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SIR DAVID BREWSTER, President, in the Chair.

At the request of the Council, Professor William Thomson of Glasgow delivered the following Address on the Forces concerned in the Laying and Lifting of Deep-Sea Cables.

THE forces concerned in the laying and lifting of deep submarine cables attracted much public attention in the years 1857–58.

An experimental trip to the Bay of Biscay in May 1858, proved the possibility, not only of safely laying such a rope as the old Atlantic cable in very deep water, but of lifting it from the bottom without fracture. The speaker had witnessed the almost incredible feat of lifting up a considerable length of that slight and seemingly fragile thread from a depth of nearly $2\frac{1}{2}$ nautical miles.* The cable had actually brought with it safely to the surface, from the bottom, a splice with a large weighted frame attached to it, to prevent untwisting between the two ships, from which two portions of cable with opposite twists had been laid. The actual laying of the cable a few months later, from mid ocean to Valencia on

* Throughout the following statements, the word mile will be used to denote (not that most meaningless of modern measures, the British statute mile) but the nautical mile, or the length of a minute of latitude, in mean latitudes, which is 6073 feet. For approximate statements, rough estimates, &c., it may be taken as 6000 feet, or 1000 fathoms.

one side, and Trinity Bay, Newfoundland, on the other, regarded merely as a mechanical achievement, took by surprise some of the most celebrated engineers of the day, who had not concealed their opinion, that the Atlantic Telegraph Company had undertaken an impossible problem. As a mechanical achievement it was completely successful; and the electric failure, after several hundred messages (comprising upwards of 4359 words) had been transmitted between Valencia and Newfoundland, was owing to electric faults existing in the cable before it went to sea. Such faults cannot escape detection, in the course of the manufacture, under the improved electric testing since brought into practice, and the causes which led to the failure of the first Atlantic cable no longer exist as dangers in submarine telegraphic enterprise. But the possibility of damage being done to the insulation of the electric conductor before it leaves the ship (illustrated by the occurrences which led to the temporary loss of the 1865 cable), implies a danger which can only be thoroughly guarded against by being ready at any moment to back the ship and check the egress of the cable, and to hold on for some time, or to haul back some length according to the results of electric testing.

The forces concerned in these operations, and the mechanical arrangements by which they are applied and directed, constitute one chief part of the present address; the remainder is devoted to explanations as to the problem of lifting the west end of the 1200 miles of cable laid last summer, from Valencia westwards, and now lying in perfect electric condition (in the very safest place in which a submarine cable can be kept), and ready to do its work, as soon as it is connected with Newfoundland, by the 600 miles required to complete the line.

Forces concerned in the Submergence of a Cable.

In a paper published in the "Engineer" Journal in 1857, the speaker had given the differential equations of the catenary formed by a submarine cable between the ship and the bottom, during the submergence, under the influence of gravity and fluid friction and pressure; and he had pointed out that the curve becomes a straight line in the case of no tension at the bottom. As this is always the

case in deep-sea cable laying, he made no farther reference to the general problem in the present address.

When a cable is laid at uniform speed, on a level bottom, quite straight, but without tension, it forms an inclined straight line, from the point where it enters the water, to the bottom, and each point of it clearly moves uniformly in a straight line towards the position on the bottom that it ultimately occupies.* That is to say, each particle of the cable moves uniformly along the base of an isosceles triangle, of which the two equal sides are the inclined portion of the cable between it and the bottom, and the line along the bottom which this portion of the cable covers when laid. When the cable is paid out from the ship at a rate exceeding that of the ship's progress, the velocity and direction of the motion of any particle of it through the water are to be found by compounding a velocity along the inclined side, equal to this excess, with the velocity already determined, along the base of the isosceles triangle.

