This slot should be considerably wider than the diameter of the pin, so that when the screw is turned backward a half turn no part of the nut will touch the carriage. At the forward end the screw should be provided with a graduated head, and also some simple form of index for indicating the whole number of turns. It is con-

¹ This is the form adopted for the Paris interferometer. For a discussion of its advantages see paper On the Manufacture of Very Accurate Straight Edges, F. L. O. Wadsworth, Jour. Franklin Inst., July, 1894, and American Machinist, Aug. 23, 1894.

² *Ibid.*

venient to have the screw itself of a comparatively coarse pitch (not finer than 1 mm., and better 2 mm.), and have at the front end a slow motion device, such as a tangent screw or worm gear that may be readily thrown in when desired to give a very slow motion to the carriage.

The head plate at the front of the bed which carries the glasses *A*, *B*, and *C* (see Fig. 7) should be thick and heavy (not less than one and one half to two times the thickness of the glass plates themselves), and, to prevent springing, should be screwed to the bed on the side only.

2. *Mountings of the Mirrors.*—The proper mounting of the mirrors is of the utmost importance, as upon this more than upon anything else depends the successful and convenient manipulation of the instrument. They must be held firmly and at the same time in such a way as to introduce no strain, and so as to allow each one to be independently adjusted in both a vertical and horizontal plane. The most convenient and efficient way which I have

Fig. 8.

Fig. 9.

found of fulfilling these conditions is to support each mirror (if rectangular, as is generally the case) on an L-shaped piece at whose vertex and ends are three inwardly projecting forks (see Figs. 8 and 9), the opening in which is slightly greater than the thickness of the mirror. The whole piece is milled out of the solid metal, and its thickness is about one and one half times the thickness of the mirror, so as to be very stiff and rigid in the direction at right angles to the plane of the mirror. Through each arm of the forks are threaded fine screws, *a*, *b*, *c*, between the points of which the glass is lightly clamped.¹ On one side the glass rests directly against the points of the screws; on the other the points are hollow, as shown in the figure, and little rounded plugs of cork are inserted to allow for any expansion or contraction, and to allow also of a slight adjustment of the glass without moving the screws on both sides.

¹ One great advantage of this form of mounting is that mirrors of different sizes may be held in the same mounting.

In the case of glasses *A* and *D* the supporting piece is screwed down to the head plate and carriage respectively, by means of screws through the lower arm of the L ; but in the case of *C* and *B* it is mounted so as to permit of a more delicate motion of adjustment than can be obtained by means of the screws *a* and *c* alone. This may be and has been done in a number of ways in different instruments, but the plan I have found most reliable and convenient is one similar to that used in the Paris interferometer for the back support of the "intermediate standards," and for the support of the compensating mirror. This is to mount the L piece on a short, stiff steel spring, a convenient form of which is a steel column, one end of which is screwed into the bed plate and the other into a lug on the side of the L near the vertex (see Fig. 8, dotted lines). The required small horizontal adjustment is effected by slightly *twisting* this column about its axis, best by the varying pressure of a very light spring acting on the end of the horizontal arm of the L ; the vertical adjustment by *bending* the column by a similar arrangement acting on the end of the vertical arm.

The most recent arrangement which I have used for this purpose, and which has been found more compact and satisfactory than any heretofore tried, is that shown in Fig. 9. It consists of a light spiral spring, one end attached to a pin in the end of the arm of the L, the other to the outer end of a hollow screw through which the spring passes freely. The thread of the screw should not be finer than 1 mm. pitch,¹ and should work very smoothly in the nut, which should be at least two diameters in length. The strength of the spring should be so adjusted that one revolution of the screw shifts a fringe by not more than twice its own width. The screws should have a run of about 2 cm., corresponding to a total movement of about 40 fringes, or for a mirror 2 cm. wide, to a linear movement of about 0.01 mm. It will be at once seen by those familiar with the inherent defects of even the very best micrometer screws, how very difficult it would be to move the mirror by an amount corresponding to a tenth of a wave (*i.e.* 0.00025 mm.), as is necessary in the final adjustment, by any screw bearing directly upon the mirror or any arm of practicable length attached thereto. Not only do the errors of the screw far exceed this amount, but what is more, the unavoidable longitudinal pressure exerted by the fingers in turning the head, moves the screw in the nut and springs the support of the latter, no matter how rigid we may attempt to make it, by an amount far greater than the desired movement. With this spring arrangement the effect of these sources of irregular motion is reduced in the same proportion that the movement of the screw for a given effect is increased, *i.e.* about 1000 times. To reduce the effect in the same proportion by

¹ Heretofore it has been customary to use screws of much finer pitch bearing against much stiffer flat steel springs

simply lengthening the arm of the L would require that this be made 20 m. long.

