XX. Hydraulic investigations, subservient to an intended Croonian Lecture on the motion of the blood

Thos Young M.D. For. Sec. R.S.

To cite this article: Thos Young M.D. For. Sec. R.S. (1809) XX. Hydraulic investigations, subservient to an intended Croonian Lecture on the motion of the blood , Philosophical Magazine Series 1, 33:130, 123-133, DOI: [10.1080/14786440908562834](https://doi.org/10.1080/14786440908562834)

To link to this article: <http://dx.doi.org/10.1080/14786440908562834>

Published online: 18 May 2009.

Submit your article to this journal [↗](#)

Article views: 4

View related articles [↗](#)

stance carefully converted into charcoal, and afterwards applied to the same purpose. The substances being so varied and numerous that contain the oxide of carbon, it could not possibly follow that the extent of their carbonaceous effects would be in the proportion of their masses under similar circumstances, nor that the charcoal or oxide that each of them afforded by distillation would bear the same relation to each other in point of purity, even where the acknowledged quantity of alloy in the state of earths, salts, &c., were the same. We therefore find, that even in the state of the raw substance equal weights or quantities of matter, calculating always upon the residuum alloys of different substances, produce results materially different, which can only be attributable to the different existing state of the oxide, or to the decomposition of the hydro-carbonate which most of them contain. The difference of the results becomes much greater when the oxide of carbon is used in the state of charcoal or coke, and the variety of the results here also obtained, where no hydro-carbonate comes into action, can only be placed to the state of oxidation of the carbon.

In a future communication I shall illustrate, by some experiments, what I have just stated.

XX. *Hydraulic Investigations, subservient to an intended Croonian Lecture on the Motion of the Blood.* By THOS. YOUNG, M.D. For. Sec. R.S.*

I. *Of the Friction and Discharge of Fluids running in Pipes, and of the Velocity of Rivers.*

HAVING lately fixed on the discussion of the nature of inflammation, for the subject of an academical exercise, I found it necessary to examine attentively the mechanical principles of the circulation of the blood, and to investigate minutely and comprehensively the motion of fluids in pipes, as affected by friction, the resistance occasioned by flexure, the laws of the propagation of an impulse through the fluid

* From Philosophical Transactions for 1808.

contained in an elastic tube, the magnitude of a pulsation in different parts of a conical vessel, and the effect of a contraction advancing progressively through the length of a given canal. The physiological application of the results of these inquiries I shall have the honour of laying before the Royal Society at a future time; but I have thought it advisable to communicate, in a separate paper, such conclusions, as may be interesting to some persons, who do not concern themselves with disquisitions of a physiological nature; and I imagine it may be as agreeable to the Society that they should be submitted at present to their consideration, as that they should be withheld until the time appointed for the delivery of the Croonian Lecture.

It has been observed by the late Professor Robison, that the comparison of the Chevalier Dubuat's calculations with his experiments is in all respects extremely satisfactory; that it exhibits a beautiful specimen of the means of expressing the general result of an extensive series of observations in an analytical formula, and that it does honour to the penetration, skill, and address of Mr. Dubuat, and of Mr. de St. Honoré, who assisted him in the construction of his expressions. I am by no means disposed to dissent from this encomium; and I agree with Professor Robison, and with all other late authors on hydraulics, in applauding the unusually accurate coincidence between these theorems and the experiments from which they were deduced. But I have already taken the liberty of remarking, in my lecture on the history of hydraulics, that the form of these expressions is by no means so convenient for practice as it might have been rendered; and they are also liable to still greater objections in particular cases, since, when the pipe is either extremely narrow, or extremely long, they become completely erroneous: for notwithstanding Mr. Dubuat seems to be of opinion, that a canal may have a finite inclination, and yet the water contained in it may remain perfectly at rest, and that no force can be sufficient to make water flow in any finite quantity through a tube less than one twenty-fifth of an inch in diameter; it can scarcely require an argument to show, that he is mistaken in both these respects.

