

THE CONDUCTIVITY OF A SPARK-GAP.

BY W. P. BOYNTON.

SOME years ago the writer made a study of the high-frequency (Tesla) induction coil,¹ from which he concluded that the resistance of the spark-gap in the primary circuit might be of the order of from 100 ohms down to 10 or possibly even 5 ohms. Results of the same order were obtained a little earlier by Trowbridge and Richards,² by a method different in detail, but depending upon the same fundamental principle, namely the damping effect upon the oscillations.

Some work recently done by the writer and Mr. Ralph S. Shelley, Assistant in the Department, leads to the conclusion that the resistance of the primary spark may be, for a part of the time at least, very much less than one ohm.

This work was undertaken primarily with the purpose of verifying experimentally the theoretical conditions of resonance of such a double circuit. For such quantitative work, it was necessary to select the quantity for which resonance was to be sought. Three naturally suggest themselves; the potential difference of the terminals of the secondary condenser, the current in the secondary circuit, and the heating effect, or "mean square" current in the secondary circuit. The previous paper furnishes the basis for the mathematical discussion.

It appears the maximum attainable secondary potential is³

$$(1) \quad \frac{2C_2}{K_2} = \frac{2V_0MK_1}{\sqrt{(L_1K_1 - L_2K_2)^2 + 4M^2K_1K_2}}$$

and hence that the adjustment of the secondary capacity giving resonance is that which makes the expression under the radical sign a minimum, namely

¹ PHYS. REV., VII., pp. 35-63, 1898; Phil. Mag. (5), 46, pp. 312-338, 1898.

² Phil. Mag. (5), 43, pp. 349-367, 1897.

³ Loc. cit., p. 42, eq. (20); also p. 52.

$$(2) \quad K_2 = \frac{L_1 L_2 - 2M^2}{L_2^2} K_1,$$

an expression independent of the resistances of the circuits, while substituting this value gives this maximum as

$$(3) \quad \frac{L_2 V_0}{\sqrt{L_1 L_2 - M^2}}.$$

The conditions for maximum current can be easily found, but are not given, as no attempt was made to test this point.

The value for the "mean square" of the secondary current previously deduced was¹

$$(4) \quad \bar{I}_2^2 = \frac{nV_0^2 M^2 K_1 K_2 (R_1 K_1 + R_2 K_2)}{R_1 R_2 (L_1 K_1 - L_2 K_2)^2 + M(R_1 K_1 + R_2 K_2)^2}.$$

For which a maximum is found when

$$(5) \quad \frac{K_1}{K_2} = -\frac{R_2}{R_1} + \frac{\frac{R_2}{R_1} + \frac{L_2}{L_1}}{\sqrt{1 + \frac{R_1 M^2}{R_2 L_1^2}}},$$

APPARATUS.

The primary capacity consisted of plates of window glass 12 x 16 inches coated on both sides with tinfoil, having a capacity of about .003 microfarad per plate. The secondary capacity consisted of two sheets of zinc, 4 x 6 feet, suspended vertically by white silk ribbons from a wooden frame work, forming an air condenser of adjustable capacity. The ratio of the capacities was measured directly by the bridge method, using intermittent current (110 volts) and telephone.

The primary consisted of five turns of heavily insulated No. 4 wire wound in an open spiral about 9 cm. long and of the same diameter; the secondary consisted of 104 turns No. 20 wire wound on a glass cylinder, forming a solenoid 13 cm. long and 4.3 cm. in diameter. Their inductances were determined by comparison with a standard of self-inductance, whose computed value, .00419 henrys, had been checked by comparison with measured capacities. The

¹ Loc. cit., p. 47, eq. (26).

Anderson-Maxwell method as modified by Fleming¹ was employed for the self-inductances, using a condenser as an intermediary, and the mutual inductance was determined from the self-inductance of the secondary coil by Maxwell's method,² adopting Fleming's modification suggested in the paper last referred to. The values were also computed as a check. They were found to be

$$L_1 = 2,000 \text{ cm.},$$

$$L_2 = 130,000 \text{ cm.},$$

$$M = 5,400 \text{ cm.}$$

The spark gaps employed were zinc knobs of about 2 cm. diameter mounted on glass standards.