In the case of the compensating glass there is no necessity for a delicate adjustment in the vertical plane, and only one spring movement, *i.e.* that acting on the end of the horizontal arm of the support, is needed. In this case we need a movement corresponding to only two or three fringes, but since this movement is used to measure fractions of a wave length, the inequalities in the motion must not exceed a quantity corresponding to $\frac{1}{100}$ wave. The pitch of the screw should therefore be much coarser than before (a double or triple thread or spiral of about 5 mm. pitch is not too coarse), and the spring still finer, so that one complete turn corresponds to a shift of one fringe. This screw should have a large head, graduated to hundredths of a turn (so that one division corresponds approximately to $\frac{1}{100}$ wave length).

Setting up and Adjusting the Instrument.

a. Silvering of Mirrors. — Mirrors *B* and *D* are full silvered on their front surface; *A* is "half silvered" on the surface nearest the compensating glass. Of the different processes of full silvering, the writer prefers above all others that of Brashear, the details concerning which were given in the *Astrophysical Journal* for March, 1895.¹ For "half silvering," *i.e.* depositing a semitransparent coat of such density as to reflect about half the incident light, I have, however, obtained the best results with the ordinary rochelle salts solution prepared as directed by Boettger.² For "half" silvering the solution should be comparatively new, and the glass cleaned with the greatest care in order to obtain a uniform film. The plate should be supported in the liquid by one of the glass "forks" described in the article above referred to, so that it may readily be lifted out of the solution from time to time and the density of the deposit examined. When of about the right thickness it has a peculiar blue color by transmitted light. A few trials will be necessary at first to enable the operator to determine by this means just when the right thickness of film has been obtained. It should then be immediately lifted from the silvering solution, plunged into a large vessel of distilled water and thoroughly rinsed, and then turned on edge on a sheet of blotting paper to dry. Great care must be taken not to touch the silvered surface with the fingers either before or after drying, as it is so thin that it is very readily rubbed off.

b. Putting Mirrors in Place. — First put *D* and *B* into their supports,

¹ "Silvering Solutions and Silvering"; F. L. O. Wadsworth, *Astrophysical Journal*, Vol. I., p. 252.

² Kohlrausch, *Physical Measurements*, p. 146.

and by means of a small steel square adjust them so that their surfaces are at right angles to the side and end of the bed, and the faces of the carriage and head plate, respectively. Then if the mechanical work on the machine has been properly done these two surfaces ought to be very nearly at right angles to each other. Next put in *A* and *C* and adjust *A* until the image of *D* seen by reflection is exactly superposed on *B*; then finally adjust *C* until it is parallel to *A*, either mechanically by the use of a pair of outside calipers (outside calipers are used instead of inside ones to avoid injuring the silver film which is on the side nearest the compensator), or optically by making the two images of a bright distant object seen by reflection from the surfaces of *A* and *C* coincide. It is, perhaps, necessary here to call attention to the fact that the thickness of the two plates *A* and *C* should be the same within a few thousands of a millimeter, and that both ought to be as nearly as possible plane parallel, *i.e.* both surfaces should be optically flat, and as nearly parallel to each other as possible. The best way is to have them both cut from one large piece.¹ If it is necessary, as sometimes happens on account of the loss or breaking of one of the "pair," to use two pieces of slightly different thickness, we may up to a certain point still obtain practically perfect compensation by inclining *C* slightly to *A*, so as to make the length of the path traversed by the rays in the glass equal in the two cases. If *C* is thicker, it is of course turned so as to make the angle of incidence smaller than on *A*; if thinner, so as to make the angle of incidence larger. It is further to be mentioned that these adjustments are only necessary in order to obtain the fringes in white light; with monochromatic light we might dispense with *C* altogether, as is sometimes done.

