THE PSYCHOLOGICAL REVIEW

ON THE ELIMINATION OF THE TWO EXTREME INTENSITIES OF THE COMPARISON STIMU- LI IN THE METHOD OF CONSTANT STIMULI

BY SAMUEL W. FERNBERGER

Clark University, Worcester, Mass.

The investigators who have been interested in the theory of the psychophysical methods, have concerned themselves with two general problems. In the first place, they have sought for a better theoretical understanding of the methods, and of the thresholds and other final values obtained by the use of them. Secondly, they have attempted to standardize each of these methods in such a form, that the time and labor required for obtaining and treating the data would be reduced to a minimum. This second problem has been of special importance for the method of constant stimuli, inasmuch as the time and labor required for experimentation by this method has been, until recently, disproportionately large. But the method of constant stimuli is exceedingly valuable, because we probably have a better theoretical understanding of it than of any other psychophysical method.

Urban attacked the problem of shortening the time required for the calculations involved in the practice of the method of constant stimuli, and in 1912 published certain tables, by the aid of which an investigator may perform the complete calculation of a threshold in less than twenty minutes, once he has obtained the data.¹ It is unlikely that one ever will be able to reduce the calculation to a briefer form than that made

¹ F. M. Urban, 'Hilfstabellen für die Konstanzmethode,' *Arch. für d. ges. Psychol.*, XXIV., 1912, 236ff.

possible by the use of these tables. Hence it would seem that any further advance towards reducing the labor involved in the practice of this method must consist in altering the experimental technique for the collecting of the data. This may be done in either of two ways; (1) by satisfying oneself with a smaller number of judgments on each comparison pair; and (2) by reducing the number of pairs employed in the determination. In either case, it is of interest to know the effect of such an alteration upon the values of the thresholds.

The question of how many comparison pairs should be employed has not been settled satisfactorily. Of course it would be desirable to use as many as possible; but both the time and the energy of the investigator have their limitations. It seems that the question has been decided chiefly for reasons of convenience; most experimenters employed seven pairs.¹ The differences between the standard stimulus and the largest and the smallest comparison stimulus of the series are as a rule so large that the judgments 'heavier' and 'lighter' respectively have a very high relative frequency. It is a fact that such results have but little influence on the values of h and c , the two quantities which determine the psychometric functions. This suggested the idea of recalculating a series of data, after eliminating the extreme comparison stimuli, and of comparing the results with those obtained in the complete series. The quantities h and c are those employed in the calculation of the thresholds, with which all of the psychophysical methods are primarily concerned; and if the above-mentioned elimination produces little change in the relation of these quantities, the values of the threshold should likewise show but slight variation. That this is actually the case was shown in a recent paper.² It was found that the values of the thresholds show very little variation for the averages of a large group of experiments. While individual values for groups of 100 judgments

¹ L. J. Martin and G. E. Müller, 'Zur Analyseder Unterschiedsempfindlichkeit,' Leipzig, 1899, 6ff. F. M. Urban, 'The Application of Statistical Methods to the Problems of Psychophysics,' Philadelphia, 1908, 5ff. S. W. Fernberger, 'On the Relation of the Methods of Just Perceptible Differences and Constant Stimuli,' *PSYCHOL. REV. Monographs*, XIV., No. 4, 6ff.

² S. W. Fernberger, 'A Simplification of the Practice of the Method of Constant Stimuli,' *Amer. Jour. of Psychol.*, XXV., 1914, 121-130.

on each comparison stimulus show at times rather large and apparently unsystematic variations, still for a large group, these variations tend to cancel one another. An examination of the variations in the measure of sensitivity,—the threshold of Volkmann,—for the extended and for the reduced series reveals a surprising uniformity. Indeed, these values are the more remarkable when we consider that the disregarding of the two extreme comparison stimuli has eliminated one third of the experimental data which forms the basis of the study. The conclusion is that, formally at least, the elimination of the two extreme comparison pairs from the classical form of the method of constant stimuli does not materially affect the values of the thresholds.

There may be a difference between data obtained in a short series and those obtained in a longer series from which the extreme differences were eliminated afterwards. In other words, it may be that the presence of very large differences influences the attitude of the subject. Hence, in order to determine whether the same relations held empirically, we devised the following experiment in the field of lifted weights.¹ In all we employed four subjects; those who kindly consented to act as subjects were Messrs. F. J. O'Brien, D. I. Pope, G. S. Snoddy and R. H. Wheeler, all graduate students in experimental psychology at Clark University. Of this group, two were already trained in our particular technique in lifted weights, as they had both acted as subjects throughout a long series of liftings during the preceding winter and spring. The other two subjects were entirely untrained in this form of experimentation. One of these latter showed great adaptability and his judgments were recorded after less than a ten minute period of practice on the first day. In the case of the other subject, nearly an hour's practice was given him before the hand movements became sufficiently automatic to warrant our recording the judgments.

