JOURNAL

OF THE

ROYAL MICROSCOPICAL SOCIETY.

AUGUST 1900.

TRANSACTIONS OF THE SOCIETY.

V.—On the “Lag” in Microscopic Vision.

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(*Read 16th May, 1900.*)

THE method employed by the “man in the street” to determine the efficiency of a clock is to test its capability of keeping time; but a scientific horologist would not be satisfied with this mere time-keeping test; he would want to know the difference between the amount of energy put into the clock and that given out by the clock—in other words, the amount of energy wasted in the clock itself by friction, etc.

Example:—The work given out by a common grandfather’s clock is the wagging of a 4 lb. pendulum, 40 in. long (roughly), and the energy put into the clock is a 10 lb. weight falling 4 in. per diem.

The ratio between these quantities is therefore $\frac{10 \times 4}{4 \times 40} = \frac{1}{4}$. Compare this with a good regulator which wags a 30 lb. pendulum 40 in. long, by means of a 5 lb. weight falling 3 in. per diem; thus $\frac{5 \times 3}{30 \times 40} = \frac{1}{80}$; the regulator therefore has twenty times the efficiency of the grandfather’s clock.

Now, we microscopists have hitherto, like “men in the street,” been content to test a Microscope objective by measuring its numerical aperture, and indicating the nature of its performance by some qualifying adjective; thus, a microscopist may be heard to say that a certain $1/4$ is a beautifully corrected lens of 0.8 N.A., whereas, with regard to another lens of precisely the same power and aperture, he will not commit himself further than to describe the lens as fair, or tolerably good. The question therefore that must present itself to every one’s mind is:—Cannot we microscopists, following the example of scientific horologists, substitute, by means of a formula, definite figures for the indefinite terms applied to objectives, such as excellent, fair, tolerably good, etc.? The scope of this paper then is to show how this may be easily accomplished. The method is as follows:—First, find out the tangent of the visual angle for a minimum visible with

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the unaided eye ; this tangent, when multiplied by a million to get rid of the decimal point, will be distinguished by the letter v . Secondly find out the tangent of the similar angle with a Microscope, care being taken that the minimum visible is not the minimum resolvable with the given objective ; in other words, the power must be so adjusted that the minimum visible is within the grip of the aperture of the objective ; this tangent, when multiplied by a million, will be known by m . Thirdly, the "lag" is the difference between the quantities represented by m and v .

Of course it is necessary that the observer measuring the various objectives should determine the value of v for his own sight. Let us suppose that an observer can divide the one-hundredth of an inch at 10 in. ; then by a Microscope with a power of 100 he ought, if there is no "lag," to be able to divide the one ten-thousandth of an inch at a 10 in. projection distance. In this case v is 1000, and m also 1000, the "lag" or the difference between these values being 0. If, on the other hand, the Microscope is capable of resolving only 8000 lines to the inch with the same power, m will be 1250 ; and as v will be 1000, as before, the "lag" will be 250. In 1875, while experimenting with some telescopes, I found that if a certain grating could just be resolved by the unaided eye at a distance of say 20 yards, the same grating could always be seen with a telescope of a power, say, 10 at a distance greater than 200 yards. Similar experiments were performed with many telescopes of various powers, the powers and distances being all very accurately measured, and in every case the telescope resolved the grating at a distance greater than the naked eye distance multiplied by the telescope's power. This shows that, so far as the telescope is concerned, the "lag" becomes a negative quantity ; if then we meet with a similar experience with respect to a Microscope, we need not be surprised.

We have seen above that the "lag" in microscopic vision may be expressed by the formula $\tan M - \tan V$; it will now only be necessary to explain the practical method of using this formula for non-mathematical readers, and to give a table of values obtained from objectives tested in this manner. To find $\tan V$, set up any suitable grating, for example an ivory scale divided into fiftieths of an inch would answer the purpose, and measure the greatest distance at which the lines can just be separated ; a bright day must be chosen for performing this experiment. Suppose that one-fiftieth of an inch can just be divided at 45 in., then $\tan V$ is $1/50$ or 0.02 ; divided by 45, this equals 0.000444 . Multiplying this by one million, we obtain the value of v as 444. To find $\tan M$, divide the magnifying power by ten times the number of lines to the inch on the test-plate resolved by the Microscope. Example :—with a power of 280 the number of lines to the inch resolved is 45,000, then $\tan M$ will be 280 divided by 450,000. This is equal to 0.000622 , which multiplied by a million = $m = 622$. In this case the "lag" $m - v = 622 - 444 = +178$.

