MUSIC OF THE MOONS.—II.

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[Continued from Vol. lxxiv, p. 354.]

The harmonic series, of which Mars and its satellites form a part, seems to have been established before the ring of greatest nebular condensation—the ring of which Earth was the centre—was broken up. In the solar system, as well as in the group of densest planets, the number 3, which represents the uneven harmonics of an organ-pipe, as well as the oscillatory divisions of a linear pendulum, holds a prominent place. For we find, at the outset, the following approximations to important nebular centres :

$3^8 = 9^4 =$	6561	$6518 =$	Neptune's secular aphelion.
3^7	2187	$2222 =$	Saturn's secular aphelion.
3^6	9^3 729	735	Cybele.
3^5	243	229	Earth's secular aphelion.
3^4	9^2 81	83	Mercury.
3^3	27	..	
3^2	9^1 9	..	
3^1	3	..	
3^0	9^0 1	1	Sun's semi-diameter.

This accordance is the more significant, because Saturn's secular aphelion is at the centre of the ring of secondary condensation, which extends from Sun's surface to Uranus's secular aphelion.

"Bode's Law,"ⁱ was based on successive differences of $2^0 \times 3$, $2^1 \times 3$, $2^2 \times 3$, etc. If we subtract 1 from each of the theoretical Bode numbers, and divide the remainders by 3, the quotients are 1, 2, 3, 5, 9, 17, etc., each of the quotients, except those for Venus and Neptune, being of the form $d_{n+1} = 2d_n - 1$; the dense-belt series being of the form $d_{n+1} = 3d_n - 1$.ⁱⁱ

In the infinite series, $\frac{1}{2} + 3^{-\infty} + 3^{-\infty+1} + \dots 3^{-1} + 3^0 + 3^1 + 3^2 + \dots$, successive sums, in the neighborhood of unity, give the following accordsances :

ⁱ This JOURNAL, Sept., 1877, p. 162.

ⁱⁱ *Ibid.*, Nov., 1877, p. 352.

Sums.	Harmonic Divisors.	Quotients.	Observed.
$\frac{1}{2} =$	$\frac{1}{2}$	27.38	$27.00 = 3^3$.
$+ 3^{-\infty}$	.	.	.
$+ .$	.	.	.
$+ .$	.	.	.
$+ 3^{-4}$	$\frac{14}{27}$	26.40	26.20 Extreme major-axis.
$+ 3^{-3}$	$\frac{5}{9}$	24.64	24.39 Mean major-axis.
$+ 3^{-2}$	$\frac{2}{3}$	20.53	20.68 Extreme secondary radius.
$+ 3^{-1}$	1	13.69	13.69 Nebular radius.
$+ 3^0$	2	6.85	6.85 Deimus.
$+ 3^1$	5	2.74	2.73 Phobus.
$+ 3^2$	14	.98	1.00 Semi-diameter of Mars.
$+ 3^3$	41	.33	.33 Oscillatory centre.
$+ 3^4$	122		120.56 Moon's major-axis.
$+ 3^5$	365		365.26 Terrestrial acceleration.
$+ 3^6$	1094		1096.20 Jupiter's semi-major-axis.

The "Extreme major-axis" is the major-axis of an ellipse, connecting the inner planets of the two outer two-planet belts at the secular aphelia of Uranus and Jupiter; the "Mean major-axis" is the sum of the mean distances of Uranus and Jupiter; the "Extreme secondary radius" is Uranus's aphelion radius, or the semi-diameter of the ring of secondary condensation; the "Nebular radius" not only represents the theoretical incipience of Mars's nebular condensation, but it also corresponds, almost precisely, with the sum of the secular perihelia of Jupiter (4.886) and Saturn (8.734), in units of Earth's semi-major-axis—the secular perihelion being the time of greatest orbital *vis viva*; "Moon's major-axis" is also Earth's "Nebular radius;" the "Terrestrial acceleration" represents the theoretical increase in the angular velocity of Earth's rotation, since its rupture from the central nucleus, or the ratio of its day to its year; "Jupiter's semi-major-axis" is measured in units of Sun's mean perihelion distance from the centre of gravity of Sun and Jupiter.

The sum of the infinite series, to and including 3^{-3} , is $\frac{5}{9}$, which represents the ratio of *vis viva* between undulatory velocity and the velocity of the particles of a medium constituted according to the Kinetic theory.¹ Alexander has shown the importance of that ratio in planeto-taxis,² and I have shown that it represents "centres of

¹ Maxwell and Preston, *Phil. Mag.*, June, 1877.