The stimuli which we presented to the subjects were hollow

¹ The experimental work for this study was performed at Clark University during the autumn of 1913. My thanks are due to the subjects for their faithful and sympathetic coöperation, and also to Professor J. W. Baird, Professor F. M. Urban and Dr. S. C. Fisher for their many helpful suggestions.

brass cylinders, 2.5 inches in diameter and 1 inch high, which were closed at one end. In none of their dimensions did they show a variation greater than 0.001 inch. A different number had been stamped on each cylinder, with a small steel die, so that the experimenter could differentiate between the different stimuli. Thus our stimuli were of the same shape as those which have been employed in earlier investigations. They differ from previously employed stimuli, however, in the manner of weighting.¹ The latter have been loaded to the proper intensity with shot and paraffine, the paraffine being inserted in order to hold the shot stationary. But the paraffine is not entirely anhydrosopic and for this reason the stimuli vary in weight. Experimenters have usually corrected any stimulus which showed a variation of more than ten milligram from its proper intensity. Our cylinders, which were weighted with solder, were entirely anhydrosopic. They have been under observation and in use for about a year and none of them ever showed a greater variation than six milligrams.

We prepared a series of stimuli of the following intensities: (a) fourteen weights of 100 grams each, of which twelve served as the standard stimuli for each pair; (b) two weights each of the following intensities: 88, 92, 96, 104 grams; (c) single weights of 84 and 108 grams. We arranged this group of weights into two complete series; one, a classical extended series of seven comparison pairs in which the comparison stimuli weighed 84, 88, 92, 96, 100, 104 and 108 grams; and the other, a reduced series with 88, 92, 96, 100 and 104 gram comparison stimuli. The second series was exactly similar to the first except that in it the two extreme intensities of the comparison stimuli were eliminated. Each of these comparison stimuli was compared with a standard stimulus of 100 grams. We shall use the expression '*extended series*' to designate the complete series of seven pairs of comparison weights; and we shall refer to the second series of five pairs as the '*reduced series*.'

The manner of lifting was the same as that employed in our

¹ F. M. Urban, *ibid.*, 1ff. S. W. Fernberger, *ibid.*, 7ff.

former study.¹ The weights were arranged about the circumference of a circular table with a revolving top. By means of this arrangement each weight could be brought in successive fashion directly under the hand of the subject, and in this way the space error was eliminated. The time error was present in the first order; that is, the standard weight was always lifted first and the comparison weight second. This error was kept constant by controlling the motions of the hand by the beats of a metronome. The metronome was set at 92 beats per minute and the lifting of each stimulus covered a period of four beats.²

The disposal of our weights was as follows. Numbers from one to twenty-four were printed about the circumference of the table. The standard stimuli were placed on the odd, and the comparison stimuli on the even numbers of the table. Hence by a complete revolution of the table, we secured a judgment on each of the comparison pairs for both the reduced and extended series. The comparison stimuli were not arranged about the table in a haphazard manner; but their order was carefully planned so that (1) the subject would acquire as little knowledge as possible regarding the objective relation of the stimuli; and (2) so that the effects of the order of presentation would be minimized.

TABLE I

Table Numbers	First Arrangement	Second Arrangement
1 and 2	92	104
3 and 4	100	92
5 and 6	88	84
7 and 8	108	100
9 and 10	96	108
11 and 12	104	88
13 and 14	84	96
15 and 16	92	104
17 and 18	100	92
19 and 20	88	100
21 and 22	96	88
23 and 24	104	96

¹ S. W. Fernberger, 'On the Relation of the Methods of Just Perceptible Differences and Constant Stimuli,' *Psychol. Rev. Monographs*, XIV., No. 4, 1913, 7-18.

² Cf. S. W. Fernberger, *ibid.*, 10f.

Two arrangements of the stimuli were used in the experiment, which are given in Table I. The first column contains the numbers on the stimulus table in pairs; the second and third columns contain the intensities of the comparison stimuli. The arrangement was changed after 200 judgments had been secured on each comparison pair. The subjects were not informed that the change was to be made; all subjects later reported not to have been aware of any alteration, which indicates that they had neither acquired any knowledge of the objective relations of the stimuli nor learned the order of their succession. Three complete series of judgments were usually secured from one subject, upon which he was allowed to rest for a brief period. This minimized the effect of fatigue. When experimentation was resumed, after the rest, the first two judgments were not recorded, this period being allowed for the hand movements of the subject to become more regular. The intensity of the weights which were lifted at the start of a series was varied. Furthermore, in order that the subject should have no knowledge of the objective relations of the stimuli which were being presented to him, the table was entirely screened from view; the hand of the subject passing through an aperture in this screen.

Immediately after the lifting of the comparison stimulus of each pair, the subjects gave a judgment in terms of the three categories lighter, equal and heavier. A lighter or heavier judgment indicated that the second weight of the pair was subjectively lighter or heavier than the first weight. The equality judgment included not only all cases of actual subjective equality between the stimuli, but also all cases in which the subject was unable to formulate a judgment of heavier or lighter,—the so-called doubtful cases.

With the arrangement of stimuli and the manner of procedure which we have described above,¹ we obtained from

¹ In the description of our experimental arrangement we have spoken as if 24 stimuli had been employed: 12 standard stimuli, 7 comparison stimuli for the extended series, and 5 for the reduced series. We did this for the sake of clearness of exposition, and of differentiating more sharply between the extended and the reduced series. In reality, however, we employed only 14 stimuli, a complete series of 7 pairs for the extended series of the classical form. For the extended series, we obtained a judgment

each subject 500 judgments on each of the comparison pairs for both the extended and the reduced series, making a total of 24,000 individual judgments. We believe that these data are sufficient in quantity to make our results conclusive. By this experimental arrangement, moreover, the experiments for the extended and the reduced series were mingled and performed at the same time, so that we may assume that they were made under similar conditions. The subjects were not informed as to the nature of the problem, or as to the intensities of the stimuli; and their reports indicated that they possessed no knowledge of the objective relations. Finally, by our choice of trained and untrained subjects, we believe that we shall be able to ascertain the relations of the thresholds both for the extended and the reduced series, for two distinct stages of progressive practice.

