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II. *On the Electron Theory of Matter and the Explanation of Fine Spectrum Lines and of Gravitation.* By G. A. SCHOTT, B.A., B.Sc., University College of Wales, Aberystwyth*.

§ 1. LORD RAYLEIGH† and Mr. Jeans‡ have recently discussed, in the pages of the Philosophical Magazine, the question of the explanation of the fineness of the spectrum lines. Jeans finds it necessary to postulate forces of unknown origin between electrons in addition to the electric and magnetic forces required by Maxwell's theory of the electromagnetic field, and makes the periods of vibration of systems of electrons depend on the action of these two types of forces; on this view spectrum lines are of dynamical origin. Lord Rayleigh, on the other hand, maintains that the relations between the spectrum lines studied by Balmer, Rydberg, and Kayser and Runge, point to a kinematical origin, but finds difficulty in explaining the existence of fine lines at all. The explanation developed in the present paper overcomes this difficulty, and at the same time leaves us free to adopt a kinematical or dynamical origin for the spectrum series. Moreover it offers some prospect of explaining gravitation on the lines of the hydrodynamic theories of Bjerknes and Pearson.

§ 2. We shall consider the case of n equidistant electrons moving in a circle with uniform velocity. Let the radius of the circle be ρ , the velocity v , the velocity of light C , $\frac{v}{C} = \beta$, the mass of an electron m , its charge e .

Further let T , P be the components, along the tangent and towards the centre, of the mechanical force on one of the electrons. The equations of motion are

$$T = \frac{d(mv)}{dt} + \frac{2e^2\beta^3}{3\rho^2(1-\beta^2)^2},$$

$$P = \frac{mv^2}{\rho}.$$

The second term in the first equation represents the reaction of the æther on the electron due to radiation.

The first term vanishes in steady motion; it is retained here for the sake of generality, so as to include the possibility of m varying while v is constant. This is essential for what follows.

The general equations of motion of an electron, of which those just written constitute a special case, can be obtained

* Communicated by the Author.

† Phil. Mag. ser. 5, xlv. p. 356; ser. 6, xi. p. 123.

‡ Phil. Mag. ser. 6, ii. p. 421; xi. p. 604.

from Maxwell's equations of the electromagnetic field by direct integration. They are identical with those obtained by Abraham * by an indirect process.

The mass m is of electromagnetic origin, at any rate in part. It can be shown by the method of dimensions that the electromagnetic part of the mass is of the form $\frac{e^2}{C^2 a} \psi(\beta)$,

where a is a length, which we shall take to be the mean radius of the electron, and $\psi(\beta)$ depends on β and on the shape of the electron. If a be supposed invariable, as is usually done, m is constant when β is so; but this restriction is unnecessary. The variation allowable in a is very small;

for $\frac{\dot{m}}{m} = -\frac{\dot{a}}{a}$, and m cannot possibly vary at a greater rate than, let us say, $\frac{1}{3}$ per cent. per annum; that is, $\frac{\dot{a}}{a}$ is certainly less, and probably very much less, than 10^{-10} .

§ 3. The force (T, P) consists of two parts:

(1) (T₁, P₁) due to electrons outside the ring;

(2) (T₂, P₂) due to the remaining electrons of the ring.

In the first place, T₁ vanishes for a system of electrons in steady motion.

For T₁ is due partly to a steady electric force, partly to a steady magnetic force. The electric part vanishes, because the lines of force of a steady electric field cannot form closed curves. The magnetic part vanishes, because the mechanical action of a magnetic field on a moving electron is always perpendicular to its velocity.

Secondly, I find for (T₂, P₂) the expressions

$$T_2 = \frac{2e^2\beta^3}{3\rho^2(1-\beta^2)^2} - \frac{e^2}{\rho^2} nU,$$

$$P_2 = -\frac{e^2}{\rho^2}(1+\beta^2)K + \frac{e^2}{\rho^2} V,$$

where

$$K = \sum_{i=1}^{i=n-1} \frac{1}{4} \operatorname{cosec} \frac{\pi i}{n},$$

$$U = 2 \sum_{s=1}^{s=\infty} [sn\beta^2 J'_{2sn}(2sn\beta) - s^2 n^2 (1-\beta^2) \int_0^\beta J_{2sn}(2snx) dx],$$

$$V = \sum_{s=0}^{s=\infty} (s + \frac{1}{2}) \cot \frac{(2s+1)\pi}{2n} [\beta(1-\beta^2) J_{2s+1}\{(2s+1)\beta\} \\ + (1+\beta^2) \int_0^\beta J_{2s+1}\{(2s+1)x\} dx].$$

* *Theorie der Elektrizität*, vol. ii. p. 123 (85).

