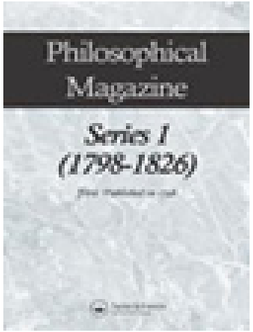


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# XLIII. An attempt towards a theory of the resistance experienced by two and four-wheeled carriages on different kinds of roads; and to determine the circumstances under which the one are preferable to the other

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For good figures of the C. *cancellatus*, Micheli may be referred to, and for the two latter, Curtis's Fl. Lond. and Sowerby's Engliſh Fungi, called in both publications PHALLUS.

I am not aware that any other Fungus will arrange with the above three ſpecies, although ſome have been placed under the genus PHALLUS, by Linnæus and other botaniſts, which have very little agreement with the P. *impudicus* and P. *caninus* of thoſe authors.

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## XLIII. *An Attempt towards a Theory of the Reſiſtance experienced by two and four-wheeled Carriages on different Kinds of Roads; and to determine the Circumſtances under which the one are preferable to the other.* By NICHOLAS FUSS, *Profeſſor of the higher Mathematics at Peterſburgh, Member of the Imperial Academy of Sciences, &c.*

[Continued from p. 122.]

### SECOND DIVISION.

### *Of the Reſiſtance on ſolid and uneven Roads.*

### I. *Four-wheeled Carriages.*

#### *Section* 19.

IF the road AB, Plate II. fig. 4. be naturally rough and ſtony, or paved artificially, or covered in ſome places with trees laid acroſs it, new impediments ariſe from theſe inequalities, and the reſiſtance found in the preceding diviſion, ſection 9, which takes place here alſo, acquires an increaſe, which is determined in the following manner:

Let the wheel touch in G and G′ two equally large and ſolid inequalities of the road, and for the fore-wheels let the angle $GOG' = 2\varphi$, but for the hind-wheels $GOG' = 2\psi$. Now as a part of the moving power OV, which we ſhall call $OU = K$, muſt be employed to overcome a part of the power of gravity acting in a direction perpendicular to AB, which is $(\frac{1}{2}P + p)$ cos. $\alpha$, and to raiſe the fore-axle together with its part of the load over the fixed point G, the momentum of theſe powers OU.GU and OR.GR muſt be equal to each other; that is, $K.GO \cos.\varphi = (\frac{1}{2}P + p) \cos.\alpha . GO \sin.\varphi$; ſo that the increaſe of reſiſtance ariſing from this impediment will be for the fore-wheels $K = (\frac{1}{2}P + p) \cos.\alpha \tang.\varphi$. A ſimilar increaſe will be found for the hind-wheels $= (\frac{1}{2}P + p') \cos.\alpha \tang.\psi$. Conſequently the whole reſiſtance on roads of the ſecond claſs for four-wheeled carriages is:

$$R = \frac{1}{2}(m+n)\lambda P \cos.\alpha + (P + p + p') \sin.\alpha + (\tfrac{1}{2}P + p) \cos.\alpha \tang.\varphi + (\tfrac{1}{2}P + p') \cos.\alpha \tang.\psi$$

II. *Two-*

## II. *Two-wheeled Carriages.*

### *Section* 20.

The resistance found in the preceding division, section 12, for carriages of this kind, acquires here an increase, which, if we make the angle GOG′ = $2\omega$, will be found in the same manner as in the preceding section = $(P+\pi)$ cos. $\alpha$ tang. $\omega$. The whole resistance for two-wheeled carriages on roads of the second class will therefore be expressed as follows:

$$R' = \left.\begin{array}{l}\mu\lambda P \cos.\alpha + (P+\varepsilon P+\pi)\sin.\alpha \\ + (P+\pi)\cos.\alpha \tan.\omega\end{array}\right\}$$