After completing our calculations we found that the results from one of our subjects were of such a nature that they must be treated separately; but nevertheless they were quite in accord with the results which we obtained from the other three subjects, so far as our main problem is concerned. We shall treat the results of this subject *in extenso* at the end of this paper.

Tables II., III. and IV. contain the observed relative frequencies of Subjects I., II. and III. respectively, both for the extended and the reduced series. These three tables show a similar arrangement. The 500 judgments obtained from each subject have been divided into five groups of 100 judgments each, in the order in which they were taken. The Roman

for each of the pairs; the top of the table was revolved so that the stimuli were presented to the subject in successive fashion. For the reduced series, the cylinders of the five central intensities of the comparison stimulus were presented in the same manner to the subject. When we came to one of the extreme intensities of the comparison stimulus, which we wished to eliminate, the table was swung through a larger arc so that the two weights—the standard and comparison stimuli of the eliminated pair—passed under the hand of the subject, and the standard stimulus of the next pair was actually presented to him. Seldom did the subjects report that they were aware of the table having been moved through a larger arc than usual. But the possession of this knowledge would not have aided our subjects, for the reason that they were ignorant of the problem and of the objective relation of the stimuli. By the use of 14 instead of 24 weights, our experimental technique was very much simplified, while apparently the validity of our method was not affected.

TABLE II

Groups	Extended Series																		Reduced Series																	
	84			88			92			96			100			104			108			88			92			96			100			104		
	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.			
I.	0.97	0.02	0.01	0.94	0.02	0.04	0.73	0.09	0.18	0.65	0.12	0.23	0.22	0.06	0.72	0.18	0.06	0.76	0.06	0.03	0.91	0.94	0.03	0.03	0.58	0.15	0.26	0.53	0.08	0.39	0.36	0.13	0.51	0.10	0.05	0.
II.	0.98	0.02	0.00	0.94	0.00	0.06	0.83	0.03	0.14	0.54	0.06	0.40	0.26	0.06	0.68	0.13	0.06	0.81	0.02	0.02	0.96	0.94	0.05	0.01	0.69	0.08	0.23	0.47	0.08	0.45	0.32	0.08	0.60	0.12	0.09	0.
III.	0.93	0.06	0.01	0.96	0.03	0.01	0.87	0.06	0.07	0.41	0.31	0.28	0.30	0.17	0.53	0.10	0.09	0.81	0.03	0.07	0.90	0.97	0.03	0.00	0.84	0.12	0.04	0.38	0.27	0.35	0.25	0.24	0.51	0.15	0.13	0.
IV.	0.97	0.03	0.00	0.94	0.05	0.01	0.81	0.14	0.05	0.36	0.33	0.31	0.21	0.23	0.56	0.11	0.16	0.73	0.08	0.08	0.84	0.87	0.10	0.03	0.81	0.14	0.05	0.36	0.28	0.36	0.29	0.22	0.49	0.10	0.23	0.
V.	0.94	0.01	0.05	0.93	0.05	0.02	0.69	0.16	0.15	0.46	0.16	0.38	0.19	0.17	0.64	0.13	0.07	0.80	0.07	0.03	0.90	0.87	0.09	0.04	0.75	0.13	0.12	0.30	0.19	0.51	0.24	0.15	0.61	0.11	0.12	0.

TABLE III

Groups	Extended Series																		Reduced Series																	
	84			88			92			96			100			104			108			88			92			96			100			104		
	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.			
I.	0.70	0.12	0.18	0.63	0.18	0.19	0.45	0.22	0.33	0.33	0.28	0.39	0.14	0.21	0.65	0.10	0.17	0.73	0.04	0.05	0.91	0.60	0.20	0.20	0.34	0.34	0.32	0.24	0.25	0.51	0.09	0.11	0.80	0.09	0.12	0.0
II.	0.71	0.14	0.15	0.70	0.15	0.15	0.51	0.28	0.21	0.36	0.29	0.35	0.10	0.40	0.50	0.05	0.29	0.66	0.02	0.11	0.87	0.69	0.23	0.08	0.32	0.41	0.27	0.18	0.39	0.43	0.06	0.36	0.58	0.03	0.29	0.0
III.	0.56	0.40	0.04	0.51	0.47	0.02	0.21	0.66	0.13	0.05	0.53	0.42	0.03	0.32	0.65	0.00	0.25	0.75	0.00	0.12	0.88	0.52	0.46	0.02	0.25	0.61	0.14	0.07	0.52	0.41	0.02	0.42	0.56	0.02	0.19	0.0
IV.	0.46	0.53	0.01	0.50	0.47	0.03	0.21	0.66	0.13	0.08	0.47	0.45	0.01	0.38	0.61	0.00	0.17	0.83	0.00	0.09	0.91	0.53	0.45	0.02	0.19	0.65	0.16	0.03	0.57	0.40	0.04	0.44	0.52	0.00	0.26	0.0
V.	0.52	0.44	0.04	0.46	0.45	0.09	0.15	0.65	0.20	0.04	0.36	0.60	0.01	0.15	0.84	0.00	0.13	0.87	0.00	0.03	0.97	0.52	0.43	0.05	0.15	0.58	0.27	0.05	0.40	0.55	0.02	0.27	0.71	0.01	0.11	0.0