It

It was therefore necessary for my purpose to substitute, for the formulæ of Mr. Dubuat, others of a totally different nature; and I could follow Dubuat in nothing but in his general mode of considering a part of the pressure, or of the height of a given reservoir, as employed in overcoming the friction of the pipe through which the water flows out of it; a principle, which, if not of his original invention, was certainly first reduced by him into a practical form. By comparing the experiments, which he has collected, with some of Gerstner's, and some of my own, I have ultimately discovered a formula, which appears to agree fully as well as Dubuat's with the experiments from which his rules were deduced, which accords better with Gerstner's experiments, which extends to all the extreme cases with equal accuracy, which seems to represent more simply the actual operation of the forces concerned, and which is direct in its application to practice, without the necessity of any successive approximations.

I began by examining the velocities of the water discharged through pipes of a given diameter with different degrees of pressure; and I found, that the friction could not be represented by any single power of the velocity, although it frequently approached to the proportion of that power of which the exponent is 1·8; but that it appeared to consist of two parts, the one varying simply as the velocity, the other as its square. The proportion of these parts to each other must however be considered as different in pipes of different diameters, the first part being less perceptible in very large pipes, or in rivers, but becoming greater than the second in very minute tubes; while the second also becomes greater for each given portion of the internal surface of the pipe, as the diameter is diminished.

If we express, in the first place, all the measures in French inches, calling the height employed in overcoming the friction f , the velocity in a second v , the diameter of the

pipe d , and its length l , we may make $f = a \frac{l}{d} v^2 + 2 c \frac{l}{d} v$;

for it is obvious, that the friction must be directly as the length of the pipe; and since the pressure is proportional

to

to the area of the section, and the surface producing the friction to its circumference or diameter, the relative magnitude of the friction must also be inversely as the diameter, or nearly so, as Dubuat has justly observed. We shall then

find that a must be $\cdot 0000001 \left(430 + \frac{75}{d} - \frac{1440}{d+12} - \frac{180}{d+\frac{1}{3}} \right)$,

and $c = \cdot 0000001 \left(\frac{900dd}{dd+1000} + \frac{1}{\sqrt{d}} \left(1050 + \frac{12}{d} + \frac{9}{dd} \right) \right)$.

Hence it is easy to calculate the velocity for any given pipe or river, and with any given head of water. For the height required for producing the velocity, independently of friction, is, according to Dubuat, $\frac{v^2}{478}$, or rather, as it appears from almost all the experiments which I have compared, $\frac{v^2}{550}$: and the whole height h is therefore equal to $f + \frac{v^2}{550}$,

or $h = \left(\frac{al}{d} + \frac{1}{550} \right) v^2 + \frac{2cl}{d} v$; and making $b = \frac{1}{al : d + 00182}$,

and $e = \frac{bcl}{d}$, $v^2 + 2ev = bh$, whence $v = \sqrt{(bh + e^2) - e}$.

In order to adapt this formula to the case of rivers, we must make l infinite; then b becomes $\frac{d}{al}$ and $bh = \frac{d}{a} \cdot \frac{h}{l} = \frac{ds}{a}$, s being the sine of the inclination, and d four times

the hydraulic mean depth; and since e is here $= \frac{c}{a}$, $v = \frac{\sqrt{(ads + cc) - c}}{a}$, and in most rivers, v becomes nearly $\sqrt{(20000 ds)}$.

In order to show the agreement of these formulæ with the result of observation, I have extracted, as indiscriminately and impartially as possible, forty of the experiments made and collected by Dubuat; I have added to these some of Gerstner's, with a few of my own; and I have compared the results of these experiments with Dubuat's calculations, and with my own formulæ, in separate columns. There are six of Dubuat's experiments, which he has rejected as irregular, apparently without any very sufficient reason, since he has accidentally mentioned, that some of them were made with great care: I have therefore calculated the velocities for these experiments in both ways, and compared the results in a separate table.

Tabular

Tabular Comparison of Hydraulic Experiments.