The notation is the usual notation of Bessel functions.

The function U is of special importance for our purpose. The series converges when $\beta < 1$, very rapidly for large values of n and small values of β . In this case it is sufficient to use the first term only. Whatever the value of β , U is positive. The equations of motion become

$$nU = -\frac{\rho^2}{e^2} \frac{d(mv)}{dt} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$P_1 - \frac{e^2}{\rho^2} (1 + \beta^2) K + \frac{e^2}{\rho^2} V = \frac{mv^2}{\rho} \quad . \quad . \quad . \quad (2)$$

§ 4. Equation (1) shows that a strictly steady motion is impossible when we restrict ourselves to the case of an invariable electron and exclude all forces not of electromagnetic origin. For given values of n , β , ρ it determines $\dot{\beta}$. Equation (2) determines ρ when β is given, but β itself remains arbitrary. This is the difficulty first pointed out by Jeans (§ 1). There is nothing to fix the constitution of the atom, if it be supposed built up of invariable electrons in orbital motion, with only electromagnetic forces between them.

The difficulty can be overcome by supposing the electron to vary slowly in radius.

In this case,

$$\frac{dm}{dt} = -m \frac{\dot{a}}{a} = -\frac{e^2}{C^2} \psi(\beta) \frac{\dot{a}}{a^2} \quad (\S 2)$$

Equation (1) becomes

$$\frac{nU}{\beta} = \frac{\rho^2 \psi(\beta)}{C} \frac{\dot{a}}{a^2} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This equation together with (2) determines β and ρ when n and $\frac{\dot{a}}{a^2}$ are given. Since U is essentially positive $\dot{a}$ must be positive.

We easily find on examination that the right-hand member is of order 10^{-24} at most for the negative electron, if $\frac{\dot{a}}{a} = 10^{-10}$; probably it is very much less. The factor $\rho^2 \psi(\beta)$ depends on β , but does not vary very rapidly with it; the function U on the other hand, for fairly large values of n , varies very rapidly with β ; in consequence an approximate knowledge of $\rho^2 \psi(\beta)$ is quite sufficient to allow us to calculate β accurately when n and $\frac{\dot{a}}{a^2}$ are given. When β is known ρ is calculated by (2).

The following table gives approximate values of β calculated for $\frac{\dot{a}}{a} = 10^{-16}$ and $\frac{\dot{a}}{a} = 10^{-46}$.

$n \dots$		10	20	50	100	1000
β	$\frac{\dot{a}}{a} = 10^{-16}$	·023	·131	·398	·566	·894
	$\frac{\dot{a}}{a} = 10^{-46}$	·00074	·023	·187	·398	·833

It is noteworthy how little difference even the order of $\frac{a}{\dot{a}}$ makes in the value of β when n is at all large.

§ 5. We shall now consider in what way we must modify the common view as to the constitution of the electron and of the æther, so as to allow of the expansion of the electron, which we have found necessary to account for the definiteness in the structure of the atom on the electron theory.

When the electron is invariable the energy equation may be written in the form,

$$v \cdot T = \frac{dE}{dt} + R, \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where vT is the rate of working of the mechanical force acting on the electron, E is a function of its velocity, mass, and acceleration, and R is the rate at which the electron loses energy by radiation. E includes the electromagnetic energy of the electron; its characteristic is that dE is a perfect differential while $R dt$ is not.

When the electron varies in radius (4) is no longer true, but must be completed by the addition of a term to the right-hand member of the form $\frac{e^2 \dot{a}}{a^2} f(\beta)$. This additional term

represents the rate at which work is done by the internal electromotive forces of the electron on account of the expansion. The form of $f(\beta)$, like that of $\psi(\beta)$ in § 2, depends on the assumptions made as to the structure of the electron.

We must assume the existence of internal stresses in the electron which resist its expansion.

In calculating the internal electromotive force we find that the principal term is derived from a potential Φ , the "convection potential" of Searle and Lorentz; the value of Φ is the same as it would be if the electron were moving

uniformly and without relative motion of its parts with the velocity it actually possesses. The remaining terms in the electromotive force, due to acceleration and to relative motion, are only of order $\frac{a}{\rho}$ relative to the principal term, and may be neglected.

Let (P, Q, R, S, T, U) be the stress which balances the internal electromotive force. It is given by the equations :

$$\begin{aligned}\frac{\partial P}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial T}{\partial z} &= \rho' \frac{\partial \Phi}{\partial x}, \\ \frac{\partial U}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial S}{\partial z} &= \rho' \frac{\partial \Phi}{\partial y}, \\ \frac{\partial T}{\partial x} + \frac{\partial S}{\partial y} + \frac{\partial R}{\partial z} &= \rho' \frac{\partial \Phi}{\partial z},\end{aligned}$$

where ρ' is the electric density.