## III. *Comparison of the two Kinds of Carriages.*

### *Section* 21.

If the comparison be made in general according to the five different points of view mentioned in the 16th section, it will be found that four-wheeled carriages are preferable to two-wheeled when any of the five following conditions take place:

$$1\text{st},\ \frac{m+n-2\mu}{2} < \left.\begin{array}{l}\dfrac{(\varepsilon P+\pi-p-p')\tan.\alpha + (P+\pi)\tan.\omega}{\lambda P} \\ \dfrac{-(\frac{1}{2}P+p)\tan.\phi - (\frac{1}{2}P+p')\tan.\psi}{\lambda P}\end{array}\right\}$$

$$2\text{d},\ p+p'-\pi < \varepsilon P - \left.\begin{array}{l}\dfrac{(m+n-2\mu)\frac{1}{2}\lambda P + (P+\pi)\tan.\omega}{\tan.\alpha} \\ \dfrac{-(\frac{1}{2}P+p)\tan.\phi - (\frac{1}{2}P+p')\tan.\psi}{\tan.\alpha}\end{array}\right\}$$

$$3\text{d},\ P > \frac{(p+p'-\pi)\tan.\alpha + p\tan.\phi + p'\tan.\psi - \pi\tan.\omega}{\varepsilon\tan.\alpha + \tan.\omega - \frac{1}{2}\tan\phi - \frac{1}{2}\psi - \frac{1}{2}\lambda(m+n-2\mu)}$$

$$4\text{th},\ \varepsilon > \left.\begin{array}{l}\dfrac{(m+n-2\mu)\frac{1}{2}\lambda P + (p+p'-\pi)\tan.\alpha + (\frac{1}{2}P+p)}{P\,\text{rang.}\,\alpha} \\ \dfrac{\tan.\phi + (\frac{1}{2}P+p')\tan.\psi - (P+\pi)\tan.\omega}{P\tan.\alpha}\end{array}\right\}$$

$$5\text{th},\ \text{Tang.}\,\alpha > \left.\begin{array}{l}\dfrac{(m+n-2\mu)\frac{1}{2}\lambda P + (\frac{1}{2}P+p)\tan.\phi}{\varepsilon P+\pi-p-p'} \\ \dfrac{+(\frac{1}{2}P+p')\tan.\psi - (P+\pi)\tan.\omega}{\varepsilon P+\pi-p-p'}\end{array}\right\}$$

### Example I.

### *Section* 22.

Let the road be composed of round pieces of timber six inches in diameter laid across it, and let $\alpha = 0$. Let the mean diameter of the fore and hind axle-trees of the four-wheeled carriages be $3\frac{1}{4}$ inches; that of the axles of the two-wheeled carriages $3\frac{3}{4}$; and the diameter of the wheels in the same

ſame order, 26, 39, and 45 inches; ſo that $m = \frac{1}{8}$, $n = \frac{1}{12}$, and $\mu = \frac{1}{12}$: alſo, let $P = 1000$ lib., $p = 74$ lib., $p' = 106$ lib., and $\pi = 130$ lib.; and let the coefficient of the friction be $\lambda = \frac{2}{9}$. In the laſt place, as $TG = TF = 3$ inches, we ſhall have for the wheels in the above order $TO = 16$, $\frac{45}{2}$ and $\frac{51}{2}$ inches, and becauſe ſin. $TOF = \frac{TF}{TO}$; the angle TOF, that is to ſay, $\varphi = 10° 48'$; $\psi = 7° 40'$; $\omega = 6° 45°$. From theſe elements we obtain,

$$\begin{aligned}
\tfrac{1}{2}(m + n)\,\lambda P &= 23{\cdot}148 \\
(\tfrac{1}{2}P + p)\text{ tang. }\varphi &= 109{\cdot}496 \\
(\tfrac{1}{2}P + p')\text{ tang. }\psi &= 81{\cdot}575 \\
\hline
R &= 214{\cdot}219 \\
\mu\lambda P &= 18{\cdot}518 \\
(P + \pi)\text{ tang. }\omega &= 133{\cdot}744 \\
\hline
R' &= 152{\cdot}262
\end{aligned}$$

For a horſe, then, whoſe ſtrength $M = 400$ lib. and velocity $G = 13$ feet on ſuch a road, we have $g = 3{\cdot}484$ feet, and $g' = 4{\cdot}979$ feet in a ſecond.

It might be conjectured that four-wheeled carriages, the fore-wheels being ſo ſmall, would experience a reſiſtance conſiderably greater than the two-wheeled. Were $m = n = \frac{1}{12}$, and $p = p' = 106$, we ſhould have $R = 181{\cdot}669$, and $g = 4{\cdot}238$.

### Example II.

*Section* 23.