Observer.	d.	$\frac{1}{s}$	Superf. Veloc.	v.	Dub.	Log. ratio.	Y.	Log. ratio.	a	c	$\sqrt{\frac{1}{20000d}}$
DEBUAT.	262.5	35743	15.96	12.56	10.53	.0776	11.10	.0537	424	952	11.1
	258.5	6413	31.77	26.63	28.76	.033	8.02	.0221	424	952	28.3
	92.4	21827	9.61	7.01	8.38	.0773	8.11	.0619	415	913	9.3
	75.6	27648	7.27	5.07	6.55	.111	6.27	.0923	413	887	7.5
	17.6	9288		5.70	5.26	.0120	5.97	.0291	376	465	6.1
	16.4	432		32.52	31.61	.012	0.67	0.255	374	451	27.6
	11.7	1412		14.17	13.59	.0182	14.05	.0037	360	416	12.2
	9.9	427		22.37	24.37	.037	24.41	.0379	355	414	21.7
	5.8	91		27.31	27.19	.005	27.3	.0027	332	466	23.5

Observers.	d.	L.	h.	v.	Dub.	Log. ratio.	Y.	Log. ratio.	a	c
COUPLET.	18	13200	145.08	39.16	0.51	.0148	38.49	.0075	376	469
	5	34240	25.00	5.32	5.29	.0024	5.40	.0065	326	492
			16.75	4.13	4.23	.0103	4.21	.0083		
			5.58	2.01	2.25	.0490	2.01	.0000		
BOSSUT.	2.01	2160	24	24.73	24.08	.0115	24.76	.0006	287	747
			12	16.38	16.10	.0675	16.86	.0125		
		1080	24	35.77	35.10	.0082	35.05	.0089		
		360	24	58.90	58.80	.0007	56.80	.0154		
	1.33	2160	12	12.56	12.75	.0065	13.28	.0242	270	919
		1080	24	28.08	28.21	.0020	28.84	.0116		
		360	24	48.53	49.53	.0088	48.66	.0015		
	1.	600	12	22.28	21.98	.0055	22.83	.0106	259	1063
DEBUAT.			4	12.22	11.76	.0167	11.92	.0108		
		737	23.7	28.67	29.11	.0111	30.11	.0213		
			12.2	19.99	19.95	.0009	20.67	.0145		
			4.2	10.56	10.66	.0011	10.90	.0137		
		117	36	84.95	85.52	.0029	83.12	.0069		
			18	54.21	58.17	.0014	58.41	.0012		
	24167	36.25	53.25	85.77	85.20	.0029	85.71	.0003	309	2268
			41.25	73.81	73.30	.0005	74.67	.0050		
			20.17	51.96	50.14	.0153	50.87	.0093		
			5.00	23.40	23.19	.0039	23.09	.0008		
			.83	7.58	8.22	.0420	7.22	.022		
	1667	36.25	51.25	64.37	64.95	.0031	64.02	.0021	402	2827
			38.75	54.19	55.32	.0090	54.9	.0055		
			15.29	33.38	33.17	.0028	32.67	.0094		
			2.04	10.62	10.49	.0053	9.23	.0601		
	125	34.17	42.17	45.17	46.21	.0070	45.88	.0039	518	3435
			35.33	41.61	41.71	.0010	41.55	.0006		
			11.58	26.20	25.5	.0114	24.9	.0214		
			2.08	7.32	8.35	.0572	6.98	.0206		

(Mean .0178 (Mean .0169
= L. 1.042) = L. 1.040)

Observers.	d.	l.	h.	v.	Dub.	Log. rat.	Y.	Log. rat.	a.	c.
GERSTNER at 55°50' F.	2	68	10.7	24.2	23.9	.006	24.1	.002	349	2533
			7.7	21.0	19.9	.023	19.1	.042		
			4.7	15.8	14.9	.026	13.9	.056		
			1.7	7.5	8.2	.039	6.9	.036		
			.7	2.5	5.0	.301	3.4	.133		
	133	33	10.7	27.1	23.1	.061	22.5	.081	488	3559
			7.7	23.2	19.5	.077	18.5	.098		
			4.7	15.4	14.	.024	13.5	.058		
			1.7	5.6	8.1	.160	6.7	.078		
			.7	2.3	4.6	.301	3.4	.169		
	1067	33	10.7	10.0	8.9	.051	10.1	.004	975	5700
			7.7	7.2	7.9	.012	8.2	.057		
			4.7	4.5	5.6	.095	5.6	.095		
			1.7	1.5	3.1	.316	2.5	.222		
			.7	.5	1.8	.444	1.1	.342		