EXPERIMENTAL.

A spark gap was placed in parallel with the secondary condenser, and readings of the spark length taken for a series of different distances between the plates of the condenser. With the arrangement employed, namely 8 condenser plates in the primary condenser, and a primary spark of about .13 cm., a distinct maximum of about 1.2 cm. was observed when the plates were about 8 cm. apart. A direct comparison showed the secondary capacity to be $1/71.5$ of the primary. The ratio computed by substituting the values of the inductances in formula (2) was $1/83$. This discrepancy could easily have been made to disappear by moving the plates slightly farther apart, which would have produced no observable change in the spark length. Substituting the values of the inductances in formula (3) shows that the secondary potential cannot be expected to exceed 8.56 times the primary. The ratio of the spark lengths, $1.2/.13 = 9.22$, is entirely consistent with this conclusion as the potentials are very nearly represented by a linear function of the spark length, so that

$$(6) \quad \frac{V_2}{V_0} = \frac{S_2 - a}{S_1 - a},$$

in which S_1 and S_0 represent the spark lengths and the not impos-

¹ Phil. Mag. (6), 7, p. 536. 1904.

² Elec. and Mag., Vol. II., pp. 397-398 (3d. edition).

sible value $a = .0115$ would give exactly the computed ratio, 8.56.

This preliminary test serves to indicate that the apparatus functions normally.

Measurements of the "mean square" current were made by a hot wire ammeter similar to that described in the previous paper.¹ Readings of this instrument were taken as in the case of the potential measurements for a series of distances between the plates of the secondary condenser, then these were set at the distance, seeming to correspond to the maximum current, and the ratio of primary and secondary capacities determined. This was done for different values of the primary capacity, as, shown in the following tabulation :

No. plates in primary	2	3	4	5	6	8
K_1/K_2	22	29	31	34	35.5	38

For comparison the values of K_1/K_2 are plotted from eq. (5) in the accompanying figure as a function of R_2/R_1 . A comparison of the

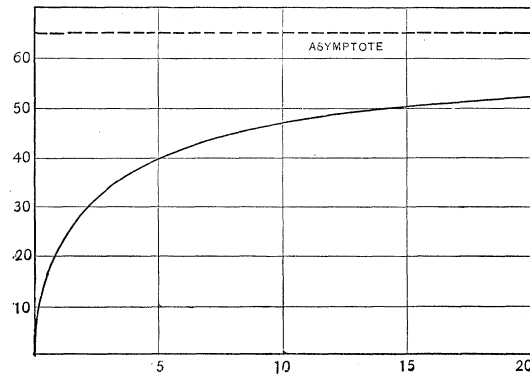


Fig. 1.

observed ratios with the tabulation indicates that the values of R_2/R_1 lie between 1 and 5, or since the measured value of R_2 is 2.07 ohms, the value of R_1 would seem to lie between 2.07 and .415 ohms. As the latter value is not very different from the ohmic resistance of the metallic circuit, the implication is that the spark

¹Loc. cit., p. 53.

gap may offer a resistance of much less than an ohm to the passage of a large oscillatory discharge.

DISCUSSION.

The mathematical theory as previously developed has assumed that all resistances are metallic, that is, constant. On this assumption an oscillatory system ought to give a wave train with a considerable number of observable oscillations. Such a system of metallic conductors having a single period of oscillation was investigated by Webster,¹ who records measurements of the period of perhaps 30 oscillations of the system.

But a conducting gas, such as a spark gap, does not have a constant resistance, such an assumption being only a first approximation. A second approximation which might prove useful would be the assumption that the resistance of such a spark has for a short time a definite, finite steady resistance, suddenly changing at the end of that short time to an infinite resistance. Apparently such an assumption would explain the various phenomena, the resonance effect being determined by the instantaneous small value of the resistance, the effect recorded in the previous paper being due to a time-average effect, while the number of spark-records obtained in various photographs would be related both to the instantaneous value of the resistance, and to the time during which this value persisted.

The data taken at this time are not sufficient for a satisfactory quantitative discussion of this new assumption, hence an attempt will be made to apply it to older data.