The angle between the equal sides of the isosceles triangle, that is to say, the inclination which the cable takes in the water, is determined by the condition, that the transverse component of the cable's weight in water is equal to the transverse component of the resistance of the water to its motion. Its tension where it enters the water is equal to the longitudinal component of the weight (or, which is the same, the whole weight of a length of cable hanging vertically down to the bottom), diminished by the longitudinal component of the fluid resistance. In the laying of the Atlantic cable, when the depth was two miles, the rate of the ship six miles an hour, and the rate of paying out of the cable seven miles an hour, the resistance to the egress of the cable, accurately measured by a dynamometer, was only 14 cwt. But it must have been as much as 28 cwt., or the weight of two miles of the cable hanging vertically down in water, were it not for the frictional resistance of the water against the cable slipping, as it were, down an inclined plane from the ship to the bottom, which therefore must have borne the difference, or 14 cwt. Accurate observations are wanting as to the angle at which the cable entered the water; but from measurements of angles at the stern of the ship, and a

* Precisely the movement of a battalion in line changing front.

dynamical estimate (from the measured strain) of what the curvature must have been between the ship and the water, I find that its inclination in the water, when the ship's speed was nearly $6\frac{1}{2}$ miles per hour, must have been about $6\frac{3}{4}^{\circ}$, that is to say, the incline was about 1 in $8\frac{1}{2}$. Thus the length of cable, from the ship to the bottom, when the water was 2 miles deep, must have been about 17 miles.

The whole amount (14 cwt.) of fluid resistance to the motion of this length of cable through it, is therefore about $\cdot 81$ of a cwt. per mile. The longitudinal component velocity of the cable through the water, to which this resistance was due, may be taken, with but very small error, as simply the excess of the speed of paying out above the speed of the ship, or about 1 mile an hour. Hence, to haul up a piece of the cable vertically through the water, at the rate of 1 mile an hour, would require less than 1 cwt. for overcoming fluid friction, per mile length of the cable, over and above its weight in water. Thus fluid friction, which for the laying of a cable performs so valuable a part in easing the strain with which it is paid out, offers no serious obstruction, indeed, scarcely any sensible obstruction, to the reverse process of hauling back, if done at only 1 mile an hour, or any slower speed.

As to the transverse component of the fluid friction, it is to be remarked that, although not directly assisting to reduce the egress strain, it indirectly contributes to this result; for it is the transverse friction that causes the gentleness of the slope, giving the sufficient length of 17 miles of cable slipping down through the water, on which the longitudinal friction operates, to reduce the egress strain to the very safe limit found in the recent expedition. In estimating its amount, even if the slope were as much as 1 in 5, we should commit only an insignificant error, if we supposed it to be simply equal to the weight of the cable in water, or about 14 cwt. per mile for the 1865 Atlantic cable. The transverse component velocity to which this is due may be estimated with but insignificant error, by taking it as the velocity of a body moving directly to the bottom in the time occupied in laying a length of cable equal to the 17 miles of oblique line from the ship to the bottom. Therefore, it must have been about 2 miles in $17 \div 6\frac{1}{2} = 2\cdot 61$ hours, or $\cdot 8$ of a mile per hour. It is not

probable that the actual motion of the cable lengthwise through the water can affect this result much. Thus, the *velocity of settling* of a horizontal piece of the cable (or velocity of sinking through the water, with weight just borne by fluid friction) would appear to be about .8 of a mile per hour. This may be contrasted with longitudinal friction by remembering that, according to the previous result, a longitudinal motion through the water at the rate of 1 mile per hour is resisted by only $\frac{1}{17}$ th of the weight of the portion of cable so moving.

These conclusions justify remarkably the choice that was made of materials and dimensions for the 1865 cable. A more compact cable (one for instance with less gutta percha, less or no tow round the iron wires, and somewhat more iron), even if of equal strength and equal weight per mile in water, would have experienced less transverse resistance to motion through the water, and therefore would have run down a much steeper slope to the bottom. Thus, even with the same longitudinal friction per mile, it would have been less resisted on the shorter length; but even on the same length it would have experienced much less longitudinal friction, because of its smaller circumference. Also, it is important to remark that the roughness of the outer tow covering undoubtedly did very much to ease the egress strain, as it must have increased the fluid friction greatly beyond what would have acted on a smooth gutta percha surface, or even on the surface of smooth iron wires, presented by the more common form of submarine cables.