c. To Find the Fringes.—First measure, by means of a thread or scale, the distances from any point on the edge of the silvered surface of *A* to the surfaces of *B* and *D*, and move the latter forward or backward until these distances are the same to within about 1–4 mm. Then cover the whole instrument with a wooden or pasteboard box (as indicated by dotted lines in Fig. 2), having two holes about 3 cm. square cut in front of the mirror *A*, as shown. It should also have an opening in front through which projects the handle and the slow-motion device for turning the screw, and another opening in the side opposite the mirror *B*, through which the screws which adjust the latter can be reached. This box is not essential if the room is very quiet and free from air currents, but it is always desirable to use it when possible, as it will frequently greatly facilitate the finding of the fringes. Next place a sodium flame at *F*, about 30 or 40 cm. from *A*,

¹ Mr. Brashear has attained such skill in the manufacture of these plane parallel pieces that he can furnish at merely nominal cost a "pair" that will always give satisfaction.

and between the two an ordinary condensing lens which will render the rays falling on *A* approximately parallel. Place the eye at *I*, in front of *A*, looking in the direction of the double arrow toward *D*, and move the flame and condenser until the whole field of the mirrors is uniformly filled with light. Place a needle point just in front of the flame near the focus of the condenser, and with the fine motion screws of *B* at the middle of their run, adjust this mirror with the screws *a* and *c*, until the two images of the point seen by reflection in *A* (one formed by reflection from *D*, the other by reflection from *B*) are in exact coincidence. (Besides the pair of images formed by reflection from the silvered surface of *A*, there will be another pair formed by reflection from the front unsilvered surface. If the glass is exactly plane parallel this second pair will also be in coincidence when the first pair is, but not otherwise. Care, therefore, should always be taken to select the right pair of images. They may be easily distinguished by their greater brilliancy.) This adjustment to coincidence needs to be performed with great care, for unless it is very exact the fringes will not appear. It is, therefore, best to use at first a telescope at *I*, by means of which the images of the point will be considerably magnified, and the coincidence more easily determined. It is also a help to change the position of the needle, holding it with its axis horizontal when adjusting in a vertical plane, by means of the screw *a* (Figs. 8 and 9), and vertical when adjusting in the other plane by means of screw *c*. When coincidence is judged to be exact, remove the telescope (if one has been used) and needle point, and place the eye about 25 or 30 cm. from the mirror *A*. If the fringes do not at once appear (generally they will appear as fine, more or less curved lines), first move the eye back and forth (there being a position of maximum visibility for each particular value of Δ and θ). If this does not bring them into view, move *D* slightly (about 0.2 mm. each time) by means of the screw, first in one direction and then in the other, and examine again after each movement by moving the eye backward and forward as before. After each movement of *D*, reexamine the images of the needle point to see that they are still in coincidence. The fringes will be certain to appear in one of these positions if the preceding directions have been carefully followed.

d. Adjusting the Fringes.—After the fringes have once been seen, it will be found an easy matter, after a little practice, to adjust them to any desired width or position by means of the fine-motion screws on *B*. A little manipulation of these screws and a study of the effect produced is worth more than any number of directions. A word, however, may be said regarding the adjustment for circular fringes, which appear when each of the mirrors is exactly perpendicular to the incident beam of light falling on it: and the finding of the central straight fringe in white light. To

produce circular fringes, the mirror D should be moved back until there is a considerable difference of path between the two rays (1000 to 2000 wave lengths), and the screws moved so as to *increase* the curvature of the fringes. The center of the system will then soon appear, and may, by a further movement of the screws, be brought to the center of the field. Then move the eye to the right and left, and if the diameters of the circles change, move the screw acting on the horizontal arm of the mirror mounting B in such direction that this change grows less and less, and finally nearly disappears. Then move the eye up and down, and if a corresponding change takes place, adjust the other screw until this also disappears. When the fringes remain the same in all parts of the field, the adjustment is perfect.