² "Smithsonian Contributions," 280.

explosive oscillation," or the centre of secondary oscillation between the primary centre of oscillation and the centre of gravity, in a homogeneous line of particles ($\frac{2}{3} - \frac{2}{3}$ of $\frac{1}{6} = \frac{5}{6}$). Adding the next term of the series, we get $\frac{2}{3}$, which represents the centre of linear oscillation. Neptune's major-axis (60.06) is, within $\frac{1}{10}$ of 1 per cent. ($3^4 - 3^3 + 3^2 - 3^1 = 60$) times Earth's mean radius vector.

These harmonies embrace orbital radii of the largest five planets of the solar system, of the inner planets, and of the asteroidal belt, together with nebular-, satellite-, and planetary-radii, for the outer and the middle planets in the theoretically primitive central belt, or the belt of greatest condensation. Can any interpretation be rightly put upon such a chain of harmonies, which does not recognize the fundamental laws of harmonic oscillation and harmonic design?

Neither of Mars's moons is of sufficient magnitude to cause any great perturbations. To this fact, perhaps, as much as to the proximity of the density-centre, we may attribute the regularity of the Mavortian system. In the solar system, as we have seen,¹ the preponderating mass of Jupiter sets up a new order of differences in the harmonic denominators; and we may find probable indications of similar influence in some of the satellite systems, and in the elementary spectra.

In the satellite system of Uranus, if we take the semi-major-axis of the outer satellite as the common numerator (22.75), we find the following harmony:

Satellites.	Distances.	Denominators.	Theoretical.
Oberon,	22.75	1.000	1.000
Titania,	17.01	1.337	$1.343 = 1 + 2 a$
Umbriel,	10.37	2.194	$2.199 = 1 + 7 a$
Ariel,	7.44	3.058	$3.055 = 1 + 12 a$
Semi-diameter,	1.00	22.750	$22.750 = 1 + 127 a$

In the Saturnian system there is a slight uncertainty in the satellite elements, except in the case of Titan, whose orbit was well determined by Bessel. It will be seen that Titan's great mass introduces a secondary harmony. The following harmonic denominators are based upon relative mean distances which would represent the orbital times, as furnished by Professor Hall:

¹ This JOURNAL, Sept., 1877, p. 167.

Satellites.	Times.	Denominators.	Theoretical.
Japetus,	79.3292 ^d	1.000	1.000
Hyperion,	21.3113	2.402	2.397 = $1 + a$
Titan,	15.9454	2.914	2.920 $1 + b$
Rhea,	4.5175	6.756	6.760 $1 + 3b$
Dione,	2.7369	9.436	9.384 $1 + 6a$
Tethys,	1.8878	12.087	12.179 $1 + 8a$
Enceladus,	1.3702	14.966	14.974 $1 + 10a$
Mimas,	.9425	19.206	19.166 $1 + 13a$
Semi-diameter,		64.359	64.360 $1 + 33b$

It is well to notice that b (1.920) is very nearly the square of a (1.397).

In the column of times, Japetus, divided by Titan, is nearly 5; Hyperion, by Rhea, 5; Dione, by Enceladus, 2; Tethys, by Mimas, 2; Titan, by Rhea, $\frac{7}{2}$; Rhea, by Dione, $\frac{5}{3}$; Hyperion, by Titan, $\frac{4}{3}$; Hyperion, by Dione, 8; Hyperion, by Mimas, $\frac{4.5}{2}$; Titan, by Mimas, 17.

The satellite system of Jupiter, our Sun's "companion star," exhibits harmonies of distance, time and mass. The mean distance of the outer satellite, Callisto, is 3³ semi-diameters of its primary (26.9984). Using this as a common numerator, we find that the other satellites are phyllotactically, as well as harmonically, arranged:

Satellites.	Distances.	Denominators.	Theoretical.
Callisto,	26.9984	1.000	
Ganymede,	15.3502	1.759	1.731 = $5a$.
Europa,	9.6235	2.807	2.769 $8a$.
Io,	6.0485	4.464	4.500 $13a$.
Semi-diameter,	1.0000	26.998	26.998 $78a$.

The harmonies of time and mass are as follows:

Satellites.	Times.	Theoretical.	Mass. Theoretical.
Callisto,	16.689 ^d	16.684 = $28t$	4266 4403 = $\frac{1}{2}m$.
Ganymede,	7.155	7.150 $12t$	8850 8806 $1m$.
Europa,	3.551	3.575 $6t$	2324 2202 $\frac{1}{4}m$.
Io,	1.769	1.788 $3t$	1733 1761 $\frac{1}{6}m$.