Take another objective, which with a power of 140 just resolves 35,000 lines to the inch; then 140 divided by 350,000 = 0.0004, which multiplied by a million = $m = 400$. The "lag" is now 400 - 444 or - 44, the negative sign indicating that the Microscope is doing relatively more than the unassisted eye. The objectives in the following table were tested with a wide-angled solid cone of axial illumination, the condenser was kept accurately centred, and the edge of the lamp flame strictly focussed upon the object. This is an important point, because if the condenser is racked so that the flame image is a little out of focus, a hollow annular cone would be substituted for a solid axial one, and the character of the illumination would be changed into one which would partake somewhat of the nature of oblique light with a slot; and the resolving power of the objective would consequently be increased. The column marked O.I. shows the optical index of the lens; it is found by dividing one thousand times the N.A. of the objective by its initial magnifying power. Example:— a 4/10 of N.A. 0.62 has an initial power of 33, its O.I. is therefore $\frac{620}{33} = 18.8$. The eye-pieces distinguished by numerals are compen-

sating; those by letters Huyghenian; that by XII is a twelve-power of my own construction. The column P contains the combined magnifying power of the objective and eye-piece. This, together with the N.A. and initial powers of the objectives, was measured in every instance, and was not derived from the values given in catalogues. The column t shows the number of thousands of lines per inch resolved. With regard to m , a simpler way of finding this quantity is to multiply P by 100 and divide by t . Example in the seventh line:— $\frac{12900}{25} = 516$.

The column marked "Lag" is, as we have seen above, $m - v$, and as $v = 437$ throughout this table, it therefore is $m - 437$.

A number obtained empirically is placed in the last column, which shows the order of merit of each objective's performance in resolving a certain number of thousands of lines to the inch with a certain magnifying power, and on account of its general utility may be preferred by some to the column marked "Lag." The number is obtained by dividing $10 v$ by $m - t - \text{O.I.}$ It will be conceded that if two objectives having precisely the same initial powers but different apertures resolve the same band with a low-power eye-piece, the preference ought to lie with the wider angled lens; by subtracting the O.I. from the denominator compensation is to a certain extent effected. Again, if two objectives of different powers have the same optical index, some allowance will have to be made for the objective which resolves the higher band; thus a 1 inch of 0.26 N.A. and a 1/4 of 0.95 N.A. have both an O.I. of 23.5; it would be very much more difficult for the 1/4 to resolve 60,000 lines to the inch than it would be for the 1 inch to resolve 15,000; compensation for this is effected by subtracting the number of thousand lines to the inch resolved from the denominator.

A column of the denominators $m - t - \text{O.I.}$ is given; 10 v is of course 4370 throughout the table.

Objective.	Focus.	O.I.	Eye-piece.	P.	t .	m .	$m-t$ -O.I.	"Lag."	Order of Merit.
Apochromatic	$1\frac{1}{2}$	26·3	6	35	10	350	314	-87	13·9
"	"	"	12	67	15	447	406	+10	10·8
Achromatic	$1\frac{1}{2}$	23·0	A	37	10	370	337	-67	13·0
Apochromatic	1	28·0	4	51	15	340	297	-97	14·7
"	"	"	6	70	20	350	302	-87	14·5
"	"	"	8	90	20	450	402	+13	10·9
"	"	"	12	129	25	516	463	+79	9·4
"	"	"	XII	120	25	480	427	+43	10·2
Achromatic	1	21·8	A	57	15	380	343	-57	12·7
"	"	"	B	87	20	435	393	-2	11·1
Apochromatic	$1/2$	32·0	4	70	20	350	298	-87	14·7
"	"	"	6	130	35	371	304	-66	14·4
"	"	"	8	180	40	450	378	+13	11·6
"	"	"	XII	250	50	500	418	+63	10·4
"	"	"	12	270	50	540	458	+103	9·5
" without screen	"	"	"	"	45	600	523	+163	8·4
"	"	"	18	360	55	654	567	+217	7·7
Achromatic	$1/2$	24·4	XII	290	50	580	506	+143	8·6
"	$1/2$	15·3	A	105	25	420	380	-17	11·5
Apochromatic	$1/4$	22·7	4	167	45	371	303	-66	14·4
"	"	"	6	250	55	455	377	+18	11·6
"	"	"	8	350	60	583	500	+146	8·7
Achromatic	$1/4$	17·0	A	250	50	500	433	+63	10·1
"	"	"	B	350	55	636	564	+199	7·8
Semi-apochromatic *	$1/4$	19·0	A	150	40	375	316	-62	13·8
"	"	"	B	200	50	400	331	-37	13·2
"	"	"	C	260	55	473	399	+36	11·0
Achromatic	$1/5$	15·8	A	283	55	515	444	+78	9·8
"	"	"	C	510	60	850	774	+413	5·6
Semi-apochromatic *	$1/6$	12·8	A	250	55	454	386	+17	11·3
"	"	"	B	333	60	555	482	+118	9·1
"	$1/6$	14·5	B	286	55	520	451	+83	9·7
"	"	"	XII	470	60	783	709	+346	6·2
Achromatic *	$1/6$	14·5	B	350	45	778	719	+341	6·1