The speaker showed models illustrating the paying-out machines used on the Atlantic expeditions of 1858 and 1865. He stated that nothing could well be imagined more perfect than the action of the machine of 1865 in paying out the 1200 miles of cable then laid, and that if it were only to be used for *paying out*, no change either in general plan or in detail seemed desirable, except the substitution of a softer material for the "jockey pulleys," by which the cable in entering the machine has the small amount of resistance applied to it which it requires to keep it from slipping round the main drum. The rate of egress of the cable was kept always under perfect control by a weighted friction brake of Appold's construction (which had proved its good quality in the 1858 Atlantic expedition) applied to a second drum carried on the same shaft

with the main drum. When the weights were removed from the brake (which could be done almost instantaneously by means of a simple mechanism), the resistance to the egress of the cable, produced by "jockey pulleys," and the friction at the bearings of the shaft carrying the main drum, &c., was about $2\frac{1}{2}$ cwt.

Procedure to Repair the Cable in case of the appearance of an electric fault during the laying.

In the event of a fault being indicated by the electric test at any time during the paying out, the safe and proper course to be followed in future (as proved by the recent experience), if the cable is of the same construction as the present Atlantic cable, is instantly, on order given from an authorised officer in the electric room, to stop and reverse the ship's engines, and to put on the greatest *safe* weight on the paying-out break. Thus in the course of a very short time the egress of the cable may be stopped, and, if the weather is moderate, the ship may be kept, by proper use of paddles, screw, and rudder, nearly enough in the proper position for hours to allow the cable to hang down almost vertically, with little more strain than the weight of the length of it between the ship and the bottom.

The best electric testing that has been practised or even planned cannot show within a mile the position of a fault consisting of a slight loss of insulation, unless both ends of the cable are at hand. Whatever its character may be, unless the electric tests demonstrate its position to be remote from the outgoing part, the only thing that can be done to find whether it is just on board or just overboard, is to cut the cable as near the outgoing part as the mechanical circumstances allow to be safely done. The electric test immediately transferred to the fresh-cut seaward end shows instantly if the line is perfect between it and the shore. A few minutes more, and the electric tests applied to the *two ends* of the remainder on board, will, in skilful hands, with a proper plan of working, show very closely the position of the fault, *whatever its character may be*. The engineers will thus immediately be able to make proper arrangements for resplicing and paying out good cable, and for cutting out the fault from the bad part.

But if the fault is between the land end and the fresh-cut seaward end on board ship, proper simultaneous electric tests on board ship and on shore (not hitherto practised, but easy and sure if properly planned) must be used to discover whether the fault lies so near the ship that the right thing is to haul back the cable until it is got on board. If it is so, then steam power must be applied to reverse the paying-out machine, and, by careful watching of the dynamometer, and controlling the power accordingly (hauling in slowly, stopping, or veering out a little, but never letting the dynamometer go above 60 or 65 cwt.), the cable (which can bear 7 tons) will not break, and the fault will be got on board more surely, and possibly sooner, than a “sulky” salmon of 30 lbs. can be landed by an expert angler with a line and rod that could not bear 10 lbs. The speaker remarked that he was entitled to make such assertions with confidence now, because the experience of the late expedition had not only verified the estimates of the scientific committee and of the contractors as to the strength of the cable, its weight in water (whether deep or shallow), and its mechanical manageability, but it had proved that in moderate weather the Great Eastern could, by skilful seamanship, be kept in position and moved in the manner required. She had actually been so for thirty-eight hours, and eighteen hours during the operations involved in the hauling back and cutting out the first and second faults, and reuniting the cable, and during seven hours of hauling in, in the attempt to repair the third fault.