TABLE IV

Groups.	Extended Series																		Reduced Series																	
	84			88			92			96			100			104			108			88			92			96			100			104		
	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.			
I.	0.98	0.02	0.00	0.95	0.01	0.04	0.82	0.05	0.13	0.70	0.15	0.15	0.17	0.12	0.71	0.13	0.06	0.81	0.06	0.03	0.91	0.91	0.02	0.07	0.55	0.15	0.30	0.37	0.21	0.42	0.11	0.11	0.78	0.13	0.04	0.
II.	0.99	0.00	0.01	0.97	0.02	0.01	0.88	0.03	0.09	0.72	0.04	0.24	0.23	0.12	0.65	0.22	0.06	0.72	0.03	0.00	0.97	0.90	0.04	0.06	0.55	0.14	0.31	0.34	0.11	0.55	0.30	0.06	0.64	0.14	0.09	0.7
III.	0.98	0.02	0.00	0.93	0.06	0.01	0.81	0.16	0.03	0.41	0.37	0.22	0.12	0.32	0.56	0.08	0.20	0.72	0.04	0.08	0.88	0.92	0.06	0.02	0.80	0.13	0.07	0.43	0.29	0.28	0.16	0.31	0.53	0.07	0.19	0.7
IV.	0.98	0.02	0.00	0.98	0.02	0.00	0.78	0.20	0.02	0.48	0.43	0.09	0.09	0.56	0.35	0.09	0.23	0.68	0.03	0.09	0.88	0.92	0.07	0.01	0.75	0.22	0.03	0.44	0.31	0.25	0.09	0.52	0.39	0.05	0.32	0.6
V.	1.00	0.00	0.00	1.00	0.00	0.00	0.89	0.08	0.03	0.67	0.23	0.10	0.08	0.51	0.41	0.04	0.21	0.75	0.00	0.02	0.98	0.90	0.08	0.02	0.81	0.16	0.03	0.52	0.30	0.18	0.16	0.39	0.45	0.04	0.19	0.7

numerals in the first columns indicate these groups. Three columns are given to each comparison weight, in which appear the frequencies of the lighter, equal and heavier judgments on that weight. Reading from left to right, the first seven groups of three columns give the observed relative frequencies for the different comparison stimuli of the extended series in the order of their intensity. The last five groups of three columns show a similar arrangement for the reduced series.

From these data, we have calculated three groups of thresholds for the directions both of increase and decrease. First, the thresholds for the extended series of seven pairs of comparison stimuli have been calculated. Second, formal values of the reduced series from the data of the extended series have been calculated, by disregarding the two extreme intensities of the comparison stimuli. Third, the thresholds from the observed relative frequencies of the reduced series of five pairs of stimuli have been calculated. We shall refer to these three types of values as the threshold values of the *extended series*, of the *reduced series calculated* and of the *reduced series observed* respectively. These thresholds were all calculated in the usual manner by an application of the method of least squares and the use of Urban's tables, noted above. By this method the investigator obtains two values— h and c —from the frequencies both of the heavier and lighter judgments for every series. These quantities determine the form and position of the curves of the psychometric functions for all three categories of judgment. Moreover the relation between them of c/h defines the value of the threshold. The threshold in the direction of increase is obtained from the frequencies of the heavier judgments. The threshold in the direction of decrease is obtained, when we employ the relative frequencies of the lighter judgments. In our present study, both thresholds were calculated for every group of 100 judgments in the extended and reduced series, both calculated and observed, for all four subjects. Thus it was necessary to calculate in all 120 thresholds. Several years ago the labor of such a task would have been entirely disproportionate to

the result obtained, but with the use of Urban's tables neither the labor nor the time involved were excessive.¹

TABLE V

Groups	S_1			S_2		
	Extended Series	Reduced Series		Extended Series	Reduced Series	
		Calculated	Observed		Calculated	Observed
I.	97.07	97.03	96.00	98.78	98.59	98.06
II.	96.81	96.90	96.19	97.80	97.86	97.85
III.	96.22	96.65	96.49	99.65	99.43	99.85
IV.	96.33	95.80	95.70	100.00	99.73	100.27
V.	95.68	95.62	93.87	98.25	98.28	98.11

TABLE VI

Groups	S_1			S_2		
	Extended Series	Reduced Series		Extended Series	Reduced Series	
		Calculated	Observed		Calculated	Observed
I.	90.50	91.76	89.29	96.53	97.27	95.42
II.	91.16	92.20	90.20	98.99	99.91	98.58
III.	86.30	87.80	87.82	98.53	98.44	98.63
IV.	85.23	87.91	87.79	98.24	98.01	99.13
V.	85.50	87.28	87.41	95.62	95.66	96.21

TABLE VII

Groups	S_1			S_2		
	Extended Series	Reduced Series		Extended Series	Reduced Series	
		Calculated	Observed		Calculated	Observed
I.	94.66	94.83	94.12	98.55	98.30	96.50
II.	98.24	98.35	94.66	99.31	99.52	96.86
III.	95.64	95.46	95.52	100.46	100.18	99.86
IV.	96.01	95.97	96.46	102.01	101.85	101.77
V.	96.76	96.76	95.64	100.90	101.08	100.50

