hibited when silver is ignited in the gas flame; a thick polished sheet of silver immediately becoming dull white when thus heated. Under the microscope, the metal appears swollen and warty. Where it has been exposed to the action of the inner flame along its circumference, this warty condition is visible to the naked eye. A stroke with the burnishing-stone, however, presses down the loosened particles and reproduces the original polish. This peculiar condition which the surface of silver assumes when it is ignited, is well known to silversmiths; it cannot be replaced by any etching with acids; and it must be remembered that what is dull white in silver appears grey in platinum.

If each commencement of this loosening is again destroyed, the crucibles will be preserved unaltered, otherwise they must gradually become brittle. Crucibles of the alloy of platinum and iridium are altered like those of platinum when they are ignited. It is, however, somewhat more difficult to reproduce the original polish of the metal by means of sand, as might be expected from the greater hardness of the alloy.

The sand used should be well worn. When examined under the microscope, no grain of it should exhibit sharp edges or corners; all the angles should be obtuse.—*Journ. für praktische Chemie*, lxxix, 117.

For the Journal of the Franklin Institute.

Triangular Beams. By Prof. D. WOOD.

As I find that the statements of different authors respecting the strength of triangular beams are very conflicting, I propose in this article to make an analytical investigation of the problem, and compare the results with experiment.

The problem may, for convenience, be thus stated:

A triangular prism being supported at its extremities; it is required to ascertain how great a weight it will sustain at the middle point without fracture.

But as we know that the strength of a beam depends upon its transverse sections, it will only be necessary to investigate the *resistance of a triangular section*.

To introduce the several points which I wish to discuss, I shall make extracts from various authors. I find in Olmstead's Natural Philosophy, p. 153 (or p. 107 of Prof. Snell's edition), the following statement:

"A triangular beam is twice as strong when resting on its broad base as when resting on its edge."

I would pass this statement with the remark that the deduction is founded on Galileo's theory, which is known to be false, were it not for the fact that many think that the statement is correct; also that beams made of some kinds of material are stronger in the former than in the latter position, but not for the reasons given by Olmstead.

1. Suppose the beam is made of wood.

The resistance of wood to compression is nearly the same as for ex-

tension. If it were the same, the beam would be equally strong in either position ; for the same strain comes upon the edge, whether it be up or down.

But if it resist more to tension than compression it would be stronger *when resting on its edge*, and *vice versa*. These conclusions are fully sustained by experiment. Mr. Couch made some experiments on triangular (Canadian) oak beams, fixed by one end and loaded at the other. The mean of four experiments gave,

with the angle upward, 315 lbs.
 “ downward, 348 “

(See Barlow's Strength of Materials, p. 118.)

2. Suppose the beam is made of cast iron.

Cast iron will resist about 6 times as much to compression as to tension (see Mahan's Civil Engineering, p. 82). Hence, if we suppose the yielding to be at the angle, the beam will be six times as strong when the angle is upward as when it is downward. I know of no experiments on triangular cast iron beams, but Barlow has made experiments which confirm us in our conclusion (see his Strength of Materials, p. 337 ; also, Mahan, p. 88). In a similar manner we might determine the relative strength of a beam in the two positions if made of any other material. If the beam is fixed at one extremity instead of being supported, then the preceding conclusions would be reversed.

I now propose to *find the strongest trapezoidal beam which can be cut from a given triangular one.*

To investigate this case, we will resort to the fundamental expression for the resistance of beams subjected to a transverse strain.

Let R = the ultimate resistance of a unit of section to compression or extension.

d_1 = distance of the farthest fibre from the neutral surface.

I = moment of inertia of the section about an axis through its centre of gravity and parallel to the base.

We then have for the moment of resistance

$$\frac{R I}{d_1}$$

[See Mosley's Mechanics and Engineering Eq., (637).]

Let ABC be the given triangle.

$ABED$ the required trapezoid.

Let $b = AB$. $v = DE$.

$a = CG =$ longest altitude of ABC . $w = CF$.

H = centre of gravity of the trapezoid $ABED$.

$d_1 = FH$. (2) $z = CH = d_1 + w$.

IJ is the neutral axis of the trapezoid.

To find CH , we have, from the equation of moments, origin at C ,

$$\frac{1}{2} a b \frac{2}{3} a = \frac{1}{2} v w \frac{2}{3} w + \frac{b+v}{2} (a-w) z.$$

$$(3) \quad \therefore \quad z = \frac{\frac{2}{3} a^2 b - v w^2}{(b+v)(a-w)}.$$

(4) From the similarity of triangles we have

$$v : w :: b : a \quad \therefore w = \frac{a}{b} v.$$

From (2) and (3) we have $d_i = z - w = \frac{2}{3} \frac{a^2 b - v w^2}{(b + v)(a - w)} w$;

by substituting w from (4) we have

$$(5) \quad d_i = \frac{1}{3} \frac{a}{b} \frac{2b^2 - bv - v^2}{b + v}.$$

The moment of inertia about IJ as an axis is equal to the moment of inertia about DE *minus* the area ABED $\times \overline{FH}^2$.