To find the fringes in white light, the fringes are first made straight and preferably vertical. The screw is then turned so as to move the mirror D towards the zero position. It is easy to determine when this position has been nearly reached by the increasing straightness of the fringes, and also by the fact that there is very little tilting or change of curvature as the eye is moved up and down. The slow-motion screw is then thrown in, the sodium light replaced by a gas flame, and the carriage moved very slowly (at such speed that about 10 fringes cross the field per second) until the colored fringes appear. This position, as determined by the graduated head of the screw, should be noted for convenience in finding these fringes again when desired. Since only some thirty fringes are distinctly visible in white light, the position of the mirror must be right to within less than 0.01 mm. in order to make them appear at all.

e. Observing and Measuring with the Fringes.—The straight fringes obtained when the two mirrors are slightly inclined to each other are in general most conveniently observed by the eye alone, without the aid of the telescope. If a telescope is used it must be focused for the central white fringe on the surface D . For the fringes to the right or left of this, corresponding to a definite difference of path Δ between the two rays, the focus of the telescope changes. This fact, together with the continual change in the curvature, and consequent apparent shifting of the fringes with a change in the position of the eye, makes their use unsuitable in measuring any large differences of path. The circular fringes are free from all these difficulties, for being formed at infinity (see theory of instrument) they are always visible for all values of Δ in a telescope focused for parallel rays; and their position and appearance are likewise independent of the position of the eye. They are therefore always to be used when large differences of path are to be observed. For measuring small movements, however, the straight fringes are more convenient, it being only necessary to count the number which pass a given point in the field (most conven-

iently a line engraved on the surface of one of the mirrors B or D) when the movement takes place. When the displacement is so rapid that the counting is impracticable, the central fringe of the colored system seen in white light must be used to mark the initial and final position, this being the only fringe of the system that can be identified.

It may appear from what has preceded that the interferometer is an instrument of such extreme delicacy that its use in ordinary laboratory measurements is impracticable. Such, however, is not the case if the instrument is properly constructed. Its stability and steadiness is then perfectly surprising, considering the minute movements which it will detect. It is far less difficult to work with, for example, than a delicate galvanometer or electrometer, and in applying the interferometer to the measurement of the deflections of these instruments, the difficulties we meet with are not those due to the interferometer proper, but those due to the unsteadiness of the suspended system of the galvanometer or electrometer. It was for the purpose of determining whether these difficulties could be overcome that the experiments already alluded to were, at Professor Michelson's suggestion, undertaken, in 1891, at Clark University, by Dr. Austin, now of the University of Wisconsin, and the writer.¹

The type of optical system adopted was that first described by Professor Michelson in his paper "Application of Interference Methods to Astronomical Measurements" (*Phil. Mag.*, July, 1890), and shown in Fig. 11

Fig. 10.

Fig. 11.

Fig. 12.

of that paper. Figures 10, 11, and 12 show how this is derived from the type of instrument for linear measurement. It is evident that this latter may be used as well for angular measurements, by mounting the mirror D on the end of the arm whose deflections are to be measured (see Fig. 10). This arrangement, however, has the great disadvantage of altering the inclination of the mirror, and thus changing the width of the fringes at the same time that they are shifted in the field by the linear change in path.² This objection can be at once overcome by the modification shown in Fig. 11,

¹ Unfortunately Dr. Austin left the University before the experiments were completed.

² This same objection applies to the type of instrument first proposed for angular measurement by Michelson in his paper, *Measurement by Light Waves*, *Amer. Journal*, Feb., 1890, Plate III., Fig. 1; also Group III., Figs. 4 and 5.

in which the mirror B is mounted parallel to D , and on the same swinging arm which carries the latter. This is accomplished by introducing an additional mirror B' , placed parallel to A and C . The effect of the angular motion of one mirror is compensated by the equal angular motion of the other, while if the point of suspension is between the two mirrors, as at O , the sensitiveness is doubled by the fact that a movement of the suspended system shortens one path at the same time that it lengthens the other. The only practical objection to the arrangement is that the mirrors B and D are not in the same plane, and are not therefore as easily mounted and managed as they would be if this were the case. The form in Fig. 12 accomplishes this last object by the use of the two additional mirrors, D and D'' .