Let the road be ſteep and compoſed of ſtones, the angle of elevation being $\alpha = 14°$, and let the pavement be of ſuch a nature, that the diſtance between the points of contact G, $G' = 3$ inches, conſequently the angle $\varphi = 5° 45'$; $\psi = 4° 6'$, and $\omega = 3° 49'$. Let the burden be $P = 1800$ lib., the mean diameter of the axle-trees of the fore-wheels $= 3\frac{1}{2}$ inches, of the hind-wheels $3\frac{1}{2}$ inches, that of the two-wheeled carriages $3\frac{1}{4}$ inches; let the diameter of the wheels in the ſame order be 30, 42, and 45 inches; ſo that $m = \frac{1}{9}$, $n = \frac{1}{12}$, and $\mu = \frac{1}{12}$. The weight of the wheels we ſhall ſuppoſe to be $p = 80$ lib. $p' = 110$ lib., and $\pi = 120$ lib. In the laſt place, let $\lambda = \frac{1}{5}$, and $\varepsilon = \frac{1}{24}$. Hence we obtain

$$\begin{aligned}
\tfrac{1}{2}(m + n)\,\lambda P\cos.\alpha &= 33{\cdot}960 \\
(P + p + p')\text{ ſin. }\alpha &= 481{\cdot}425 \\
(\tfrac{1}{2}P + p)\cos.\alpha\text{ tang. }\varphi &= 95{\cdot}750 \\
(\tfrac{1}{2}P + p)\cos.\alpha\text{ tang. }\psi &= 70{\cdot}247 \\
\hline
R &= 681{\cdot}382
\end{aligned}$$

$\mu\lambda P$

$$\begin{aligned} \mu\lambda P \cos.\alpha &= 29{\cdot}109 \\ (P + \varepsilon P + \pi) \text{ fin. } \alpha &= 482{\cdot}634 \\ (P + \pi) \cos.\alpha \text{ tang. } \omega &= 124{\cdot}282 \\ \hline R' &= 636{\cdot}025 \end{aligned}$$

Three horses, therefore, whose strength is $M = 1200$ lib. and their velocity on this kind of road $G = 10$ feet, could draw the burden $P = 1800$ lib. on a four-wheeled carriage at the rate of $g = 2{\cdot}47$ feet per second, and on a two-wheeled carriage about $g' = 2{\cdot}72$ feet.

*Section* 24.

On a horizontal road of the above nature we shall have for the same carriages and load $R = 206{\cdot}08$ and $R' = 158{\cdot}09$. For a horse therefore whose strength $M = 400$ lib., and velocity on this horizontal road $G = 15$ feet, we shall have $g = 4{\cdot}23$ and $g' = 5{\cdot}58$ feet.

## Third Division.

### *Of the Resistance on soft and even Roads.*

*Section* 25.

If the surface of the road be composed of soft sand, slate, earth, clay, dirt, &c. the wheels will sink into it, and the depth to which they sink will be greater the softer and more fluid the matter is of which the road consists.

Now as the increase of resistance depends on the depth to which the wheels sink, and as this depth depends on the nature of the matter, that is to say, its hydrostatic power, which can be determined only by experiment, I shall assume that a prismatic body the thickness of which is $dd$ square inches and its weight 9 lib. sinks merely by its gravity $c$ inches deep in the fluid matter, or displaces $ddc$ cubic inches of it before it comes into equilibrium: this matter, then, in regard to hydrostatic power produces the same effect as would be produced by an aqueous fluid a cubic foot of which weighs $\frac{12^3 q}{ddc}$ lib.

A load Q, Pl. II. fig. 5. will press down both wheels on the axis O till they have displaced $\frac{ddcQ}{q}$ cubic inches of the fluid matter. If FR be the depth to which the wheels sink, and $\pi'$ the circumference of a circle whose diameter $= 1$, the superficial content of the sunken segment GFG'R $= GO^2 \left(\frac{\pi' \angle GOR}{180^\circ} - \frac{1}{2} \text{ fin. } GOG'\right)$. And if the fellies be

be $b$ inches broad, we ſhall have for both wheels:

$$2\,b\,GO^2\left(\frac{\pi'.\sphericalangle GOR}{180^\circ} - \tfrac{1}{2}\text{ fin. }GOG'\right) = \frac{ddcQ}{q}.$$

Hence we find,

$$\frac{\pi'.GOR}{90^\circ} - \text{fin. }GOG' = \frac{ddcQ}{bq.GO^2};$$

from which equation the angle GOR neceſſary for determining the reſiſtance is found.

