On Atmospheric Refraction and its Bearing on the Transmission of Electromagnetic Waves Round the Earth's Surface

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1913 Proc. Phys. Soc. London 26 318

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XXXIII. *On Atmospheric Refraction and its Bearing on the Transmission of Electromagnetic Waves Round the Earth's Surface.* By J. A. FLEMING, M.A., D.Sc., F.R.S.

RECEIVED MAY 28, 1914.

APART altogether from true diffraction or from any reflective or refractive effects due to atmospheric ionisation the normal refraction of the atmosphere due to the decreasing density of the constituent gases as we rise above the earth's surface exercises a bending effect upon rays of light, and therefore also upon electric radiation sent out from any point in various directions. If a ray of light is sent out from a source on the earth's surface, starting in a direction tangential to the terrestrial spheroid, it follows a curved path of varying curvature with concavity directed towards the earth, as it passes outwards through the atmosphere. This curvature is such that, very roughly speaking, a tangent to the ray is bent through an angle of about half a degree in its course.

Conversely, the atmospheric refraction bends incoming rays from stars or celestial bodies so that their apparent zenith distances are less than the true ones.

It becomes, therefore, an interesting matter to inquire under what conditions this ordinary atmospheric refraction taken alone could operate to carry a ray, starting from any point on the earth's surface in a tangential direction, round the earth parallel to its surface. The question is not by any means a new one, and has already been considered in connection with some astronomical problems.* It has, however, become increasingly clear of late years that the full explanation of the phenomena of long-distance radio-telegraphy, such, for instance, as the transmission of wireless signals a quarter of the way round the earth from Ireland to Argentina, is only to be found in atmospheric action on the transmitted waves. The most complete mathematical investigations, such as those of Prof. H. M. Macdonald,† March and Rybczynski,‡ show that it is not possible for true diffraction to contribute more than a certain

* See Dr. August Schmidt, "Die Strahlenbrechung auf die Sonne," Stuttgart, 1891. Also Miss Agnes Clerke, "Problems in Astrophysics," p. 165.

† "The Transmission of Electric Waves around the Earth's Surface," "Proc." Roy. Soc., A., Vol. XC., p. 50, 1914.

‡ "Ann der Phys," Vols. XXXVII. and XLI.

fraction of the total effect at the distant receiving station. On the other hand, whilst there are indications that some of the effect is propagated along or through the earth's crust and follows round any irregularities in it, it is also clear that the major part cannot arrive in this manner or it would not be so dependent upon changes in the state of the atmosphere as regards solar illumination, as we find it to be.

We are, therefore, thrown back upon the consideration of the question of the relation of the atmosphere to electric wave propagation through it. The extraordinary vagaries of long electric wave transmission and the manner in which our power of transmitting such waves varies from hour to hour and day to day through the year points to some continually fluctuating atmospheric condition as a principal factor in it. Such a condition is found in the ever-varying ionisation of the atmosphere under solar light.

Dr. W. H. Eccles has provided the foundation for a valid theory of ionic refraction and an explanation of the manner in which ionisation of the upper air can cause an electric ray to be bent round so as to follow more or less the earth's curvature in its path.*

Also he has considered the influence of what may be called the normal refraction of the atmosphere, and suggested some modes of variation of refractive index with height above the earth's surface which would tend to fulfil the requirements of circular propagation of the ray round the earth.†

The whole question of long-distance radiotelegraphy is, therefore, intimately bound up with fundamental questions of meteorology and atmospheric composition, and it may be well to refer briefly to our knowledge on the subject. A very valuable summary of information on it from the point of view of chemistry and spectroscopy was given by Sir James Dewar in his Presidential Address to the British Association at Belfast in 1902, and also in a Friday Evening Discourse at the Royal Institution on April 11, 1902, on "Problems of the Atmosphere." (See "Proc." Roy. Inst., Vol. XVII., p. 223.)

He showed that, broadly speaking, the atmosphere may be divided into two regions. First, a lower region in which winds

* See Dr. W. H. Eccles, "Proc." Roy. Soc., A., Vol. LXXXVII., p. 79, June, 1912.

† See "British Association Report," Birmingham, 1913. "Proc." Sec. G., and "The Electrician," Sept. 19th, 1913. "Atmospheric Refraction in Wireless Telegraphy."

and convective motions keep the constituents so well mixed that although there is a gradual decrease of density as we ascend the chemical composition is not very much changed, except in the percentage of water vapour and carbon dioxide.

This convective region extends to a height of 10 or 20 miles, more or less.

Above that height we pass into a second region in which convection ceases to operate and differences of density in the gaseous constituents and other more obscure actions cause a separation to take place. The denser gases oxygen and nitrogen gradually disappear at some height dependent on the temperature gradient, and above this the atmosphere consists practically of hydrogen and helium, with possibly the other more volatile atmospheric gases—neon, krypton, &c.—present in some degree.