Let $Ff = x$.

$de = y$.

Then from the similar triangles CDE and cde we have

$$y : b :: x + w \text{ or } x + \frac{a}{b} v : a.$$

$$\therefore y = \frac{bx + av}{a}.$$

Hence, the moment of inertia about DE as an axis is

$$\begin{aligned} \int_0^{x=a-w} x^2 y dx &= \int_0^{\frac{a}{b}(b-v)} \frac{x^2}{a} (bx + av) dx = \frac{b}{a} \left[\frac{x^4}{4} \right]_0^{\frac{a}{b}(b-v)} + \left[\frac{v}{3} x^3 \right]_0^{\frac{a}{b}(b-v)} \\ (6) \quad &= \frac{a^3}{12b^3} (3b^4 - 8b^3v + 6b^2v^2 - v^4). \end{aligned}$$

Area of the trapezoid is

$$\frac{b+w}{2} (a-v) = \frac{a}{2b} (b^2 - w^2),$$

which multiplied by $FH^2 = d_i^2$ gives

$$(7) \quad \frac{1}{18} \frac{a^3}{b^3} (b^2 - v^2) \left(\frac{2b^2 - bv - v^2}{b + v} \right)^2,$$

which subtracted from (6) gives the moment of inertia about IJ; which by reduction will become

$$\begin{aligned} I &= \frac{1}{36} \frac{a^3}{b^3} \left[b^4 - v^2(8b^2 - 16bv + 9v^2) - 8 \frac{b-v}{b+v} v^4 \right] \\ (8) \quad &= \frac{1}{36} \frac{a^3}{b^3} \left[\frac{b^5 + b^4v - 8b^3v^2 + 8b^2v^3 - bv^4 - v^5}{b + v} \right]. \end{aligned}$$

If $v = 0$ this becomes $= \frac{1}{36} a^3 b$, which is the moment of inertia of a triangle about one axis passing through its centre of gravity and parallel to its base.

By substituting (5) and (8) in (1) we have

$$(9) \quad R \frac{I}{d} = \frac{1}{12} R \frac{a^3}{b^3} \left[\frac{b^5 + b^4v - 8b^3v^2 + 8b^2v^3 - bv^4 - v^5}{2b^2 - bv - v^2} \right]$$

which is to be a maximum;

∴ we have

$$(10) \quad \frac{d u}{a v} = 0 = v^6 + 2 b v^5 - 5 b^2 v^4 - 8 b^3 v^3 + 19 b^4 v^2 - 10 b^5 v + b^6.$$

By discussing this equation we find that it has three roots, each equal b , which gives a minimum: Divide Eq. (10) by $(v - b)^3$, and we have

$$(11) \quad v^3 + 5 b v^2 + 7 b^2 v - b^3 = 0$$

$$(12) \quad \text{make} \quad v = \frac{z - 5b}{3} \text{ and (11) becomes}$$

$$z^3 - 12 b^2 z - 92 b^3 = 0,$$

which solved by Cardan's formula gives

$$(13) \quad z = 5.39118 b, \quad \text{and this in (12) gives}$$

$$(14) \quad v = 0.13093 b, \quad \text{and this in (4) gives}$$

$$(14) \quad w = 0.13093 a.$$

By substituting v , Eq. (13), in (9) it becomes

$$(15) \quad R \frac{I}{d} = 0.545625 \frac{R a^2 b}{12}.$$

Making $v = 0$ in (9), we have the moment of the full triangular

$$(16) \quad \text{section} \quad = 0.5 \frac{R a^2 b}{12}.$$

Taking Eq. (16) from (15) we have

$$0.04562 \frac{R a^2 b}{12}.$$

Hence, if the angle of the prism be taken off to .13 of its depth, the prism will be $0.04562 \frac{R a^2 b}{12}$ stronger, or 1.09125 times as strong, which is a gain of over 9 per cent.

Haupt, in his valuable work on Bridge construction, p. 51, says: "It is found that the prism becomes one-thirty-seventh part stronger when the angle is taken off to one-tenth of the depth."

By making $v = .1$ in (9) it becomes

$$(17) \quad 0.54385 \frac{R a^2 b}{12},$$

which is $0.04385 \frac{R a^2 b}{12}$ stronger than the full section. Dividing this by (16) we find that it is 0.0877, or more than one-twelfth part stronger.