## I. *Four-wheeled Carriages.*

### *Section* 26.

As the wheels in this caſe have ſunk to the points G,G′ of their circumference, there ariſes in the point G an impediment to the motion, which will be greater or leſs according as the ſurface of the road is covered with more or leſs fluid matter; that is, according to the greater or leſs extent of the angle which the ſides of as ſteep an accumlation as poſſible of this matter forms with the level ſurface. This angle, which gives the moſt convenient meaſure of the fluidity, I ſhall in future call $\beta$.

Now if GFG′ were an excavation in a perfectly ſolid road, G would oppoſe the moving power OV in conſequence of the power of gravity $OQ = S$ with a power $OR = S \cos.\ \alpha$, the momentum of which is $OR.GR = S \cos.\ \alpha.\ GO$ fin. GOR. If the part of the moving power employed for this purpoſe be K, its momentum $K.GU = K.GO \cos.\ GOR$; ſo that $K = S \cos.\ \alpha$ tang. GOR. If the angle for the fore-wheels be $GOR = \xi$, and for the hind-wheels $GOR = \eta$, the reſiſtance for the former $= (\tfrac{1}{2}P + p) \cos.\ \alpha.$ tang. $\xi$, and for the latter $= (\tfrac{1}{2}P + p') \cos.\ \alpha$ tang $\eta$; conſequently, the increaſe of reſiſtance for the four-wheeled carriages $= \cos.\ \alpha\ [(\tfrac{1}{2}P + p)$ tang. $\xi + (\tfrac{1}{2}P + p')$ tang. $\eta]$. With this power G oppoſes a reſiſtance as a fixed point, but, as a fluid, it yields and oppoſes a leſs power the greater its fluidity, the coefficient of which is fin. $\beta$. The increaſe of reſiſtance on roads of this kind is therefore $= \cos.\ \alpha$ fin. $\beta\ [(\tfrac{1}{2}P + p)$ tang. $\xi + (\tfrac{1}{2}P + p')$ tang. $\eta]$; conſequently the whole reſiſtance for four-wheeled carriages:

$$\left.\begin{aligned} R = \tfrac{1}{2}(m + n)\,\lambda P \cos.\ \alpha + (P + p + p')\text{ fin. }\alpha \\ + \cos.\ \alpha \text{ fin. }\beta\ [(\ P + p)\text{ tang. }\xi + (\tfrac{1}{2}P + p)\text{ tang. }\eta.] \end{aligned}\right\}$$

## II. *Two-wheeled Carriages.*

### *Section* 27.

If we make the angle $GOR = \vartheta$, it will be found in the

 ſame

same manner as in the preceding section, that the increase of resistance on roads of this class is $= (P + \pi) \cos. \alpha \sin. \beta \tan. \vartheta$; and the whole resistance for two-wheeled carriages will be,

$$R' = \left.\begin{array}{l} \mu \lambda P \cos. \alpha + (P + \varepsilon P + \pi) \sin. \alpha \\ + (P + \pi) \cos. \alpha, \sin. \pi, \tan. \vartheta. \end{array}\right\}$$

III. *Observations on these Formulæ R and R'.*

*Section* 28.

These expressions for R and R' as above found contain, as we shall soon see, a complete solution of the problem, as they answer for all the three classes of roads.

1st, For solid and even roads where $\xi = 0$, $\eta = 0$, $\vartheta = 0$, and $\beta = 90°$, they give for R and R' the same values as those found for resistance in the first division.

2d, For solid and uneven roads where $\xi = \varphi$, $\eta = \psi$, $\vartheta = \omega$, and $\beta = 90°$, we have the same values for R and R' as those found in the second division.

3d, For roads with standing water on a solid bottom (which, however, must not reach to the spokes, because in this case the resistance must be determined in another manner), where $\beta = 0$, we shall have R and R' as in the first division. In this case, no perceptible increase of resistance arises: on the other hand, on such roads G and consequently V is less than on roads of the first class.