Sir James Dewar gave at his Royal Institution lecture above mentioned diagrams representing the atmospheric composition at various heights on certain assumed temperature gradients. He stated that on any permissible temperature gradient hypothesis the atmosphere above 60 miles or so would be substantially composed of hydrogen and helium.

If we assume a homogeneous atmosphere and neglect the variation of gravity with height, and assume temperature to be constant or to decrease upwards according to some straight line law, it is a comparatively easy matter to calculate the density at any height in a column of gas in equilibrium under the forces of gravity and its own elasticity. Thus, if p_0 and q_0 are the pressure and density of the gas at the earth's surface, and p and q the same at any height h above it, and if T is the absolute temperature, g the acceleration of gravity and G the gas constant, then we have

$$p = GTq. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If we consider a horizontal slice of thickness, δh , of a vertical column of the gas of unit cross-section, then the equation of equilibrium of the element is

$$-\frac{dp}{dh}\delta h = gq\delta h, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or

$$\frac{dq}{q} = -\frac{g}{GT}dh. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If R is the mean radius of the earth, and if the mean value of gravity at the surface is 980, then $g = 980R^2/(R+h)^2$ nearly.

Since the variation of gravity is only about 5 per cent. in a height of 100 miles we make no great error in considering it as a constant. There is more difficulty in finding a function of h to express the temperature. One of the results brought to light by modern observations with sounding balloons is that the atmosphere may be divided into two regions in which the distribution of temperature is very different. In the lower region there is a continual fall in temperature up to a height of about 10 km., or 6 miles or so, at the rate of about $6^{\circ}\text{C}.$ per kilometre. Above this level the temperature becomes constant for a certain unascertainable range. Beyond that it is reasonable to suppose there is a second gradient of temperature falling towards the temperature of space. The lower region in which the first temperature gradient takes place, which is more or less identical with the region in which convection and winds keep the atmospheric gases well mixed, is now called the *troposphere*. The region of more constant temperature is called the *stratosphere*. The height at which the stratosphere is reached depends on the latitude, being lower in polar than equatorial regions.

If we consider the temperature and gravity constant at various heights the integral of equation (3) is

$$q = q_0 e^{-\frac{g}{GT}h}, \dots \dots \dots (4)$$

showing that the density decreases in accordance with an exponential law.

If we assume a linear temperature gradient of $\beta^{\circ}\text{C}.$ per kilometre, so that T is a function of the height h of the form,

$$T = T_0 \left(1 - \frac{\beta}{T_0} h \right), \dots \dots \dots (5)$$

where $\beta = 6$ nearly, and T_0 is the absolute temperature at the earth's surface, then equation (3) takes the form

$$\begin{aligned} \frac{dq}{q} &= -\frac{g}{GT_0} \frac{dh}{(1 - \beta h/T_0)} \\ &= \frac{g}{G\beta} \frac{d(1 - \beta h/T_0)}{(1 - \beta h/T_0)}. \dots \dots \dots (6) \end{aligned}$$

The integral of this is

$$q = q_0 \left(1 - \frac{\beta}{T_0} h \right)^{g/G\beta}, \dots \dots \dots (7)$$

subject to the condition that h must not be greater than T_0/β .

If β is taken as the temperature fall in absolute degrees per kilometre, then since the dimensions of g/G are the inverse of a length, and since 1 km. = 10^5 cm., we have to multiply $g/G\beta$ by 10^5 if we take the unit of length as the kilometre. Again, if we take gravity as 980 and the absolute density of air as $1/800$ at a pressure of 10^6 dynes per square centimetre, then we have

$$\frac{g}{G\beta} = \frac{980 \times T_0 \times 10^5}{800 \times \beta \times 10^6} = \frac{49}{400} \frac{T_0}{\beta},$$

the kilometre being the unit of length.

If we take the average temperature of the air at the earth's surface to be $300^\circ \text{abs.} = T_0$, and if $\beta = 6^\circ$ per kilometre then $g/G\beta = 49/8 = 6.125$. This is rather a high value for T_0 , but we take 300 to get a round number.

We have then

$$q = q_0 \left(1 - \frac{h}{50} \right)^{6.125} \dots \dots \dots (7a)$$

This last expression, which takes account of the temperature gradient, is really more simple than the exponential expression (4), but (7) is limited in range to T_0/β km., at which $q = 0$, whereas (4) gives $q = 0$ at an infinite distance. It is clear, however, that β is not constant throughout the whole height of the atmosphere.