Barlow, in his Strength of Materials, p. 116, intimates "that it is the commonly received notion, that if the vertex, or upper edge of a triangular prismatic beam, be cut off to one-third the depth, the piece will be stronger than before."

To compare this "received notion" with theory, make $v = \frac{1}{3}$ in (9), and we have

$$(18) \quad 0.465608 \frac{R a^2 b}{12},$$

which being divided by (16) gives 0.93101, or according to theory

this trapezoid beam should be only ninety-three-hundredths as strong as the triangular one from which it is cut.

This conclusion is sustained to a remarkable degree of exactness by the experiments of Mr. Couch as given by Barlow, p. 118.

The mean of seven experiments made by Mr. Couch on the strength of triangular oak beams with the angle upward, is 306 lbs.

The mean of two on trapezoidal beams with the narrow side up, made from triangular prisms of the same size used in the preceding experiments by cutting off the angle to one-third the depth, is 284.5 lbs., which is only four-hundredths lb. less than .93 of 306 lbs.

By comparing expressions (15), (16), (17), and (18), and calling the strength of the triangular beam unity, we have the following results:

Strength of the triangular beam,	.	.	1.
Strength of the same beam if the edge be cut off to $\frac{1}{3}$ the depth,	.	.	0.93101
" " " " " 1-10 "	.	.	1.0877
" " " " " so as to give a maximum strength,	.	.	1.0912

In order to explain this apparent paradox, we must remember that the condition is not that the triangular beam shall be fractured completely through, but that it shall not be fractured at all. Now the *greatest strain* comes upon the edge, but there is only one fibre there to resist it; but after a small portion of the edge is removed there are many fibres along the line DE at the same distance from the neutral surface, each of which will sustain the same part of the *greatest strain*.

If the triangular beam were loaded so as to just commence fracturing at the edge, we might increase the load 9 per cent. and increase the fracture to only thirteen-hundredths of its depth; but if the load be increased beyond this amount, it will break the beam completely in two.

Again, Haupt says (p. 51), "It is found that the strength of the triangular prism is to that of a rectangular prism having the same base and altitude, as 339 : 1000, or nearly as 1 : 3."

The moment of resistance of the triangular section is, Eq. (16),

$$\frac{1}{24} R a^2 d.$$

The moment of resistance of a rectangular section is well known to be (found from (1)),

$$\frac{1}{6} R a^2 d.$$

Hence, theoretically, they are to each other as 1 : 4.

I will close this article by comparing the maximum trapezoidal beam with a rectangular one having the same base and altitude.

Let d = altitude = $0.87 a$, nearly;

$$\therefore a^2 = \frac{d^2}{.7569},$$

which substituted in (15) gives

$$0.72 \frac{R d^2 b}{12}.$$

Hence the resistance of the maximum trapezoidal beam is that of a rectangular one having the same base and altitude as $\frac{1}{1.2}$ of $0.72 : \frac{1}{8}$, or as 360 : 1000.

Experiments made by M. Starke, with the machine of the Polytechnic Institution of Vienna, upon various Cast Steels made at the Imperial and Royal Foundry at Reichraming in Styria.

	Breaking weight per square millimetre.	
No. 1. Common Steel of Reichraming,	92.49 kilog.	Defect in the fracture.
" "	83.29	
" "	85.39	
2 "	51.89	
" "	81.91	
" "	96.84	
3 "	112.42	Blister in surface of rupture.
" "	105.32	
" "	86.44	
4. English Steel, (known as Huntsman's,)	84.34	
" "	86.51	
" "	76.99	
5. Steel made with Tungsten,	102.26	Broke at a section greater than the minimum.
" "	109.27	
" "	117.10	

NOTE.—The kilogramme = 2.2 lbs. The millimetre = 0.0394 inch.

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Superheated Steam.

The readers of the *Journal* have no doubt read much on this subject in the last few years, and there can be no doubt that, if superheated steam is properly used, a saving of fuel may be effected. Many of the statements published, however, have made the success too great. It will be remembered that, a short time since, the *Artizan* published some very flattering accounts of the performance of the English Pacific Mail steamers, whose machinery was put in by Randolph, Elder & Co. The engines had double cylinders, and used *highly superheated steam* (450 to 500 degrees). It is now found that this high heat is very destructive to the engines, injuring the cylinders, pistons, valve faces, and valves. So great has the injury been, that they have commenced to take out and much reduce the number of superheating tubes.

B.

Artificial Making of Ice.

M. Carré takes two iron retorts strong enough to bear a pressure of 8 atmospheres; two mercury-bottles for instance. In one he places a very concentrated solution of ammonia; and then connects it by a tube with the other retort which is empty. A furnace heats the first retort, the second is buried in a vessel containing water at the temperature of