Should the simultaneous electric testing on board and on shore prove the fault to be 50 or 100 or more miles from the ship, it would depend on the character of the fault, the season of the year, and the means and appliances on board, whether it would be better to complete the line, and afterwards, if necessary, cut out the fault and repair, or to go back at once and cut out the fault before attempting to complete the line. Even the worst of these contingencies would not be fatal to the undertaking with such a cable as the present one. But all experience of cable-laying shows that almost certainly the fault would either be found on board, or but a very short distance overboard, and would be reached and cut out with scarcely any risk, if really prompt measures, as above described, are taken at the instant of the appearance of a fault,

to stop as soon as possible with safety the further egress of the cable.

The most striking part of the Atlantic undertaking proposed for 1866, is that by which the 1200 miles of excellent cable laid in 1865 is to be utilised by completing the line to Newfoundland.

That a cable lying on the bottom in water two miles deep can be caught by a grapnel and raised several hundred fathoms above the bottom, was amply proved by the eight days' work which followed the breakage of the cable on the 3d of August last. Three times out of four that the grapnel was let down, it caught the cable, on each occasion after a few hours of dragging, and with only 300 or 400 fathoms more of rope than the 2100 required to reach the bottom by the shortest course. The time when the grapnel did not hook the cable it came up with one of its flukes caught round by its chain; and the grapnel, the short length of chain next it, and about 200 fathoms of the wire-rope, were proved to have been dragged along the bottom, by being found when brought on board to have interstices filled with soft light gray ooze (of which the speaker showed a specimen to the Royal Society). These results are quite in accordance with the dynamical theory indicated above (see Appendix II.), according to which a length of such rope as the electric cable, hanging down with no weight at its lower end, and held by a ship moving through the water at half a mile an hour, would slope down to the bottom at an angle from the vertical of only 22° ; and the much heavier and denser wire-rope that was used for the grappling would go down at the same angle with a considerably more rapid motion of the ship, or at a much steeper slope with the same rate of motion of the ship.

The only remaining question is: How is the cable to be brought to the surface when hooked? The operations of last August failed from the available rope, tackle, and hauling machine not being strong enough for this very unexpected work. On no occasion was the electric cable broken.* With strong enough tackle, and a

* The strongest rope available was a quantity of rope of iron wire and hemp spun together, able to bear 14 tons, which was prepared merely as *buoy-rope* (to provide for the contingency of being obliged, by stress of weather or other cause, to cut and leave the cable in deep or shallow water), and was accordingly all in 100 fathoms-lengths, joined by shackles with swivels. The

hauling machine, both strong enough, and under perfect control, the lifting of a submarine cable, as good in mechanical quality as the Atlantic cable of 1865, by a grapnel or grapnels, from the bottom at a depth of two miles, is certainly practicable. If one attempt fails, another will succeed; and there is every reason, from dynamics as well as from the 1865 experience, to believe that in any moderate weather the feat is to be accomplished with little delay, and with very few if any failing attempts.

The several plans of proceeding that have been proposed are of two classes—those in which, by three or more ships, it is proposed to bring a point of the cable to the surface without breaking it at all; and those in which it is to be cut or broken, and a point of the cable somewhat eastward from the break is to be brought to the surface.

With reference to either class, it is to be remarked that, by lifting simultaneously by several grapnels so constructed as to hold the cable without slipping along it or cutting it, it is possible to bring a point of the cable to the surface without subjecting it to any strain amounting to the weight of a length of cable equal to the depth of the water. But so many simultaneous grapplings by ships crossing the line of cable at considerable distances from one another would be required, that this possibility is scarcely to be reckoned on practically, without cutting or breaking the cable at a point westward of the points raised by the grapnels. On the other hand, with but three ships the cable might, no doubt, be brought to the surface at any point along the line, without cutting it, and without subjecting it at any point to *much* more strain than the weight corresponding to the vertical depth, as is easily seen when it is considered that the cable was laid generally with from 10 to 15 per cent. of slack. And if the cable is cut at some point not far westward of the westernmost of the grapnels, there can be no doubt but it could be lifted with great ease by three grapnels hauled up simultaneously wire and hemp rope itself never broke, but on two of the three occasions a swivel gave way. On the last occasion, about 900 fathoms of Manilla rope had to be used for the upper part, there not being enough of the wire buoy-rope left; and when 700 fathoms of it had been got in, it broke on board beside a shackle, and the remaining 200 fathoms of the Manilla, with 1540 fathoms of wire-rope and the grapnel, and the electric cable which it had hooked, were all lost for the year 1865.

by three ships. The catenaries concerned in these operations were illustrated by a chain with 15 per cent. of slack hauled up simultaneously at three points.