Tables V.-VII. contain the values of the thresholds for subjects I., II. and III. respectively. Each table gives these values for each of the three forms of calculation for every group

¹ For a detailed description of this form of calculation cf.: S. W. Fernberger, 'On the Relation of the Methods of Just Perceptible Differences and Constant Stimuli,' *Psychol. Rev. Monographs*, XIV., No. 4, 1913, 29-38. F. M. Urban, 'Hilfstabellen für die Konstanzmethode,' *Arch. f. d. ges. Psychol.*, XXIV., 1912, 236ff. S. W. Fernberger, 'A Simplification of the Practice of the Method of Constant Stimuli,' *Amer. Jour. of Psychol.*, XXV., 1914, 124ff.

of 100 judgments. As before, the Roman numerals of the first columns refer to the subgroups of 100 judgments. In the next three columns are found the values assumed by the threshold in the direction of decrease (S_1); the first of these refers to the extended series, the second to the reduced series calculated, and the third column to the reduced series observed. The next three columns show a similar arrangement for the values assumed by the threshold in the direction of increase (S_2).

For any one subject, these values of the thresholds are very similar on the whole, but they are by no means identical. The individual values show certain unsystematic variations, such as are generally found in a series of this sort. No regular tendencies seem to be present for the thresholds in any one set, derived from any one of the three sources. A closer scrutiny of the three thresholds for the extended and the reduced series for any one of the subjects reveals an interesting similarity among the results. Whenever a subgroup shows a variation in either direction from the preceding subgroup for the extended series, a variation in the same direction occurs for the reduced series, both calculated and observed, in corresponding subgroups. This relation does not hold absolutely for all our subjects; still in the case of subject III. (Table VII.), it holds with but a single exception (Series V, S_1 , reduced series observed).

The relations between the series become more apparent when we consider the quantities which are obtained directly from the values of the thresholds. The interval of uncertainty is defined as the distance between the two thresholds ($S_2 - S_1$). The threshold of Volkman (which is recognized as the measure of sensitivity), is one half of this value [$(S_2 - S_1)/2$]. The point of subjective equality is defined as the average or mean of the two thresholds [$(S_2 + S_1)/2$]. These three values depend upon the thresholds and are easily obtained when once the thresholds are calculated. Now a psychophysical method is essentially a prescription for the collection of data and for their evaluation, so that the result enables one to compare the sensitivity of two subjects, or that of the same subject at dif-

ferent times or under different conditions. Hence the measure of sensitivity and the point of subjective equality are the final goals of these methods; the first gives the measure of sensitivity, while the second enables one to ascertain the extent of the effect of the constant errors. Hence these quantities may be used as a basis for discussing the variations under consideration.

TABLE VIII

Groups	Interval of Uncertainty			Measure of Sensitivity			Point of Subjective Equality		
	Ex- tended Series	Reduced Series		Ex- tended Series	Reduced Series		Ex- tended Series	Reduced Series	
		Calcu- lated	Ob- served		Calcu- lated	Ob- served		Calcu- lated	Ob- served
I.	1.71	1.56	2.06	0.85	0.78	1.03	97.92	97.81	97.03
II.	0.99	0.96	1.66	0.50	0.48	0.83	97.30	97.38	97.02
III.	3.43	2.78	3.36	1.72	1.39	1.68	97.94	98.04	98.17
IV.	3.67	3.93	4.57	1.84	1.96	2.28	98.16	97.76	97.99
V.	2.57	2.66	4.24	1.28	1.33	2.12	96.96	96.95	95.99
Average...	2.47	2.38	3.18	1.24	1.19	1.59	97.66	97.59	97.24

TABLE IX

Groups	Interval of Uncertainty			Measure of Sensitivity			Point of Subjective Equality		
	Ex- tended Series	Reduced Series		Ex- tended Series	Reduced Series		Ex- tended Series	Reduced Series	
		Calcu- lated	Ob- served		Calcu- lated	Ob- served		Calcu- lated	Ob- served
I.	6.03	5.51	6.13	3.02	2.76	3.06	93.52	94.52	92.36
II.	7.83	7.71	8.38	3.92	3.86	4.19	95.08	96.06	94.39
III.	12.23	10.64	10.81	6.12	5.32	5.40	92.42	93.12	93.22
IV.	13.01	10.10	11.34	6.50	5.05	5.67	91.74	92.96	93.46
V.	10.12	8.38	8.80	5.06	4.19	4.40	90.56	91.47	91.81
Average...	9.84	8.47	9.09	4.92	4.24	4.54	92.66	93.63	93.05

TABLE X

Groups	Interval of Uncertainty			Measure of Sensitivity			Point of Subjective Equality		
	Ex- tended Series	Reduced Series		Ex- tended Series	Reduced Series		Ex- tended Series	Reduced Series	
		Calcu- lated	Ob- served		Calcu- lated	Ob- served		Calcu- lated	Ob- served
I.	3.89	3.47	2.38	1.94	1.74	1.19	96.60	96.56	95.31
II.	1.07	1.17	2.20	0.54	0.58	1.10	98.78	98.94	95.76
III.	4.82	4.72	4.34	2.41	2.36	2.17	98.05	97.82	97.69
IV.	6.00	5.88	5.31	3.00	2.94	2.66	99.01	98.91	99.12
V.	4.14	4.32	4.86	2.07	2.16	2.43	98.83	98.92	98.07
Average	3.98	3.91	3.82	1.99	1.96	1.91	98.25	98.23	97.19

Tables VIII.-X. show the values of these three quantities for subjects I., II. and III. respectively. As previously, the first columns indicate the subgroups of 100 judgments each. The next three columns give the values assumed by the interval of uncertainty for all three modes of procedure. In the first of these appear the values for the extended series. The next column contains these same values for the reduced series calculated. The third column contains the same values obtained for the reduced series observed. The next three columns contain a similar arrangement for the measure of sensitivity; and the last three, for the point of subjective equality. The bottom row of each table contains the averages of the numbers in the various columns.