When ρ' is a function of Φ alone the stress reduces to a hydrostatic pressure

$$p = - \int \rho' d\Phi.$$

This occurs when ρ' is constant.

When the electricity is distributed over the surface of the electron there is a discontinuity of pressure at the surface.

When the electron is an oblate spheroid of eccentricity β , with its minor axis in the line of motion, the pressure in either case is constant at the surface. This is the case with the forms of electron proposed by Lorentz and by Bucherer.

§ 6. The stress just considered may be accounted for in two ways :—

(1) We may suppose it due to attractions between the elementary parts of the electron. In that case the attractions cannot follow the Newtonian law ; otherwise two negative electrons would have no electrostatic action on each other, since the new hypothetical force would balance the electrostatic repulsion between the electrons when at rest. Hence the new force cannot be used to account for gravitation, and we require to postulate two types of actions at a distance in addition to the electromagnetic actions. This explanation is equivalent to Jeans's assumption of additional forces between electric charges. These forces not only require explanation themselves, but are hardly consonant with an electron theory.

(2) We may suppose the stress due to interaction between the electron and the æther.

The simplest assumption is that the electron is an oblate

spheroid (§ 5), that the stress is a pressure, and that the surface of the electron is a surface of constant pressure.

This assumption has the advantage that the resultant action of the æther on the electron vanishes, so that the electron in moving through the æther experiences no resistance on account of its expansion. Nevertheless the æther resists the expansion of the electron with a pressure, which, as we have seen, depends on the size of the electron and on its velocity of translation.

The electron in expanding exerts an equal pressure on the surrounding æther and must be supposed to push it outwards. We can only form a mechanical conception of this action in two ways :—

(1) The æther fills the space occupied by the electron ; in this case it must be expansible.

(2) The electron is an empty space surrounded by æther ; in this case the æther need not be expansible.

§ 7. In the first case an expanding electron produces a pressure at a distant point varying approximately inversely as the distance, and in consequence acts on a second distant electron with a force inversely as the square of the distance, but the force is a repulsion. This is in accordance with the view expressed by Maxwell that gravitational attraction cannot be transmitted by means of stress in a medium. If the æther be expansible, its bulk-modulus must be so great that inequalities of pressure due to expansion are carried away by waves almost instantaneously. The first case becomes practically identical with the second for our purpose.

§ 8. Let us assume for the sake of definiteness that the æther is a continuous isotropic medium, which possesses inertia and rotational elasticity, and may, or may not, possess volume elasticity. This is the type assumed by Larmor.

In such a medium permanent rotational motions are impossible ; any permanent motion is irrotational.

The expanding electron in this æther virtually constitutes a hydrodynamic source ; two sources attract each other with a force proportional directly to their strengths and to the density of the medium, and inversely to the square of the distance between them. Therefore two expanding electrons attract each other according to the same law.

If a, a' be their radii, and μ the density of the medium, the hydrodynamic attraction is $\frac{4\pi\mu a^2 a'^2 \ddot{a}\ddot{a}'}{r^2}$.

When we consider two systems of n, n' electrons respectively, at a distance large compared with the dimensions of either

system, the hydrodynamic attraction between the two systems is

$$nn' \frac{4\pi\mu a^2 a'^2 \dot{a}\dot{a}'}{r^2}.$$

The electromagnetic masses of the systems are given by

$$M = nm, M' = nm',$$

where m, m' are the masses of the electrons.

$$\text{Hence the attraction is } \frac{MM'}{r^2} \frac{4\pi\mu a^2 \dot{a}\dot{a}'^2}{mm'}.$$

This is the law of gravitation. The last factor is the gravitation constant. In order to account for gravitation quantitatively it must have the value 6.7×10^{-8} .

§ 9. When the electrons are in motion the values of m, m' and those of $\dot{a}, \dot{a}'$ depend on the velocity. This makes the gravitation constant depend on the velocity. To avoid the difficulty several ways can be suggested :

(1) The mass m of the electron, and the pressure at its surface, differ from their values when the electron is at rest by quantities of the order β^2 ; when the velocity of the electron is not a large fraction of the velocity of light, the difference is very small.