(Mean .129 = L.1346.098 = L.1254)

Y. at 60°.	$\frac{1}{42}$	8.50	32.4	11.40	0	∞	13.36	.032	2956	13882
	$\frac{1}{123}$	3.42	30.0	.53			.52	.008	13404	452100
		1.17	5.8	.27			.30	.046		

(Mean .029 = L.1.068)

DUBUAT.	2	255.25	36.35	86.31	84.8	.011	79.7	.035	287	747
	1	24	36.25	122.5	117.8	.018	110.8	.007	259	1063
			.7	106.45	101.1	.022	104.1	.010		
			18	81.85	82.2	.015	84.8	.000		
			9	59.25	57.5	.015	59.7	.004		
	4		27.03	118.67	111.5	.027	118.5	.000		

(Mean .017 = L.1.041.009 = L.1.022)

It appears from this comparison, that in the forty experiments extracted from the collection, which served as a basis for Dubuat's calculations, the mean error of his formula is $\frac{1}{34}$ of the whole velocity, and that of mine $\frac{1}{35}$ only; but if we omit the four experiments, in which the superficial velocity only of a river was observed, and in which I have calculated the mean velocity by Dubuat's rules, the mean error of the remaining 36 is $\frac{1}{35}$, according to my mode of calculation, and $\frac{1}{37}$ according to Mr. Dubuat's; so that, on the whole, the accuracy of the two formulæ may be considered as precisely equal with respect to these experiments. In the six experiments which Dubuat has wholly rejected, the

the mean error of his formula is about $\frac{1}{45}$, and that of mine $\frac{1}{45}$. In fifteen of Gerstner's experiments, the mean error of Dubuat's rule is one third, that of mine one fourth; and in the three experiments which I made with very fine tubes, the error of my own rules is one fifteenth of the whole, while in such cases Dubuat's formulæ completely fail. I have determined the mean error by adding together the logarithmic ratios of all the results, and dividing the sum by the number of experiments. It would be useless to seek for a much greater degree of accuracy, unless it were probable, that the errors of the experiments themselves were less than those of the calculations; but if a sufficient number of extremely accurate and frequently repeated experiments could be obtained, it would be very possible to adapt my formula still more correctly to their results.

In order to facilitate the computation, I have made a table of the coefficients a and c for the different values of d , all the measures being still expressed in French inches.

Table of Coefficients for French Inches.

d	a $\cdot 17 \times$	c $\cdot 17 \times$	d	a $\cdot 17 \times$	c $\cdot 17 \times$	d	a $\cdot 17 \times$	c $\cdot 17 \times$
∞	430	900	15	370	427	$\cdot 7$	249	1278
500	427	943	10	354	414	$\cdot 6$	248	1384
400	426	946	9	350	421	$\cdot 5$	249	1524
300	423	950	8	345	433	$\cdot 4$	257	1717
200	421	951	7	340	440	$\frac{1}{3}$	268	1895
100	416	923	6	335	462	$\cdot 3$	279	2008
90	415	911	5	325	512	$\frac{1}{4}$	303	2225
80	413	896	4	319	540	$\cdot 2$	349	2532
70	410	872	3	305	617	$\frac{1}{6}$	402	2827
60	408	840	2.5	296	687	$\cdot 15$	440	3026
50	406	792	2	288	751	$\frac{1}{7}$	458	3116
40	400	719	1.5	275	866	$\frac{1}{8}$	518	3405
30	393	618	1	259	1063	$\frac{1}{9}$	589	3693
25	387	560	$\cdot 9$	255	1123	$\cdot 1$	646	3985
20	380	492	$\cdot 8$	252	1193			

For example, in the last experiment, where d is 1, l 4, and h 27.1, we have $a \pm \cdot 0000259$, $b = \frac{1}{al : d + \cdot 00182} =$

