349. The Polar Circle

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Source: *The Mathematical Gazette*, Vol. 6, No. 94 (Oct., 1911), pp. 153-154

Published by: The Mathematical Association

Stable URL: <http://www.jstor.org/stable/3603254>

Accessed: 27-06-2016 02:29 UTC

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The order of rays in the second pencil is determined by the consideration that P, Q, R, S are the poles of $O_1O_2, L_1L_2, M_1M_2, N_1N_2$ as to a conic V , one of the four with respect to which U_1, U_2 are polar reciprocals.

A. C. DIXON.

Let $PQRS$ be a quadrilateral, ABC its diagonal triangle, and let Ox_1, Ox_2 be two lines conjugate for the range of conics inscribed in the quadrilateral. Let one of them, Ox_1 , meet the lines PQ, QR, RS, SP in the points p, q, r, s ; and let the other, Ox_2 , meet the lines BC, CA, AB , in the points a, β, γ .

Then will the range $(pqrs) = \text{the range } (Oa\beta\gamma)$. (Correlative of Dr. W. P. Milne's theorem, *Math. Gaz.*, Jan. 1911, p. 386.)

Of the range of conics, if we draw the two a, b , touching respectively the conjugate lines Ox_1, Ox_2 , they will have O for a common point. Let a touch the lines PQ, QR, RS, SP in the points a_p, a_q, a_r, a_s .

Then the polar of P for a passes through C . Let this polar meet Ox_1 in y . Then, since P is the pole of Ca_p , and O is the pole of Oy , therefore y is the pole of PO .

Therefore Py, PO are conjugate lines for a(1)

$\therefore$ the range $(pqrs)$ on the tangent Ox_1
 $=$ conic-pencil of points of contact (a_p, a_q, a_r, a_s)
 $=$ range on the tangent PQ
 $= (a_p QTP)$.

Now the eleven-tangent conic, which is the envelope of the polars of O for the range of conics, touches the lines BC, CA, AB, Ox_1, Ox_2 , and also touches the line Py , since $P(yOa_p a_s)$ is harmonic by (1).

Let Py meet BC in k , and CA in l .

Then the range made by the four tangents Ox_1, BC, CA, AB on the two tangents Ox_2 and Py are equicross,

$$\text{i.e. } (Oa\beta\gamma) = (yklP) = C(yklP) = (a_p QTP) = (pqrs).$$

JOHN J. MILNE.

349. [K. 2. d.] The Polar Circle.

1. In the $\triangle ABC$, orthocentre H , draw $AA_1 \perp^f$ to BC . Then, since $HA \cdot HA_1 = \text{rad.}^2$ of Polar Circle (r.c.), it follows that this circle belongs to the coaxal system, whose limiting points are A and A_1 , and whose r.a. is $B'C'$, bisecting AB, AC .

Hence, if P be any point on the p.c.,

$$PA^2 = 2 \cdot AH \cdot PM;$$

and if Q be any point whatever,

$$QA^2 - \text{Power of } Q \text{ for p.c.} = 2 \cdot AH \cdot QN;$$

$$PM, QC \text{ being } \perp^{\text{re}} \text{ on } B'C'.$$

2. I have shown elsewhere that, if Q be the centre of an in-conic, with foci S, S' , and axes $2p, 2q$,

$$\text{then } 4R \cdot QN = SA \cdot S'A.$$

$$\therefore 2 \cdot AH \cdot QN = SA \cdot S'A \cdot \cos A$$

$$= \frac{1}{2}(SA^2 + S'A^2 - 4p^2), \text{ by a well-known theorem,}$$

$$= (QA^2 - p^2 + q^2).$$

$$\therefore \text{Power of } Q \text{ for p.c.} = p^2 + q^2,$$

or the p.c. and Director Circle cut orthogonally.

[My proof (analytical) that $4R \cdot QN = SA \cdot S'A$ is somewhat long and cumbrous. I should be glad to meet with a short proof by Pure Geometry.]

W. GALLATLY.

350. [Vol. v. p. 396.] I think that Mr. Milne has made a slip in stating that Apollonius' "Inclinationes" was restored by Anderson in 1612. In *The Ladies' Diary*, 1840, Appendix, pp. 49-61, is given an "Abstract of the Principal Geometrical Writings of Alexander Anderson, of Aberdeen." The title of the first is given as follows:

Alexandri Andersoni, Aberdonensis Supplementum, Apollonii Redivivi (of Ghetaldus). *Sive Analysis Problematis hactenus desiderati ad Apollonii Pergaei doctrinam περι νέσεων*, à Marino Ghetaldo Patritio Ragusino hujusque, non ita pridem restitutum. In qua exhibetur mechanice aequalitatum tertii gradus sive solidarum, in quibus magnitudo omnino data, aequatur homogeneae sub altero tantum coefficiente ignoto. Huic subnexa est variorum problematum practice eodem autore. Paris, 1612. In 4to. Pp. 36.

In this book there is no treatment of the problem of inclinations.

R. C. ARCHIBALD.

Dr. Archibald is quite correct. I was misled by a reference in Hutton's *Mathematical Dictionary* (1796), vol. i., under the heading Apollonius, where he says, "Alex. Anderson, Supplem. Apol. Redivivi. Inclinationes. Paris, 1612, 4to." Not having the book, I was unable to observe the golden rule: "Verify your references."

J. J. MILNE.

[Barlow's *New Mathematical and Philosophical Dictionary* (1814) gives the identical reference, with the exception of "Red." for "Redivivi." W. J. G.]

351. [A. 1. a.] *Some Interesting Formulae in Arithmetical Progression.*

With the usual notation, $S_1 = \frac{n}{2}(2a + n - 1)b$.

Let $M = \frac{a+b}{2}$, and let S_2, S_3 be the sums of the squares and cubes respectively of the terms of S_1 .

$$\text{Then } S_1 = n(a-b) + \frac{n(n+1)}{2}b,$$

$$S_2 = n(a-b)^2 + n(n+1)b(a-b) + \frac{n(n+1)(2n+1)}{6}b^2,$$

$$S_3 = \left[n(a-b) + \frac{n(n+1)}{2}b \right] \left[(a-b)^2 + (n+1)b(a-b) + \frac{n(n+1)}{2}b^2 \right]$$

$$= S_1[bs_1 + a(a-b)],$$

$$\text{and } = [bn^2 + (2a-b)n][(bn^2 + 2a-bn)b + 2a(a-b)]/4.$$