Let us first compare the values obtained for the extended series with those obtained for the reduced series calculated. The results which are yielded by this comparison are very similar to the results of our former calculation.¹ If one then compare the measure of sensitivity for the different subgroups, one finds relatively great, and by no means regular, variations. The averages for subjects I. and III. are very similar indeed; the differences between the measures of sensitivity as obtained from the extended series, and those obtained from the reduced series calculated are respectively -0.05 and -0.03 grams. In the case of subject II., this difference is considerably greater, being -0.68 grams. It will be noted, however, that the measure of sensitivity of this subject is relatively very large.

When the values assumed by the measure of sensitivity for the extended series and those for the reduced series observed are compared, one finds a similar state of affairs. The values for the subgroups show greater variations here than they did when the values of the extended series and of the reduced series calculated were compared. For subject I., a comparison of the averages for the extended and for the reduced series observed reveals the fact that the variation is larger; the difference being $+0.35$ grams. For subject II., the difference between the values for the reduced series observed and that

¹ S. W. Fernberger, 'A Simplification of the Practice of the Method of Constant Stimuli,' *Amer. Jour. of Psychol.*, XXV., 1914, 125ff.

for the extended series is smaller than the calculated value in the same comparison; the difference being -0.38 . We find for subject III. again a very close approximation; the value of the measure of sensitivity for the reduced series observed being only 0.08 grams less than that for the extended series. It might be of interest to note here that subjects I. and II. were our untrained observers, while Subject III. possessed a rather high degree of training in the technique of this experiment.

When the magnitudes of the point of subjective equality for the reduced series, both calculated and observed, are compared with those for the extended series, unsystematic variations such as we should expect are again found. A closer analysis shows that there existed in addition certain systematic variations; these will appear more clearly when we consider the averages for each subject. In the case of subjects I. and III., the points of subjective equality for the reduced series, both calculated and observed, are smaller than those for the extended series. In both cases the values of the reduced series observed show greater variation from those of the extended series than do the values of the reduced series calculated. In the case of subject II., the values of both of the reduced series are larger than the value for the extended series.

It is doubtful whether the method of constant stimuli may ever be employed properly for anything except an extended study in which more or less highly trained subjects are employed. The effect of progressive practice would seem to preclude the use of this method for short series; and this effect would also prevent the application of the method for anthropometric purposes. In an extended study, it is proper that a large number of judgments be obtained from several subjects. After the results for each subject have been treated separately, it is proper that the combined results for all subjects be considered; with the exception of those which must be eliminated because of abnormal variations. For example *cf.* the case of subject IV. whose results are to be treated separately. Table XI. contains the averages of these three final values for our subjects I., II. and III. The columns give, in order, the values

obtained for the extended series, for the reduced series calculated, and for the reduced series observed, in the order mentioned. The first line contains the values for the interval of uncertainty, and the second and third lines contain respectively the values of the measure of sensitivity and of the point of subjective equality.

TABLE XI

	Extended Series	Reduced Series	
		Calculated	Observed
Interval of uncertainty	5.43	4.92	5.36
Measure of sensitivity	2.72	2.46	2.68
Point of subjective equality	96.16	96.48	95.82

Our results reveal the fact that the value of the measure of sensitivity for the extended series is slightly larger than the values for the reduced series, calculated and observed. The difference between the value of the measure of sensitivity for the extended series and that for the reduced series calculated is -0.26 grams; while the difference between the value of the measure of sensitivity for the extended series and that for the reduced series observed is only -0.04 grams. The difference between the measure of sensitivity for the extended series and the reduced series observed may be disregarded as we might expect a difference of at least 0.26 grams, since such is the difference between the values of the extended series and the reduced series calculated. It is, of course, a mere matter of chance that the value of the reduced series observed should more nearly approximate the value for the extended series than does that for the reduced series calculated. The difference between the value of the extended series and that of the reduced series calculated is slightly larger than the corresponding difference obtained in our former study.¹ But in the present case, these variations are so small that they may be entirely disregarded in an extended study.

When we consider the averages of the points of subjective equality for our three subjects, we find that the value for the reduced series calculated is greater than that for the extended

¹ S. W. Fernberger, 'A Simplification of the Practice of the Method of Constant Stimuli,' *Amer. Jour. of Psychol.*, XXV., 1914, 128.

series, the difference being $+ 0.32$ grams. This signifies that the higher values for subject II. more than cancelled the smaller values found for subjects I. and III. When we compare the value of the reduced series observed with that of the extended series, we find that the former is somewhat smaller; the difference being $- 0.34$ grams. Again we believe that variations so minute as these may be entirely disregarded.