4th, When one of the angles $\xi$, $\eta$, or $\vartheta$, is a right angle; that is, when P is so great that the wheels sink up to the axle in the soft surface, the resistance becomes infinitely great, except in that case only where the fluid is very thin, and $\beta = 0$; in which case instead of an infinitely great resistance we shall have only an indefinite increase $= 0.\infty$, which cannot be determined from the grounds mentioned in the third observation.

In the last place, it is to be observed in regard to these expressions,

5th, That when the soft tender surface has a solid bottom, and is not so deep that the wheels can penetrate into it according to the laws of hydrostatics, the angle GOR must not be determined according to the absolute sinking, but according to the depth of the solid bottom.

6th, That for less fluid matters, such as moist sand, moist earth, stiff clay, dirt, &c. in the expression for the resistance of four-wheeled carriages, R the last member for the hind-wheels $(\frac{1}{2} P + p') \cos. \alpha$, $\sin. \beta$, $\tan. \eta$, vanishes, because these wheels run in the ruts formed by the fore-wheels.

IV. *Com-*

## IV. *Comparison between the two Kinds of Carriages.*

### *Section* 29.

On roads of this class, four-wheeled carriages are to be preferred to two-wheeled, when one of the five following conditions takes place:

$$1\text{st, } \frac{m+n-2\mu}{2} < \left.\begin{array}{l} \dfrac{(\varepsilon P+\pi-p-p')\text{ t. }\alpha+\text{fin. }\beta\,[(P+\pi)\text{ t. }\vartheta}{\lambda P} \\ \dfrac{-(\frac{1}{2}P+p)\text{ tang. }\xi-(\frac{1}{2}P+p')\text{ tang. }\eta]}{\lambda P} \end{array}\right\}$$

$$2\text{d, } p+p-\pi < \varepsilon P-\left.\begin{array}{l} \dfrac{(m+n-2\mu)\,\frac{1}{2}\lambda P+\text{fin. }\beta\,[(P+\pi)\text{ t. }\vartheta}{\text{tang. }\alpha} \\ \dfrac{-(\frac{1}{2}P+p)\text{ tang. }\xi-(\frac{1}{2}P+p')\text{ tang. }\eta]}{\text{tang. }\alpha} \end{array}\right\}$$

$$3\text{d, } P > \frac{(p+p'-\pi)\text{ t. }\alpha+\text{fin. }\beta\,(p\text{ t. }\xi+p'\text{ t. }\eta-\pi\text{ t. }\vartheta)}{\varepsilon\text{ t. }\alpha+\text{fin. }\beta\,[\text{t. }\vartheta-\frac{1}{2}\text{t. }\xi-\frac{1}{2}\text{t. }\eta]-\frac{1}{2}\lambda\,(m+n-2\mu)}$$

$$4\text{th, } \varepsilon > \left.\begin{array}{l} \dfrac{(m+n-2\mu)\,\frac{1}{2}\lambda P+(p+p'-\pi)\text{ tang. }\alpha+\text{fin. }\beta}{P\text{ tang. }\alpha} \\ \dfrac{[(\frac{1}{2}P+p)\text{ tang. }\xi+(\frac{1}{2}P+p')\text{ tang. }\eta-(P+\pi)\text{ tang. }\vartheta]}{P\text{ tang. }\alpha} \end{array}\right\}$$

$$5\text{th, Tang. }\alpha > \left.\begin{array}{l} \dfrac{(m+n-2\mu)\,\frac{1}{2}\lambda P+\text{fin. }\beta\,[(\frac{1}{2}P+p)\text{ t. }\xi}{\varepsilon P+\pi-p-p'} \\ \dfrac{+(\frac{1}{2}P+p')\text{ tang. }\eta-(P+\pi)\text{ tang. }\vartheta]}{\varepsilon P+\pi-p-p'} \end{array}\right\}$$

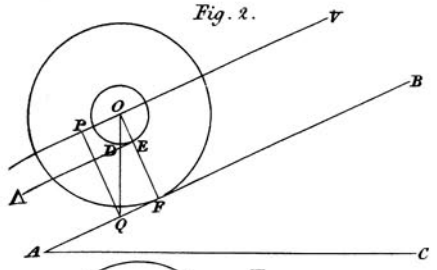
That these five conditions are general, and contain the conditions found for the two classes before treated, section 16 and section 17, it is superfluous to mention, after what has been said in the first two remarks of the preceding section.