The plan which seemed to the speaker surest and simplest is to cut the cable at any chosen point, far enough eastward of the present broken end to be clear of entanglement of lost buoy-rope, grapnels, and the loose end of the electric cable itself; and then, or as soon as possible after, to grapple and lift at a point about three miles farther eastward. This could be well and safely done by two ships, one of them with a cutting grapnel, and the other (the Great Eastern herself) with a holding grapnel. The latter, on hooking, should haul up cautiously, never going beyond a safe strain, as shown by the dynamometer. The other, when assured that the Great Eastern has the cable, should haul up, at first cautiously, but ultimately, when the cable is got well off the bottom by the Great Eastern, the western ship should move slowly eastwards, and haul up with force enough to cut or break the cable. This leaves three miles of free cable on the western side of the Great Eastern's grapnel, which will yield freely eastwards (even if partly lying along the bottom at first), and allow the Great Eastern to haul up and work slowly eastwards, so as to keep its grappling rope, and therefore ultimately the portions of electric cable hanging down on the two sides of its grapnel, as nearly vertical as is necessary to make sure work of getting the cable on board. This plan was illustrated by lifting, by aid of two grapnels, a very fragile chain (a common brass chain in short lengths, joined by links of fine cotton thread) from the floor of the Royal Society. It was also pointed out that it can be executed by one ship alone, with only a little delay, but with scarcely any risk of failure. Thus, by first hooking the cable by a holding grapnel, and hauling it up 200 or 300 fathoms from the bottom, it may be left there hanging by the grapnel-rope on a buoy, while the ship proceeds three miles westwards, cuts the cable there, and returns to the buoy. Then, it is an easy matter, in any moderate weather, to haul up safely and get the cable on board.

The use of the dynamometer in dredging was explained; and the forces operating on the ship, the conditions of weather, and the means of keeping the ship in proper position during the process of slowly hauling in a cable, even if it were of strength quite insuffi-

cient to act, when nearly vertical, with any sensible force on the ship, were discussed at some length. The manageability of the Great Eastern, in skilful hands, had been proved to be very much better than could have been expected, and to be sufficient for the requirements in moderate weather. She has both screw and paddles—an advantage possessed by no other steamer in existence. By driving the screw at full power ahead, and backing the paddles, to prevent the ship from moving ahead, or (should the screw overpower the paddles), by driving the paddles full power astern, and driving at the same time the screw ahead with power enough to prevent the ship from going astern, “steerage way” is *created* by the lash of water from the screw against the rudder; and thus the Great Eastern may be effectually steered without going ahead. Thus she is, in calm or moderate weather, almost as manageable as a small tug steamer, with reversing paddles, or as a rowing boat. She can be made still more manageable than she proved to be in 1865, by arranging to disconnect either paddle at any moment; which, the speaker was informed by Mr Canning, may easily be done.

The speaker referred to a letter he had received from Mr Canning, chief engineer of the Telegraph Construction and Maintenance Company, informing him that it is intended to use three ships, and to be provided both with cutting and with holding grapnels, and expressing great confidence as to the success of the attempt. In this confidence the speaker believed every practical man who witnessed the Atlantic operations of 1865 shared, as did also, to his knowledge, other engineers who were not present on that expedition, but who were well acquainted with the practice of cable-laying and mending in various seas, especially in the Mediterranean. The more he thought of it himself, both from what he had witnessed on board the Great Eastern, and from attempts to estimate on dynamical principles the forces concerned, the more confident he felt that the contractors would succeed next summer in utilising the cable partly laid in 1865, and completing it into an electrically perfect telegraphic line between Valencia and Newfoundland.