The complete instrument as designed on these lines is shown in Fig. 12 *a*. It was built partly by Mr. Claflin, of the Claflin Dental Mfg. Co., of Worcester, and partly by Mr. Francis and myself in the machine shop of the university. Some features of construction differ from those since adopted as more desirable (see earlier part of this paper). For example, the fine adjustment of B (in this case applied to B') is accomplished by means of very fine watch screws, bearing directly against the mirror frame, and moved the required minute amount by long steel levers (one of which can be seen in the photograph) inserted in the heads of the pins.¹ In order to adapt the optical part of the instrument to suspended systems of different magnitude, the separating mirror A and mirror D' are mounted on one carriage, D'' on another, and B' on the third. These may be slid along to any part of the long V-shaped bed and clamped in any desired position. The mirror B' is mounted on a short auxiliary slide, and moved parallel to itself a small amount by a screw which takes the place of the screw which moves D in the linear type of instrument. To impart a slow motion to this screw, the nut in which the main screw takes its bearings is itself threaded, the pitch of this thread being very slightly greater than that of the screw. When this nut, therefore, is turned, as it may be when desired by the large milled head seen in the photograph, the screw is moved in or out at each revolution of the latter by an amount equal to the difference in the pitch of the thread on the outside of the nut and that on the inside. The screw itself is kept from revolving when the nut is turned by a friction collar which is not, however, tight enough to prevent its being turned directly by the small head when a rapid motion of B' is desired.

The whole bed may also be rotated about a heavy steel pivot at the center (by means of the very fine screw with large milled head seen at the back of the instrument), in order to bring these parts of the interferometer system parallel to the suspended system. The compensator C is mounted,

¹ The objections to this method have already been stated; see page 486.

Fig. 12 a.

not on the carriage with B' , but just in front of it (between B and B' instead of between B' and A as in the diagrams in Figs. 2 and 12), on a heavy brass rod screwed into the base of the instrument. The spiral spring which twists this rod is attached, the one end to a light lever projecting from it, the other to the graduated sliding bar seen near the front. The strength of spring is so adjusted that a motion of this bar of from 15 to 20 cm. is necessary in order to twist the compensator glass sufficiently to introduce a difference of path of one wave length.

The suspended system consisted of the two small mirrors B , D , which were about 5 mm. square, mounted on a stiff though light arm made of glass fibers about 5 cm. long, and suspended at its center from a quartz fiber about 25 cm. long. It is of course impossible to adjust either of the suspended mirrors during the course of the experiment, and it was therefore necessary to be extremely careful in mounting them originally so that their surfaces lie very nearly in the same plane. This is done by laying them face down upon a piece of optical glass and examining the fringes between this surface and the faces of the mirrors.

The whole suspended system was inclosed in an air-tight chamber which consisted of a small cast-brass box with heavy walls, about 4 cm. square and 6 cm. long. The heavy brass tube¹ which carried the fiber was soldered into the top of the box and the front was closed by a thick plate of optical glass, cemented on with a mixture of ordinary glue and molasses. The top of the brass tube could also be closed by means of a brass cap which could be slipped over the head to which the upper end of the fiber was attached, and cemented in place after the latter had been adjusted. The tube and box were clamped in a heavy cast-iron stand, as shown in the plate, and the two parts of the instrument, the inclosed suspended system mounted as just described, and the optical "head," as it may be called, placed together on a very heavy stone slab, supported on rubber blocks. Blocks of stone, not shown in the plate, were also placed around the feet of the iron stand and under the brass box so as to steady the latter from below, as well as at the point where it was clamped above. It was an easy matter to obtain the fringes when the suspended system was held stationary, but it was only after many trials that they were obtained with a free system. The most convenient method of procedure was finally found to be as follows: The suspended system was first arrested in as nearly as possible its position of rest by means of a wire fork, mounted on a short brass rod passing through a hole in the side of the brass box, as shown in Fig. 13. After the fringes have been obtained in white light by the method already described, the system was gradually released by turning the rod by the long arm, δ ,

¹ A glass tube cemented into the box was first used, but afterward replaced by the metal as far more satisfactory.