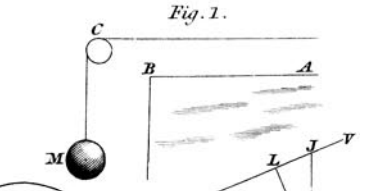
### *Application to some determinate Cases.*

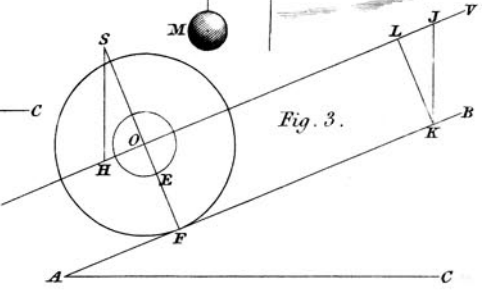
#### EXAMPLE I.

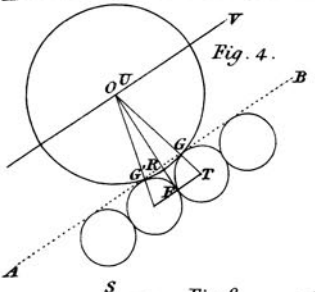
### *Section* 30.

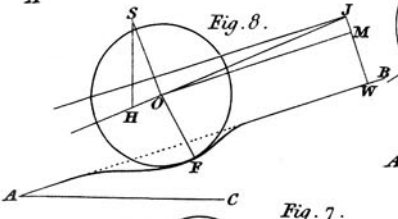
I poured very dry coarse red sand, Pl. II. fig. 6. after I had found the angle $\beta = 30° 42$ (that is to say, AB = 16 inches and CD = $4\frac{3}{4}$ inches), into a vessel, and, smoothing the surface of it after each experiment by shaking it so as to obtain the same degree of softness, I took a steel rod weighing 5 lib., which I had caused to be constructed for magnetic experiments, and which was just an inch square, and pushed it several times into the sand in a gentle manner, to prevent the acquired velocity from making it sink deeper than the equilibrium sought for. Sixteen experiments of this kind, in which the greatest difference did not amount to half a line,

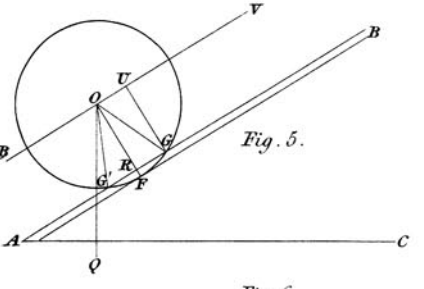


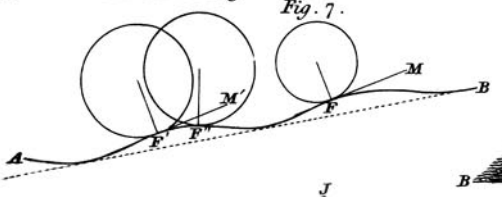



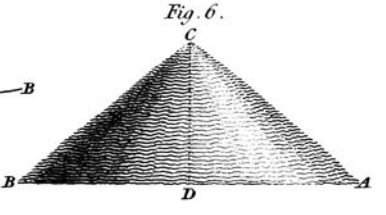



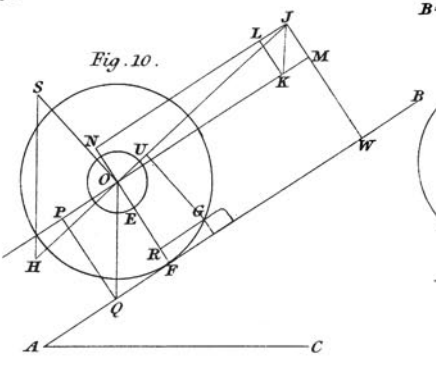



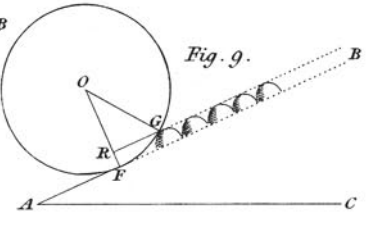




Lowry sculp.

gave as a medium of the sinking $\frac{7}{24}$ inches; so that $ddc = \frac{7}{24}$ inches, and $q = 5$ lib.