(2) An atom, if built up out of electrons, probably contains electrons moving with very different velocities; the gravitational attraction between two atoms is therefore an average taken for electrons with very different velocities. The table of values of β for rings of electrons given in § 4 shows that for rings of considerable numbers of electrons the value of β does not alter much with the number of electrons in the ring, and for these rings the values of β are large; the gravitational attraction diminishes as β approaches the value unity, because the pressure at the surface of the electron diminishes and because the mass of the electron increases. Hence rings of few electrons, which move slowly, produce an effect which is practically the same as that for electrons at rest; rings of many electrons, which move quickly, produce a smaller effect, but the difference is very slight for a considerable difference in the number of electrons in the ring. The mean gravitation constant for the whole atom need not differ very much for atoms containing very different numbers of electrons, and therefore of very different atomic weights.

Whether this be the reason why the gravitation constant is the same for all kinds of atoms or not, the same difficulty arises in Lorentz's theory of gravitation. Lorentz supposes

electrons of opposite sign to attract each other more than those of like sign repel each other. The gravitation between atoms is proportional to the number of electrons in each, while their masses depend on the masses of the electrons, and therefore on their velocities as well.

§ 10. While we may admit that the hypothesis of the expanding electron affords a qualitative explanation of the law of gravitation, yet we can hardly rest satisfied unless it gives for the gravitation constant a value of the right order. After many trials I have not succeeded in devising a mechanical model of the electron which shall do this in a satisfactory manner. When we treat the problem as that of a source in an incompressible liquid with the pressure at infinity equal to a given constant, we find that a is an elliptic function of the time which varies between two definite limits.

Since $\frac{\dot{a}}{a}$ must be small, a must be very close to one of the two

limits; thus there are possible two electrons of different sizes, each varying very slowly in size; their radii being different, so also are their electromagnetic masses; by identifying them with the negative and positive electrons we can account for the difference in mass of the two kinds of electrons. In order to account for the smallness of $\frac{\dot{a}}{a}$ we

must suppose the density μ of the æther to be very large. In order to get the proper value of the gravitation constant, we must suppose μ to be of the order 10^{100} , whether we assume gravitation to be due to one or both kinds of electrons.

In itself a large value of μ is not an objection. But it leads to a difficulty, which however is not peculiar to the present view, but is shared by every theory which assumes a mechanical action between the electron and the æther. A sphere moving through a liquid has an effective mass, greater than its true mass, by one half the mass of liquid displaced. If this were true of the electron, its effective mass would be enormously greater than its electromagnetic mass. For the negative electron this is not the case; it moves through the æther without experiencing any appreciable resistance beyond that due to electromagnetic radiation. Yet we require it to force back the æther while it expands.

§ 11. The conclusions of this investigation are as follows:—

(1) In order to account for the definiteness of atomic structure, as evidenced by the fineness of the spectrum lines, on an electron theory without the assumption of forces between electrons other than electromagnetic, it is necessary

and sufficient to assume that the electron expands at a certain slow rate (§ 4).

(2) In order to satisfy the principle of the conservation of energy it is necessary to assume that the expansion is resisted by an internal stress, which in the simplest case is a hydrostatic pressure (§ 5).

(3) In order to explain this pressure in accordance with the spirit of the electron theory, that is, without action at a distance, we must assume that it is due to the action of the æther on the electron (§§ 6, 7).

(4) The reaction of the electron on the æther produces a pressure in the æther at the surface of the electron; if this is not exactly balanced by the pressure at infinity the electron expands, presses back the æther and acts like a source in a liquid. Hence electrons gravitate towards each other, as do sources in a liquid (§ 8).

III. *The General Determination of the Optical Constants of a Crystal by means of Refraction through a Prism.* By G. F. HERBERT SMITH, M.A., Assistant in the Mineral Department of the British Museum*.

BY far the simplest method of obtaining the three principal indices of refraction of a crystalline substance is that of total-reflexion, since observations need be made on only one face or section of the crystal; but this method is necessarily restricted to substances of lower refractive power than the constant medium (usually a hemisphere of dense glass) or the liquid used to obviate the intervention of a film of air. In other cases, however, recourse must be had to refraction through a prism formed by a pair of natural or artificial faces on the crystal, inclined to one another at not too large an angle. If possible, it is arranged that the plane bisecting the angle between these faces is one of symmetry in the crystal, so that the method of minimum deviation and the corresponding simple formula are available. But it is often the case in natural crystals that advantage cannot be taken of this simple method: the crystal may not have suitable natural faces inclined at a small enough angle, and may be so small itself that artificial faces cannot be ground on to it. We propose to show that, even in such cases, it is possible by taking observations sufficiently far round a zone to determine the optical constants.

* Communicated by the Author. The substance of the paper was discussed by him at meetings of the Mineralogical Society, June 19, 1900, and November 17, 1903.