From the above findings we conclude that an investigator using the method of constant stimuli is justified in using a less extended series of comparison stimuli than the classical series of seven pairs. For we have shown that the elimination of the two extreme intensities of the comparison stimuli does not produce any considerable effect upon the values either of the measure of sensitivity or of the point of subjective equality, when we employ a long series of experiments. Such variations as may occur seem to be of a chance character and they tend to cancel one another. Moreover, these variations are so small that they may be entirely disregarded. In our former study, we showed that the above conclusions hold from formal considerations alone; our present study indicates that the same conclusions hold upon the basis of an extended body of experimental data. The elimination of the two extreme intensities obviously reduces the time and labor required for the accumulation of the data by nearly one third. This reduction becomes all the more important when we consider that the method of constant stimuli can be employed only in an extended series of experiments.

We shall now turn to a consideration of the data which we obtained from subject IV. This subject possessed at the outset the same amount of training as subject III. A study of his reactions in the former experiment¹ reveals the fact that he did not show the variations from the normal which appeared in his reactions in the present study. Table XII. contains the observed relative frequencies of the different judgments for this subject; Table XIII. contains his values of the thresholds;

¹ The results of this study have not yet been published. The experiments were performed during the winter and spring of 1913, and in them an attempt was made to secure an introspective analysis of the comparison consciousness involved in the judgment of small differences in lifted weights.

TABLE XII

Groups	Extended Series																		Reduced Series																																																					
	84						88						92						96						100						104						108						88						92						96						100						108					
	84			88			92			96			100			104			108			88			92			96			100			108			88			92			96			100			108																							
	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.	l.	e.	h.																														
I.	0.97	0.00	0.03	0.89	0.01	0.11	0.77	0.01	0.22	0.56	0.02	0.42	0.27	0.01	0.72	0.13	0.02	0.85	0.04	0.00	0.96	0.86	0.00	0.14	0.58	0.01	0.41	0.39	0.00	0.61	0.34	0.00	0.66	0.22	0.00	0.9																																				
II.	0.98	0.00	0.02	0.92	0.00	0.08	0.72	0.00	0.28	0.68	0.00	0.32	0.38	0.00	0.62	0.23	0.00	0.77	0.09	0.00	0.91	0.92	0.00	0.08	0.57	0.00	0.43	0.42	0.00	0.58	0.23	0.00	0.77	0.18	0.00	0.																																				
III.	0.77	0.06	0.17	0.76	0.02	0.22	0.56	0.13	0.31	0.57	0.08	0.35	0.29	0.12	0.59	0.18	0.08	0.74	0.18	0.07	0.75	0.78	0.04	0.18	0.65	0.07	0.28	0.42	0.12	0.46	0.30	0.05	0.65	0.25	0.02	0.																																				
IV.	0.68	0.00	0.32	0.87	0.01	0.12	0.68	0.05	0.27	0.41	0.01	0.58	0.35	0.05	0.60	0.30	0.07	0.63	0.30	0.01	0.69	0.78	0.02	0.20	0.73	0.00	0.27	0.33	0.08	0.59	0.33	0.02	0.65	0.29	0.02	0.																																				
V.	0.76	0.06	0.18	0.84	0.04	0.12	0.64	0.05	0.31	0.42	0.07	0.51	0.38	0.01	0.61	0.20	0.02	0.78	0.23	0.06	0.71	0.73	0.04	0.23	0.67	0.05	0.28	0.43	0.06	0.51	0.29	0.06	0.65	0.20	0.06	0.																																				

and Table XIV. contains the values assumed in his case by the interval of uncertainty, by the measure of sensitivity and by the point of subjective equality for all three of our classes of comparison. These tables are arranged in the same fashion as were the corresponding ones which we have discussed above. For the first two subgroups of 100 judgments each, our objective conditions were precisely the same as those for the other observers. In his first subgroup of 100 reactions, Subject IV.

TABLE XIII

Groups	S_1			S_2		
	Extended Series	Reduced Series		Extended Series	Reduced Series	
		Calculated	Observed		Calculated	Observed
I.	96.52	96.53	95.40	96.77	96.82	95.47
II.	98.12	98.14	95.11	98.12	98.14	95.11
III.	94.96	95.15	95.43	98.06	97.81	97.18
IV.	96.41	96.70	95.82	97.79	97.92	96.66
V.	95.80	95.91	94.93	97.53	96.92	96.55

TABLE XIV

Groups	Interval of Uncertainty			Measure of Sensitivity			Point of Subjective Equality		
	Ex- tended Series	Reduced Series		Ex- tended Series	Reduced Series		Ex- tended Series	Reduced Series	
		Calcu- lated	Ob- served		Calcu- lated	Ob- served		Calcu- lated	Ob- served
I.	0.30	0.24	0.07	0.15	0.12	0.04	96.67	96.65	95.44
II.	0.00	0.00	0.00	0.00	0.00	0.00	98.12	98.14	95.11
III.	3.10	2.66	1.75	1.55	1.33	0.88	96.56	96.48	96.30
IV.	1.38	1.22	0.84	0.69	0.61	0.42	97.10	97.31	96.24
V.	1.73	1.01	1.62	0.86	0.50	0.81	96.66	96.42	95.74
Average III., IV., VI.....	2.07	1.63	1.40	1.03	0.81	0.70	96.77	96.74	96.09

gave an exceedingly small number of equality judgments. The interval of uncertainty is directly dependent upon the relative number of equality judgments, and in his first 100 judgments his values for the extended series, for the reduced series calculated and for the reduced series observed were respectively 0.30, 0.24 and 0.07 grams. In spite of these abnormal results we continued with the same objective experimental arrangement and we endeavored by means of non-suggestive questioning to ascertain the factor or factors which were responsible

for the abnormalities. The results of this questioning will be given later.