Let the road consist of such soft sand; let the inclination $\alpha = 4°$; the load $P = 1200$ lib., the weight of the wheels $p = 84$ lib. $p' = 110$ lib., $\pi = 132$ lib.; their diameter in the same order as before, $31\frac{1}{2}$, 42, and 49 inches. Also let $m = \frac{1}{9}$, $n = \frac{1}{12}$, $\mu = \frac{1}{14}$, $\lambda = \frac{1}{5}$; $OS = 2\frac{6}{7}$ feet and $OI = 12$ feet; so that $\varepsilon = \frac{1}{60}$. In the last place, let $b = 3$ inches, consequently,

$$\frac{\pi'\xi}{90°} - \text{sin. } 2\xi = 0{\cdot}05361, \text{ and } \xi = 19° \, 47'$$

$$\frac{\pi'\eta}{90°} - \text{sin. } 2\eta = 0{\cdot}03130, \text{ and } \eta = 16° \, 30'$$

$$\frac{\pi'\vartheta}{90°} - \text{sin. } 2\vartheta = 0{\cdot}04315, \text{ and } \vartheta = 18° \, 23'$$

| | | |
|---|---|---|
| $\frac{1}{2}(m + n)\lambda P \cos. \alpha$ | = | 23·276 |
| $(P + p + p')$ sin. $\alpha$ | = | 97·242 |
| $(\frac{1}{2}P + p)$ cos. $\alpha$, sin. $\beta$, tang. $\xi$ | = | 125·303 |
| $(\frac{1}{2}P + p')$ cos. $\alpha$, sin. $\beta$, tang. $\eta$ | = | 107·112 |
| | R = | 352·933 |
| $\mu\lambda P \cos. \alpha$ | = | 17·101 |
| $(P + \varepsilon P + \pi)$ sin. $\alpha$ | = | 94·311 |
| $(P + \pi)$ cos. $\alpha$, sin. $\beta$, tang. $\vartheta$ | = | 225·450 |
| | | 336·862 |

If we suppose for two horses $M = 800$ and $G = 11$ feet, we shall have $g = 3{\cdot}696$ feet and $g' = 3{\cdot}861$ feet in a second.

However well the two-wheeled carriages may be constructed, however small the burden, the inclination of the road, and the height of the centre of gravity, the resistance is only a little less than for four-wheeled carriages. To show how much depends on the proportion of the wheels, and particularly in regard to carriages of this kind, I shall in the following examples change only the wheels of the two-wheeled carriages.

## Example II.

### *Section* 31.

Let every thing be as before, only for the two-wheeled carriages let $GO = 16$ inches, $\pi = 85$ lib., and $\mu = \frac{7}{64}$; therefore $\vartheta = 24° \, 15'$, and

| | | |
|---|---|---|
| $\mu\lambda P \cos. \alpha$ | = | 26·186 |
| $(P + \varepsilon P + \pi)$ sin. $\alpha$ | = | 91·032 |
| $(P + \pi)$ cos. $\alpha$, sin. $\beta$, tang. $\vartheta$ | = | 294·808 |
| | | 412·026 |

Therefore

Therefore we ſhall have as before R = 352·933 lib., but R′ = 412·026 lib.; conſequently $g = 3·696$ feet, and $g' = 3·102$ feet.

## Example III.

### *Section* 32.

After a ſeries of ſimilar experiments with ſomewhat coarſer and very moiſt red ſand, I found $\beta = 43° \; 19'$, and $c = \frac{5}{12}$ inch at a medium. But it is to be obſerved that the impreſſions made by immerſing the ſteel rod did not diſappear as from the above dry ſand, but remained after it was drawn out; and therefore after each experiment I was obliged to make the ſand even and ſoft by ſhaking it, and to render its ſurface ſmooth by a ſlight preſſure proportioned to the exact meaſurement of the depth; by which means it loſt a little of its natural ſoftneſs. When I placed the rod again in the remaining impreſſion it did not become deeper. Hence follows what we are taught by experience in general in regard to moiſt ſand, moiſt earth, thick mud, &c. that the hind-wheels when they revolve in the ruts formed by the fore-wheels do not make them deeper, and conſequently experience no reſiſtance from ſinking down; ſo that in the value of R, ſection 26, found for four-wheeled carriages, in ſuch caſes the laſt member $(\frac{1}{2} P + p)$ cos. $\alpha$, ſin. $\beta$, tang. $\eta$ vaniſhes.