In the indefinite integral of the square of an oscillatory function of the form

$$e^{-\alpha t}(A \cos \beta t + B \sin \beta t)$$

the principal term is²

$$-\frac{(A^2 + B^2)e^{-2\alpha t}}{4\alpha}.$$

If the integration be taken not from 0 to ∞ , but from 0 to a finite limit τ , the definite integral will be

¹ PHYS. REV., VI., pp. 297-314, 1898.

² Loc. cit., p. 44.

$$(7) \quad (1 - e^{-2a\tau}) \frac{A^2 + B^2}{4a},$$

which becomes, if $2a\tau$ is small

$$(8) \quad 2a\tau \frac{A^2 + B^2}{4a} = (A^2 + B^2)\tau/2.$$

Applying this result to the secondary potential, and to the secondary and primary currents,

$$(9) \quad \int_0^\tau V_2^2 dt = \frac{(A_2^2 + B_2^2)\tau}{2K_2^2} + \frac{(C_2^2 + D_2^2)\tau}{2K_2^2},$$

which becomes, neglecting B_2^2 and D_2^2 as small, and noting that $C_2^2 = A_2^2$

$$(10) \quad \int_0^\tau V_2^2 dt = A_2^2 \tau / K_2^2.$$

$$(11) \quad \int_0^\tau I^2 dt = (a^2 + \beta^2)(A^2 + B^2)\tau/2 + (\gamma^2 + \delta^2)(C^2 + D^2)\tau/2,$$

which becomes for the secondary current, neglecting a^2 , γ^2 , B_2^2 and D_2^2 as small and remembering that $C_2^2 = A_2^2$

$$(12) \quad \int_0^\tau I_2^2 dt = (\beta^2 + \delta^2)A_2^2 \tau / 2,$$

and for the primary current, where C_1 preponderates over the other coefficients.

$$(13) \quad \int_0^\tau I_1^2 dt = \delta^2 C_1^2 \tau / 2.$$

If, for the sake of definiteness, we assume that τ is the time required for m half-oscillations of the primary current, then evidently

$$(14) \quad \delta\tau = \pi m,$$

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from which

$$(15) \quad \begin{aligned} \int_0^\tau V_2^2 dt &= \pi m A_2^2 / \delta K_2^2, \\ \int_0^\tau I_2^2 dt &= \pi m (\beta^2 + \delta^2) A_2^2 / 2\delta, \\ \int_0^\tau I_1^2 dt &= \pi m \delta C_1^2 / 2. \end{aligned}$$

Substituting the values of β , δ , C_1 , and A_2 , and introducing n , the number of discharges per second of the primary condenser, the following average values are obtained :

$$\begin{aligned} \overline{V}_2^2 &= \frac{\pi n m M^2 K_1^2 V_0^2 \sqrt{L_1 K_1 + L_2 K_2 + \sqrt{(L_1 K_1 - L_2 K_2)^2 + 4 M^2 K_1 K_2}}}{\sqrt{2} [(L_1 K_1 - L_2 K_2)^2 + 4 M^2 K_1 K_2]}, \\ \pi n m M^2 K_1 K_2 V_0^2 (L_1 K_1 + L_2 K_2) \\ (16) \quad \overline{I}_2^2 &= \frac{\sqrt{L_1 K_1 + L_2 K_2 + \sqrt{(L_1 K_1 - L_2 K_2)^2 + 4 M^2 K_1 K_2}}}{2 \sqrt{2} (L_1 L_2 - M^2) [(L_1 K_1 - L_2 K_2)^2 + 4 M^2 K_1 K_2]}, \\ \pi n m K_1 V_0^2 \sqrt{L_1 K_1 + L_2 K_2 - \sqrt{(L_1 K_1 - L_2 K_2)^2 + 4 M^2 K_1 K_2}} \\ \overline{I}_1^2 &= \frac{[L_1 K_1 - L_2 K_2 + \sqrt{(L_1 K_1 - L_2 K_2)^2 + 4 M^2 K_1 K_2}]}{16 K_2 (L_1 L_2 - M^2) [(L_1 K_1 - L_2 K_2)^2 + 4 M^2 K_1 K_2]}. \end{aligned}$$