APPENDIX I.

Descriptions of the Atlantic Cables of 1858 and 1865.

(Distance from Ireland to Newfoundland, 1670 Nautical Miles.)

Old Atlantic Cable, 1858.

Conductor.—A copper strand, consisting of seven wires (six laid round one), and weighing 107 lbs. per nautical mile.

Insulator.—Gutta percha laid on in three coverings, and weighing 261 lbs. per knot.

External Protection.—Eighteen strands of charcoal iron wire, each strand composed of seven wires (six laid round one), laid spirally round the core, which latter was previously padded with a serving of hemp saturated with a tar mixture. The separate wires were each 22 gauge; the stand complete was No. 14 gauge.

Circumference of Finished Cable, 2 inches.

Weight in Air, 20 cwt. per nautical mile.

Weight in Water, 13·4 cwt. per nautical mile.

Breaking Strain, 3 tons 5 cwt., or equal to 4·85 times the cable's weight in water per mile. Hence the cable would bear its own weight in nearly five miles depth of water, or 2·05 times the—

Deepest Water to be encountered, 2400 fathoms, being less than 2½ nautical miles.

Length of Cable Shipped, 2174 nautical miles.

New Atlantic Cable, 1865.

Conductor.—Copper strand consisting of seven wires (six laid round one), and weighing 300 lbs. per nautical mile, embedded for solidity in Chatterton's compound. Diameter of single wire ·048 = ordinary 18 gauge. Gauge of strand ·144 = ordinary No. 10 gauge.

Insulation.—Gutta percha, four layers of which are laid on alternately with four thin layers of Chatterton's compound. The weight of the entire insulation 400 lbs. per nautical mile. Diameter of core ·464 of an inch; circumference of core 1·46 inches.

External Protection.—Ten solid wires of diameter ·095 (No. 13 gauge) drawn from Webster and Horsfall's homogeneous iron, each wire surrounded separately with five strands of Manilla yarn, saturated with a preservative compound, and the whole laid spirally

round the core, which latter is padded with ordinary hemp, saturated with preservative mixture.

Circumference of Finished Cable, 3·534 inches.

Weight in Air, 35 cwt. 3 qrs. per nautical mile.

Weight in Water, 14 cwt. per nautical mile.

Breaking Strain, 7 tons 15 cwt., or equal to eleven times the cable's weight in water per mile. Hence the cable will bear its own weight in eleven miles depth of water, or 4·64 times the—

Deepest Water to be encountered, 2400 fathoms, or less than $2\frac{1}{2}$ nautical miles.

Length of Cable Shipped, 2300 nautical miles.

II.

Let W be the weight of the cable per unit of its length in water; T the force with which the cable is held back at the point where it reaches the water (which may be practically regarded as equal to the force with which its egress from the ship is resisted by the paying-out machinery, the difference amounting only to the weight in air of a piece of cable equal in length to the height of the stern pulley above the water); P and Q the transverse and longitudinal components of the force of frictional resistance experienced by the cable in passing through the water from surface to bottom; i the inclination of its line to the horizon; D the depth of the water.

The whole length of cable from surface to bottom will be $\frac{D}{\sin i}$; and the transverse and longitudinal components of the weight of this portion are therefore $\frac{WD}{\sin i} \cos i$, and WD respectively. These are balanced by $P \frac{D}{\sin i}$ and $T + Q \frac{D}{\sin i}$.