* Short-tube objectives.

Looking at the figures in the Table, which deals with 14 objectives, we see that the "lag" is increased in every instance by deeper eye-pieces. In the first lens, an apochromatic of $1\frac{1}{2}$ in. focus, the "lag" is increased from -87 to $+10$ by changing the eye-piece from 6 to 12. The achromatic $1\frac{1}{2}$ with an A or 5 eye-piece does not perform so well as the apochromatic, its order of merit being $13\cdot0$ against $13\cdot9$ of the apochromatic with a 6 eye-piece. The fall in the merit of an apochromatic 1 in. with eye-pieces ranging from 4 to 12 is as much as from $14\cdot7$ to $9\cdot4$. The apochromatic $1\frac{1}{2}$ shows a similar result, viz. from $14\cdot7$ to $9\cdot5$. The XII eye-piece has an advantage of about $1\cdot0$ over the compensating. The Gifford screen also raises the order of merit about $1\cdot0$. The difference in the performance between an apochromatic $1\frac{1}{2}$ and an achromatic of nearly the same aperture in resolving the same band with the XII eye-piece, is about $2\cdot0$. An achromatic $1\frac{1}{2}$ with a very low optical index can only score $11\cdot5$ points with an A eye-piece—in other words, it is about 3 behind the apochromatic.

The apochromatic $1\frac{1}{4}$ falls off more rapidly than the $1\frac{1}{2}$, its merit with the 8 eye-piece being only $8\cdot7$; but the achromatic on the other hand is only $7\cdot8$, and that too upon a lower band. The position maintained by the semi-apochromatics in the table is most interesting. Take for example the $1\frac{1}{4}$; it begins with an order of merit only a little behind that of the apochromatic, and holds its own remarkably well under the deeper eye-pieces; it resolves 55,000 with $11\cdot0$ marks as against $11\cdot6$ for the apochromatic, and $7\cdot8$ for the achromatic. An achromatic $1\frac{1}{5}$ with some uncorrected spherical aberration falls as low as $5\cdot6$ for 60,000 lines. Two semi-apochromatic $1\frac{1}{6}$ ths come next, the one which has the lower optical index being the better corrected lens of the two. The above 13 objectives may all be described as picked lenses in their respective classes, but a bad achromatic $1\frac{1}{6}$ is put last by way of comparison; it can obtain only $6\cdot1$ marks in resolving 45,000 lines per inch.

The last point shown by this method of testing lenses is an important one, for it demonstrates that the advantage lies with short-tube objectives. It is to be regretted that the table is not as complete as it might have been, but a sufficient number of short-tube lenses were not at hand for comparison.

We see then that lenses which hitherto were thought capable of bearing the deepest eye-pieces without showing the least sign of breaking down are proved incapable of standing even medium power eye-pieces without loss; and we also learn that all super-amplification of the objective image, whether by eye-piece or tube-length, will be accompanied by a fall in the number of marks; and lastly we note the very high place taken by cheap semi-apochromatic objectives.

Assuming that there is no "lag," the power sufficient to resolve any given number of lines to the inch may be found by multiplying v by t and dividing the product by 100. Example:—What is the

power necessary, when there is no "lag," to resolve 50,000 lines per inch? Here 437 multiplied by 50 = 21,850, dividing by 100 the answer 218 is obtained

The N.A. necessary to resolve any given number of lines to the inch with a large solid axial cone and Gifford's screen, may be found by dividing t or the number of thousands of lines to the inch by 80. Example :—What is the N.A. necessary to resolve 50,000 lines per inch? Here $t = 50$, and $\frac{50}{80} = 0.625$, the N.A. required.*

The test-plate was a 12-band plate, ruled to 60,000 lines per inch, by Mr. H. J. Grayson, of Melbourne, mounted in realgar of refractive index 2.5.

* See this Journal, 1893, p. 17.