In these approximate equations the "effective" readings appear to be conditioned not upon the resistance of the spark-gap, but upon m , the number of half-oscillations it permits to pass. The first two quantities also are very simply related, having the ratio

$$(17) \quad \frac{\overline{V}_2^2}{\overline{I}_2^2} = \frac{2}{(\beta^2 + \delta^2) K_2^2} = \frac{2 K_1 (L_1 L_2 - M^2)}{K_2 (L_1 K_1 + L_2 K_2)}.$$

Substituting numerical values in this equation we have, reducing to practical units and extracting the square root,

$$(18) \quad \frac{\overline{V}_2}{\overline{I}_2} = 5,300.$$

Fig. 2 shows data of the discharge with a 2 mm. spark,¹ plotted as functions of the excitation, \overline{I}_2 being in hundredths of an ampere, \overline{V}_2 in hundreds of volts, while the ratio $\overline{V}_2/\overline{I}_2$, computed from the curves, not the points, shown by the broken line, appears very nearly constant, ranging for the most part between 5,500 and 5,000.

Equations (16) may also be used for the computation of m . Substituting the values of the known constants, and reducing to practical units, they give

¹ Loc. cit., pp. 57-58; Table III., Series II. and III.

$$\begin{aligned}
 m &= 3,340 \overline{V_2^2}/V_0^2, \\
 (19) \quad m &= 9.42 \times 10^{10} \overline{I_2^2}/V_0^2, \\
 m &= 5.18 \times 10^8 \overline{I_1^2}/V_0^2.
 \end{aligned}$$

Fig. 3 gives values for m from the data just referred to. This indicates a number of half-oscillations varying from 2 to 8, as shown by data from the secondary circuit, or including one point not in the figure, up 16 as shown by data from the primary circuit. As an interesting coincidence, upon reference to the original spark

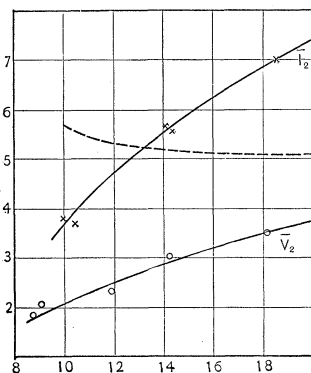


Fig. 2.

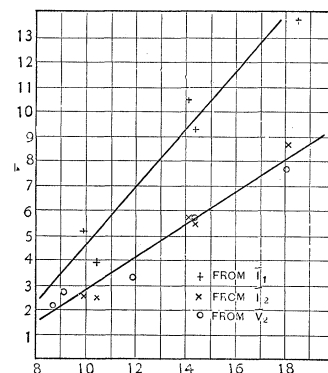


Fig. 3.

photographs, one can count in many of them 8 striations, rarely 9, but often less. These however were taken under such different conditions that their evidence on this evidence on this point is not conclusive.

CONCLUSION.

Trowbridge and Richards¹ specifically speak of the "spark from a large condenser" as the condition of low resistance of spark gap. The justice of this remark is apparent from the fact that in the earlier experiments with a spark from a condenser of .0016 M.F. capacity only the data taken with the 2 mm. spark were available for the purposes of this discussion, the others showing irregular and abnormally large values of the ratio $\overline{V_2}/\overline{I_2}$, while in the present investigation capacities from 4 to 15 times as large were used,

¹ Loc. cit.

namely, from .006 to .023 M.F., when the low resistance effect became strikingly manifest.

The two sets of equations deduced appear to be adapted to the treatment of the two types of experimental data, the old equations perhaps better expressing the conditions when the condenser is small or the spark long and thin ; the newer ones better adapted to the case where the primary condenser is large, and the spark relatively short.

An attempt to find a maximum value of $\overline{I_2^2}$ in equation (16) by differentiation gave forms too complicated for solution ; but the method of trial indicates a maximum in the neighborhood of $K_1/K_2 = 90$ for the later apparatus. This by no means accords with the experimental results, which run from 22 to 38 ; but as the resonance is conditioned upon instantaneous rather than average values, it is not surprising that the newer formulæ, which take account primarily of the time of continuance, should fail to agree with experiment.

The complete mathematical theory would of course treat the resistance of the spark gap as a variable, a function possibly of the time, and thus indirectly a function of the effective current, or the energy of the field.

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