In the second subgroup of 100 experiments, we obtained the even more remarkable finding that not a single equality judgment was given during the whole 1,200 judgments which were obtained in our extended and reduced series. In this case the thresholds in the directions of increase and decrease coincide, and hence we obtain the result that the interval of uncertainty is zero, as is also the measure of sensitivity. This means that if we compare, with our standard stimulus, any varying comparison stimulus whatsoever, the latter would be judged heavier or lighter with a probability of 0.5 or over. And furthermore, this holds even though the variation be infinitely small. Such a state of affairs is obviously impossible from either theoretical or experimental considerations.

Accordingly after the first two subgroups of 200 judgments had been completed, we altered one factor in our objective experimental arrangement by reducing the interval between our different comparison stimuli from 4 to 2 grams. Hence for the last 300 judgments—the last three subgroups in the tables—Subject IV. was given an extended series of seven comparison stimuli whose intensities were 92, 94, 96, 98, 100, 102 and 104 grams; and he was given a reduced series of five stimuli whose intensities were 94, 96, 98, 100 and 102 grams. Our object in employing such a series of comparison weights was to obtain a greater frequency of equality judgments if this could be secured by means of the introduction of a greater number of stimuli within the limits of the normal interval of uncertainty. In this object we were successful in a certain measure, as appears from the size of the intervals of uncertainty for the last three subgroups of 100 judgments each. Nevertheless, when the reduced interval was employed, the values which were obtained for the measure of sensitivity were considerably smaller than the normal values obtained with the use of a four gram interval between the comparison stimuli. Furthermore, it will be observed that the relative frequencies for the different categories of judgments on the various comparison weights do not by any means conform to

the usual type of curves of the psychometric functions.¹ This is particularly true as regards the curve of the equality judgments. These then are our reasons for disregarding the data which we obtained from subject IV. in our consideration of the primary problem of this study, namely, the effect of the elimination of the two extreme intensities of the comparison stimuli from the classical series of the method of constant stimuli.

We believe that the explanation of the abnormal character of the results which we obtained from subject IV., is to be found in the subjective attitude of the observer. We instructed our observers that they should judge whether the second stimulus presented in each pair was subjectively lighter, equal or heavier than the first. On the basis of our questioning, however, we believe that this subject did not accept the above task but set for himself an *Aufgabe* of invariably detecting a difference between the stimuli which were presented to him. For this reason, he approached the problem with a different subjective attitude from that of the other subjects,—a difference in attitude which would account for his anomalous results.

Another feature becomes evident when we compare subject IV.'s interval of uncertainty for the first two subgroups with his interval of uncertainty for the last three subgroups. The fact that the employment of a two-gram interval between the comparison stimuli gave markedly greater values of the interval of uncertainty indicates clearly that the size of the interval between the comparison stimuli has a profound inverse influence upon the size of the interval of uncertainty. Our present data are not sufficiently extensive to justify us in making positive assertions regarding this inverse influence, but they do justify us in emphasizing the importance of this phenomenon as a problem for future research. If in the light of such research, this inverse influence proves to be a genuine

¹ F. M. Urban, 'The Application of Statistical Methods to the Problems of Psychophysics,' Philadelphia, 1908, 106ff. F. M. Urban, 'The Method of Constant Stimuli and its Generalizations,' *PSYCHOL. REV.*, XVII., 1910, 229-259.

finding, it must clearly be regarded as indicating an inadequacy of the method of constant stimuli.¹

The above results and discussion may be summarized as follows:

1. The subjective attitude with which the subject approaches the problem in lifted weights, or the *Aufgabe* which he sets himself, has a profound influence upon the size of his interval of uncertainty.

2. In an extended study by the method of constant stimuli, it is possible to eliminate the two extreme intensities of the comparison stimuli from the classical arrangement of seven pairs, without thereby affecting the values either of the measure of sensitivity or of the point of subjective equality to any marked extent. In a former study we showed that this statement holds upon the basis of purely formal considerations; and our present study verifies these earlier theoretical findings upon the basis of psychological experimentation. For the trained subjects the effect of the elimination of the two extreme intensities is slightly less than it is for the untrained subjects. But in either case the differences between the extended and the reduced series as regards the value of the measure of sensitivity and the point of subjective equality are so small that they may be entirely disregarded. Moreover, when we average the final values which were obtained for all of our subjects, we find that these differences tend to disappear almost entirely.

3. The elimination of the two extreme intensities of the comparison stimuli from the classical series employed with the method of constant stimuli obviously reduces the time and labor necessary for the accumulation of an adequate body of data by nearly one third.

¹ Since writing this paper, we find that the same fact has been noted by Warner Brown in the case of the ascertaining of the stimulus threshold for salt sensations. The Judgment of Very Weak Sensory Stimuli with Special Reference to the Absolute Threshold of Sensation for Common Salt. *University of California Publications in Psychology*. I., No. 3, 1914, 229-235.