outside the box, the bed being rotated by means of the tangent screw already spoken of, so as to follow the suspended system as it swings one way or the other, and keep the fringes in the field.¹ At first, as soon as the system was completely free, it began to swing and tremble so that it was very difficult to follow the fringes. The difficulties due to vibration were finally very considerably reduced, and at times completely overcome, by floating the stone carrying the whole apparatus in mercury, in a box very slightly larger than the stone itself, the stone being kept from touching the box by silk threads.² The observing telescope was also removed from the base of instrument and mounted on a separate stand,³ so that after the preliminary adjustments had been made, nothing on the floating stone itself

Fig. 13.

magnets and consequent loss of sensibility. Probably the best system of damping which would leave the sensibility unchanged would be that indicated by dotted lines in Fig. 13. It consists of two strips of copper, or better, pure silver foil, hung from the ends of the suspended system with the plane of the strips at right angles to the mirrors. These strips swing edgewise between the poles of two powerful horseshoe or electro magnets. The brass box containing the system was not large enough to allow this

b The difficulties due to the small constant swinging of the system to and fro were largely overcome by proper damping. In the preliminary experimental work this was effected by attaching to the suspended system a long light bar magnet and placing strong bar magnets on the stone outside. Such an arrangement would not in general answer because of the directive force of the

¹ Any motion of translation, or of rotation about a horizontal axis, of the system is of course without effect on the fringes.

² What seems to be an even better method of support to prevent vibration has recently been described by Professor Brace. See *Astrophysical Journal*, Vol. V., p. 214, March, 1897.

³ The telescope may of course be placed at any desired distance from the instrument itself.

arrangement to be tried, but I feel no doubt that it would prove effective in holding the system at rest, or so nearly so that there would be no difficulty in noting the position of the fringes as they slowly swing to and fro. A considerable movement of the fringes is no disadvantage, — is in fact even an advantage, provided only the swing is steady and not so large as to throw the central white fringe (which is the only one which can be identified) out of the field. The zero point or position of rest is in such a case determined by the method of oscillations, just as in the case of the ordinary telescope and scale method of reading. To avoid parallax and other errors, the excursions of the central fringe are best noted by means of a scale engraved directly on one of the mirrors *D* or *B*.

These experiments having demonstrated the practicability of this method of reading deflections, I next began the construction of a large D'Arsonval galvanometer, designed with special reference to this new method of observation. Before this was finished, however, I was called upon to devote my whole time and energy to the design and construction of the large Paris interferometer, and immediately after its completion and installation in the laboratory of the International Bureau at Breteuil. I went to Washington to accept a position in the Smithsonian Astrophysical Observatory, where I had no opportunity to continue these experiments. It is only within the last few months, since my transfer to the Yerkes Observatory, that I have had an opportunity to resume the long-deferred completion of the galvanometer, the unfinished parts of which I have fortunately preserved.

It is as yet too soon, perhaps, to predict anything in regard to the degree of sensitiveness which it will be possible to practically attain with this instrument. A preliminary estimate, based on the dimensions of field-magnets and proposed coil and mirror system, would indicate that it would be possible to attain a sensitiveness such that a deflection of 0.1 fringe corresponds to a current of between 10^{-16} and 10^{-17} amperes. If this degree of sensitiveness can be attained, particularly with the D'Arsonval type of instrument, it will permit of certain problems of great theoretical interest being attacked for the first time experimentally.

There is one point of considerable importance in the theory of galvanometers in this connection. It might be urged by some, that this method of reading considerably increased the mass and, more particularly, the moment of inertia of the optical parts of the suspended system. This is true, but the remedy is simply to increase the mass of the rest of the suspended system in the same ratio. This is exactly the reverse of the modern practice of using very light systems to secure great sensitiveness; but as I have elsewhere¹ shown, this increased sensitiveness results not at all from

¹ Description of a Very Sensitive Form of Thomson Galvanometer and some Methods of Galvanometer Construction, *Phil. Mag.*, Vol. 38, p. 553, December, 1894.

the lightness of the system, but from the higher degree of astaticism reached. Given the same degree of astaticism in the heavy system, and it would be the more sensitive of the two, although (and this is the practical difficulty generally attending the use of the heavy system) the time of swing will be much longer. With the interferometer method of reading, however, this slowness of swing is fortunately just what is wanted.