Hence

$$P = W \cos i, \quad Q = \left(W - \frac{T}{D} \right) \sin i \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

To find the corresponding components of the velocity of the cable through the water, which we shall denote by p and q , we have only to remark that the actual velocity of any portion of the cable in the water may be regarded as the resultant of two veloci-

ties,—one equal and parallel to that of the ship forwards, and the other obliquely downwards along the line of the cable, equal to that of the paying out, obliquely downwards along the line of the cable (since if the cable were not paid out, but simply dragged, while by any means kept in a straight line at any constant inclination, its motion would be simply that of the ship). Hence, if v be the ship's velocity, and u the velocity at which the cable is paid out from the ship, we have

$$p = v \sin i, \quad q = u - v \cos i \quad . \quad . \quad . \quad (2.)$$

Now, as probably an approximate, and therefore practically useful, hypothesis, we may suppose each component of fluid friction to depend solely on the corresponding component of the fluid velocity, and to be proportional to its square. Thus we may take

$$P = W \frac{p^2}{\mathfrak{p}^2}, \quad Q = W \frac{q^2}{\mathfrak{q}^2} \quad . \quad . \quad . \quad (3.)$$

where $\mathfrak{p}$ and $\mathfrak{q}$ denote the velocities, transverse and longitudinal, which would give frictions amounting to the weight of the cable; or, as we may call them, the transverse and longitudinal *settling velocities*. We may use these equations merely as introducing a convenient piece of notation for the components of fluid friction, without assuming any hypothesis, if we regard $\mathfrak{p}$ and $\mathfrak{q}$ as each some unknown function of p and q . It is probable that $\mathfrak{p}$ depends to some degree on q , although chiefly on p ; and *vice versa*, $\mathfrak{q}$ to some degree on p , but chiefly on q . It is almost certain, however, from experiments such as those described in “Beaufoy's Nautical Experiments,” that $\mathfrak{p}$ and $\mathfrak{q}$ are each *very nearly* constant for all practical velocities.

Eliminating p and q between (1), (2), and (3), we have

$$W \cos i = W \left(\frac{v \sin i}{\mathfrak{p}} \right)^2,$$

which gives

$$\mathfrak{p} = \frac{v \sin i}{\sqrt{\cos i}} \quad . \quad . \quad . \quad (4.)$$

and

$$(WD - T) \sin i = W D \left(\frac{u - v \cos i}{\mathfrak{q}} \right)^2 \quad . \quad . \quad . \quad (5.)$$

which gives

$$\mathfrak{q} = (u - v \cos i) \sqrt{\frac{WD}{(WD - T) \sin i}} \quad . \quad . \quad . \quad (6.)$$

These formulæ apply to every case of uniform towing of a rope under water, or hauling in, or paying out, whether the lower end reaches the bottom or not, provided always the lower end is free from tension; but if it is not on the bottom, D must denote its vertical depth at any moment, instead of the whole depth of the sea. To apply to the case of merely towing, we must put $u = 0$; or, to apply to hauling in, we must suppose u negative.

It is to be remarked that the inclination assumed by the cable under water does not depend on its longitudinal slip through the water (since we assume this not to influence the transverse component of fluid friction), and that, according to equation (4), it is simply determined by the ratio of the ship's speed to the transverse "settling velocity" of the cable.

The following table shows the ratio of the ship's speed to the "transverse settling velocity" of the cable for various degrees of inclination of the cable to the horizon:—

Inclination of Cable to Horizon.		Ratio of Ship's Speed to "transverse set- tling velocity" of Cable.	Inclination of Cable to Horizon.		Ratio of Ship's Speed to "transverse set- tling velocity" of Cable.
i		$\frac{v}{p} = \frac{\sqrt{\cos i}}{\sin i}$	i		$\frac{v}{p} = \frac{\sqrt{\cos i}}{\sin i}$
angle whose sine is $\frac{1}{8\frac{1}{2}}$	5°	11.4518	45°		1.1892
	6° 45'	8.4784	50		1.0466
	10	5.7149	51° 50'		1.0000
	15	3.7973	55		.9232
	20	2.8343	60		.8165
	25	2.2013	65		.7173
	30	1.8612	70		.6224
	35	1.5779	75		.5267
	40	1.3616	80		.4231
			85		.0875

If the inclination of the cable had been exactly 6° 45' when the speed of the Great Eastern was exactly $6\frac{1}{2}$ miles per hour, the value of p for the Atlantic cable of 1865 would be exactly $6\frac{1}{2} \div 8.478$, or .765 of a mile per hour.