Let us ſuppoſe then, as in the firſt example, ſection 30, that the angle $\alpha = 4°$ the weight of the wheels, $p = 84$ lib., $p' = 110$ lib., $\pi = 132$ lib.; and their ſemidiameter in the ſame order, GO $= 15\frac{3}{4}$, 21, $24\frac{1}{2}$ inches, and $h = 3$ inches; alſo let $m = \frac{1}{9}$, $n = \frac{1}{12}$, $\mu = \frac{1}{14}$, $\lambda = \frac{1}{5}$, $\varepsilon = \frac{1}{60}$, and $g = 5$ lib. If the load be P = 900 lib., and $ddc = \frac{5}{12}$ cubic inches, we ſhall have

$$\frac{\pi' \xi}{90°} - \text{ſin. } 2\xi = 0·05980, \text{ and } \xi = 20° \; 32'$$

$$\frac{\pi' \eta}{90°} - \text{ſin. } 2\eta = 0·03527, \text{ and } \eta = 17° \; 10'$$

$$\frac{\pi' \vartheta}{90°} - \text{ſin. } 2\vartheta = 0·04776, \text{ and } \vartheta = 19° \; 10'$$

$$\begin{aligned}
\tfrac{1}{2}(m + n)\,\lambda P \cos.\,\alpha &= 17·457 \\
(P + p + p')\ \text{ſin. } \alpha &= 76·313 \\
(\tfrac{1}{2} P + p) \cos.\,\alpha, \text{ ſin. } \beta, \text{ tang. } \xi &= 136·878 \\
R &= 230·648 \\
\mu\lambda P \cos.\,\alpha &= 12·826 \\
(P + \varepsilon P + \pi)\ \text{ſin. } \alpha &= 73·035 \\
(P + \pi) \cos.\,\alpha, \text{ ſin. } \beta, \text{ tang. } \vartheta &= 243·414 \\
R' &= 329·275
\end{aligned}$$

If for a horſe on this road M = 400 lib. and G = 12½ feet, we ſhall have $g = 3{\cdot}012$ and $g' = 1{\cdot}162$ feet in a ſecond.

Alſo if the hind-wheels do not run exactly in the ruts formed by the fore-wheels, and if the member $({}^{1}P + p')$ cos. $\alpha$, ſin. $\beta$, tang. $\eta = 118{\cdot}389$, cannot be entirely omitted, we ſhall ſtill have $R < R'$ even when only the fourth part of it vaniſhes.

[To be continued.]

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## XLIV. *Some Account of* Edward Jenner, *M. D.*

THIS gentleman, who has diſtinguiſhed himſelf ſo much in the annals of medicine by bringing forward the vaccine inoculation to public notice, and who has been thought worthy of national remuneration on that account, is a ſon of the Rev. Stephen Jenner, formerly vicar of Berkley in Glouceſterſhire, a man highly reſpected by all thoſe who had the pleaſure of his acquaintance. Edward was born about the year 1749, and received his education at Cirenceſter in the ſame county. Having made a conſiderable progreſs in claſſical learning, and ſhowing an early attachment to the ſtudy of phyſic, he was placed under the care of Mr. Ludlow, an eminent ſurgeon at Sodbury, a large market-town between Briſtol and Wotton under Edge. After remaining with this gentleman ſome time, during which he applied with aſſiduity, and made rapid improvement, he repaired to London to complete his medical education, and became a houſe-pupil to the celebrated John Hunter. In this ſituation he continued two years, and availed himſelf with great ſucceſs of the inſtruction of ſo able a maſter. At the expiration of this period he removed to Berkley, where he commenced practice, and met with conſiderable encouragement. In the mean time he ſtill kept up his intimacy with Mr. Hunter by a regular correſpondence; and the frequent mention which Mr. Hunter has made of him in his works is a ſtriking inſtance of the favourable opinion which he entertained of his abilities. An ingenious paper on the natural hiſtory of the cuckow, in a letter addreſſed to Mr. Hunter, was communicated by him to the Royal Society, and was inſerted in the Philoſophical Tranſactions for 1788. Several other papers of his on intricate ſubjects in natural hiſtory were publiſhed about the ſame time. In 1778 he married miſs Catherine Kingſcote of Kingſcote in Glouceſterſhire, by whom he