In conclusion it may be of interest to point out certain changes and modifications in the apparatus which have suggested themselves as desirable. In the first place, in making another instrument of this kind I would have all the mirrors larger, particularly the moving ones. In the instrument just described the mirrors *A*, *B'*, *C*, *D'*, and *D''* were each about 10 mm. $\times$ 15 mm. and *B* and *D*, as already stated, about 5 mm. square. It would be better to have the first-named mirrors 5 mm. larger each way and the two last of about the same width as before, but at least twice as long, so as to allow greater amplitude of swing without causing the central fringe to pass out of the field.¹ The tangent screw which moves the bed in azimuth should be finer and act on the end of a longer arm, in order to be able to follow the movement of the mirror more slowly and steadily. It would be better for the same reason to have the steel pin upon which the bed turns, at one end of the latter instead of at the center. Both the bed and the mountings of the mirrors should be heavier (following more nearly the proportions given in the first part of this article) and the movable carriages on which the latter are mounted longer, and arranged to clamp more rigidly to the bed.

In the preceding form of instrument the difference in path Δ , introduced by a given angular movement θ , of the system is $2b\beta$, b being the distance *BD* between the centers of the mirrors. If we consider that the smallest movement that can be measured with certainty in a freely suspended system is one twentieth of a fringe,² the smallest angle that can be measured with the interferometer is

$$\theta_o = \frac{1}{40} \frac{\lambda}{b}.$$

The smallest movement that can be observed with a mirror as ordinarily used is probably not more than one fifteenth its angular resolving power.³

¹ Or what amounts to the same thing, without allowing the point *p*, in Fig. 4, to get beyond the edge of the mirrors.

² With the type of instrument with fixed mirrors (linear measurement) a movement of one two-hundredth of a fringe may be detected under favorable conditions.

³ See paper, "Manufacture of very Accurate Straight Edges," already referred to, p. 6, also review of Boys' determination of the Newtonian Constant of Gravitation, *Astrophysical Journal*, Vol. III., p. 303, April, 1896.

This, as is well known, is

$$\alpha = \frac{\lambda}{b},$$

and the smallest angular movement is therefore

$$\theta_m = \frac{1}{15} \frac{\lambda}{b}.$$

The interferometer system therefore is not so very much more accurate than a mirror of the same horizontal aperture used with the telescope and scale. The principal advantage of the former, as has been pointed out by Michelson, is that with it we need to use only two small pieces at the ends of a diameter instead of the whole mirror.¹ We could also do this in the case of the telescope without loss of resolving power provided we could make these two pieces form parts of one and the same optical surface, and also give sufficient light; but in general neither of these conditions could be fulfilled. When they can be, the interferometer does not offer sufficient increase in accuracy to compensate for the greater difficulties encountered in its use.² It occurred to me, however, during the course of these experiments, that we can further increase the attainable accuracy of the interferometer method by employing the principle of multiple reflection. One very simple and symmetrical arrangement which suggested itself is that shown in Fig. 14, in which, as before, *B* and *D* are the movable mirrors and *F*, *F'*, *F''* are three additional fixed mirrors. Each ray is reflected from the movable mirrors three times, and in such directions that the total difference in path produced by a given movement θ is now $6b\theta$, and the resolving power is therefore, on the same assumption as before,

Fig. 14.

$$\theta_o = \frac{1}{120} \frac{\lambda}{b},$$

or eight times as great as for the mirror. But the greatest advantage of such a system is that we may, for a given resolving power, make the length of the arm supporting the two mirrors one third as great as in the more elementary form, and therefore reduce the amount of inertia to less than one tenth of what it was in the latter.

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March, 1897.

¹ This is an advantage only when the distance *b* is very large; at least fifteen times the height of the mirrors *B* and *D*.

² A fuller discussion of the comparative advantages of the different methods of observing small angular movements will be found in a paper, "On the Conditions which Determine the Ultimate Optical Efficiency of Methods for Observing Small Rotations," which will soon be published in the *Phil. Mag.*

Fig. 12 a.