516, $c = \cdot 0001063$, $e = bcl : d = \cdot 22$, and $v = \sqrt{(bh + e^2)} - e = 118\cdot46$, which agrees with the experiment within $\frac{1}{500}$ of the whole. I had at first employed for a the formula $\frac{430}{1 + 1} : + \frac{57}{d} + \frac{1}{6dd}$, but I found that the value, thus determined, became too great when d was about 20, and too small in some other cases. Coulomb's experiments on the friction of fluids, made by means of the torsion of wires, give about $\cdot 00014$ for the value of c , which agrees as nearly with this table, as any constant number could be expected to do. I have however reason to think, from some experiments communicated to me by Mr. Robertson Buchanan, that the value of a , for pipes about half an inch in diameter, is somewhat too small; my mode of calculation, as well as Dubuat's, giving too great a velocity in such cases.

If any person should be desirous of making use of Dubuat's formula, it would still be a great convenience to begin by determining v according to this method; then, taking $b = \frac{l}{h - v^2 : 478}$, or rather, as Langsdorf makes it, $b = \frac{l}{h - v^2 : 482}$, to proceed in calculating v by the formula $v = 148\cdot5 (\sqrt{b - \text{H. L. } \sqrt{(b + 1\cdot6)}} - \cdot 001)$, since this determination of b will, in general, be far more accurate than the simple expression $b = \frac{l + 45d}{h}$, and the continued repetition of the calculation, with approximate values of v , may thus be avoided. Sometimes, indeed, the values of v found by this repetition will constitute a diverging instead of a converging series, and in such cases we can only employ a conjectural value of v , intermediate between the two preceding ones.

Having sufficiently examined the accuracy of my formula, I shall now reduce it into English inches, and shall add a second table of the coefficients, for assisting the calculation. In this case, a becomes $\cdot 0000001 (413 + \frac{75}{d} - \frac{1440}{d + 12\cdot8} - \frac{180}{d + 355})$, $c = \cdot 0000001 (\frac{900dd}{dd + 1136} + \frac{1}{\sqrt{d}}$
(1085

$\{1085 + \frac{13.21}{d} + \frac{1.0563}{dd}\}$, and $b = \frac{1}{al : d + .00171}$, e being $\frac{bcl}{d}$, and $v = \sqrt{(bh + e^2) - e}$, or $= \sqrt{\left(\frac{ds}{a} + \frac{cc}{aa}\right) - \frac{c}{a}}$, as before; and in either case the superficial velocity of a river may be found, very nearly, by adding to the mean velocity v its square root, and the velocity at the bottom by subtracting it.

Table of Coefficients, for English Inches.

d	a $\cdot 17 \times$	c $\cdot 17 \times$	d	a $\cdot 17 \times$	c $\cdot 17 \times$	d	a $\cdot 17 \times$	c $\cdot 17 \times$
∞	413	900	15	354	430	$\cdot 7$	243	1322
500	410	944	10	339	413	$\cdot 6$	243	1433
400	409	948	9	336	421	$\cdot 5$	245	1578
300	406	951	8	331	433	$\cdot 4$	254	1779
200	404	951	7	327	449	$\frac{1}{3}$	268	1963
100	399	918	6	322	471	$\cdot 3$	280	2082
90	398	903	5	312	507	$\frac{1}{4}$	305	2307
80	396	885	4	306	556	$\cdot 2$	354	2631
70	393	860	3	292	635	$\frac{1}{6}$	409	2943
60	391	825	2.5	284	694	$\cdot 15$	447	3150
50	389	772	2	277	774	$\frac{1}{7}$	466	3251
40	383	698	1.5	266	894	$\frac{1}{8}$	528	3558
30	377	597	1	251	1099	$\frac{1}{9}$	599	3866
25	371	526	$\cdot 9$	248	1161	$\cdot 1$	657	4183
20	364	482	$\cdot 8$	245	1234			

II. Of the Resistance occasioned by Flexure in Pipes or Rivers.

Mr. Dubuat has made some experiments on the effect of the flexure of a pipe in retarding the motion of the water flowing through it; but they do not appear to be by any means sufficient to authorise the conclusions which he has drawn from them. He directs the squares of the sines of the angles of flexure to be collected into one sum, which, being multiplied by a certain constant coefficient, and by the square of the velocity, is to show the height required for overcoming the resistance. It is, however, easy to see, that such a rule must be fundamentally erroneous, and its coin-

cidence with some experiments merely accidental, since the results afforded by it must vary according to the method of stating the problem, which is entirely arbitrary. Thus it depended only on Mr. Dubuat to consider a pipe bent to an angle of 144° as consisting of a single flexure, as composed of two flexures of 72° each, or of a much greater number of smaller flexures; although the result of the experiment would only agree with the arbitrary division into two parts, which he has adopted. This difficulty is attached to every mode of computing the effect either from the squares of the sines or from the sines themselves; and the only way of avoiding it is to attend merely to the angle of flexure as expressed in degrees. It is natural to suppose, that the effect of the curvature must increase, as the curvature itself increases, and that the retardation must be inversely proportional to the radius of curvature, or very nearly so; and this supposition is sufficiently confirmed by the experiments which Mr. Dubuat has employed in support of a theory so different. It might be expected, that an equal curvature would create a greater resistance in a larger pipe than in a smaller, since the inequality in the motions of the different parts of the fluid is greater; but this circumstance does not seem to have influenced the results of the experiments made with pipes of an inch and of two inches diameter: there must also be some deviation from the general law in cases of very small pipes having a great curvature, but this deviation cannot be determined without further experiments. Of the 25 which Dubuat has made, he has rejected ten as irregular, because they do not agree with his theory: indeed four of them, which were made with a much shorter pipe than the rest, differ so manifestly from them, that they cannot be reconciled: but five others agree sufficiently, as well as all the rest, with the theory which I have here proposed, supposing the resistance to be as the angular flexure, and to increase besides almost in the same proportion as the radius of curvature diminishes, but more nearly as that power of the radius of which the index is $\frac{7}{8}$. Thus if p be the number of degrees subtended at the centre of flexure, and q the radius of curvature of the axis of the pipe in French inches, we shall

shall have $r = \frac{pv^2}{200000q}$ nearly, or, more accurately, $r = \frac{.0000045 pv^3 q^{\frac{1}{2}}}{q}$. These calculations are compared with the whole of Dubuat's experiments in the following table.

Table of Experiments on the Resistance occasioned by Flexure.

p	q	v^2	r	B.	Y. 1	Y. 2
238	3.22	15030	4.75		6.71	6.98
		11330	3.50		5.06	5.26
		7199	2.33		3.21	3.34
		3510	1.08		1.56	1.62
216	6.12	7216	2.49	2.49	2.42	2.52
144			1.50	1.66	1.61	1.67
72			.75	.83	.80	.83
196.5			1.50	1.66	1.16	1.31
147.4	.53		1.12	1.24	.87	.93
98.3			.75	.83	.58	.65
49.1			.37	.41	.29	.33
112.5			6.00		7.68	6.36
99	3.22		5.90		6.74	5.60
288		3415	1.50	1.55	1.52	1.58
288		3415	1.50	1.57	1.52	1.58
144			.75	.78	.76	.79
72	6.12		.37	.39	.38	.39
196.5			.75	.78	.55	.62
112.5			1.50		3.63	3.00
720			5.90		5.72	5.95
288	3.22	5125	5.90	5.90	5.72	5.95
		3458	1.64	1.59	1.54	1.60
		860	.41	.40	.38	.40
		821	.39	.38	.37	.38
288	4.10	3448	1.33		1.21	1.30
		7449	2.90		2.59	2.78
294.8	9.9		8.64		8.08	8.62
360	4.1					
112.5	1.1					

In the last three experiments, the diameter of the pipe was two inches. The radius of curvature is not ascertained within the tenth of an inch, as Dubuat has not mentioned the thickness of the pipes. The mean error of his formula in fifteen experiments, and of mine in twenty, is $\frac{1}{25}$ of the whole.

[To be continued.]