Throughout the region of the troposphere the formula (7) gives a ratio for q/q_0 numerically less than that given by (4). Thus, if $h = 5$ km., and if T_0 and β have the values given above, we have $q/GT = 0.1225$, and (4) gives us

$$\frac{q_0}{q} = e^{0.6125} = 1.845,$$

whereas (7a) gives

$$\frac{q_0}{q} = \left(\frac{10}{9} \right)^{6.125} = 1.898.$$

If $h = 10$ km., then (4) gives the ratio $q_0/q = 3.404$, whilst (7a) gives it 3.920.

This is as it should be, because lowering the temperature causes the density to degrade more rapidly.

We can next consider the atmospheric condition above the so-called convective region. In the latter the ingredients of the atmosphere are kept so well mixed by winds that the

relative proportions will not be altered with height, although the absolute density will be reduced.

If, then, we insert in (7a) the value $q_0=0.001293$, this being the density of air at the earth's surface, it will give us the density q at any height below 10 km. fairly accurately.

Above the convective region Dalton's law will begin to assert itself, and the constituent gases will attenuate independently, each gas attenuating in density according to the law as expressed by equations (4) or (7). Since the gas constant varies inversely as the density of the gas it follows that the denser gases fade away soonest.

Thus, if we take the average temperature gradient throughout a range of 150 km. to be 2° per kilometre then in formula (7) we have to put $\beta=2$. Also $g/G\beta=q_0 14,700$; if we take the surface temperature as 300° abs. Therefore, we have

$$\frac{q}{q_0} = \left(1 - \frac{h}{150} \right)^{q_0 14,700},$$

or
$$\log_{10} \frac{q}{q_0} = q_0 14,700 \log_{10} \left(1 - \frac{h}{150} \right)$$

Consider the atmospheric composition at a height of 100 km.

Then, $1 - \frac{100}{150} = \frac{1}{3}$. But $\log_{10} 1 = 0$, and $\log_{10} 3 = 0.47712$.

Hence $\log_{10} \frac{q_0}{q} = 7013.7 q_0$.

Table I. below gives the absolute densities of various atmospheric gases at 10^6 dynes per square centimetre pressure and 0°C .

TABLE I.		
Gas.	Relative density.	Absolute density.
Hydrogen	1	0.00008837
Helium.....	2	0.00017674
Neon	10	0.0008837
Nitrogen	14	0.0012393
Oxygen	16	0.0014107
Argon.....	20	0.0017674
Krypton	40.88	0.0036125
Xenon	64	0.0056428
Air	—	0.001293

If we multiply these absolute densities by 7013.7 and find the anti-logarithms we have the values of q_0/q , that is, the ratio

of the density at the surface to the density at a height h km., as follows :—

TABLE II.

Gas	q_0/q at 100 km.
Hydrogen	4.167
Helium.....	17.36
Neon	1,570,000
Nitrogen	475,500,000
Oxygen	8,257,000,000

The above figures show without further explanation that at a height of 100 km. or 60 miles the atmosphere consists, as stated by Sir James Dewar, substantially of hydrogen and helium. At 50 km. height there is, however, a very large proportion of nitrogen and some oxygen.

Sir James Dewar recalls an observation of Pickering on the spectrum of a meteorite just entering the earth's atmosphere, which showed strongly the lines of hydrogen and helium, as confirmatory of the above conclusions as to the composition of the atmosphere at great heights.

These two formulæ (4) and (7) are therefore practically in agreement with the formulæ of Heinrich and Ferrel, from which the charts were prepared, given in Sir James Dewar's Royal Institution Discourse (*loc. cit.*). By the summation of expressions such as (7), with various values for β , we could represent the change in density due to any assigned mode of temperature variation.

We are now in a position to consider the refractive effects of the atmosphere at various heights. In gases the refractivity ($\mu-1$) is connected with the density q with fair accuracy by Gladstone and Dale's law $(\mu-1)/q=A=a$ constant. The constant A may be called Gladstone and Dale's constant. Its value for various gases are given in Table III. We can then calculate the curvature of a ray passing through the atmosphere as follows :—

The atmosphere may approximately be considered as formed of concentric layers of gas having refractive indices which decrease as the altitude increases. Let us take the centre of the earth as origin and consider the curved path of any ray travelling to or from the earth (*see* Fig. 1). Then if a tangent is drawn at any point P to this curve, and if l is the length of the perpendicular let fall from the origin on the tangent, and μ the refractive index at the point of contact of the tangent with the curve, it can easily be shown that all along the path of the ray the product $l\mu$ is constant (*see* Parkinson's "Optics," p. 109).

$(r-R)$ is the height above the earth's surface. The symbol α stands for g/GT , where g is the acceleration of gravity, G is the gas constant, and T the absolute temperature.

From (10) we have

$$\varrho = r \frac{dr}{dl} = r \frac{dr}{d\mu} \frac{d\mu}{dl}, \quad \dots \quad (11)$$

and from (8)

$$\frac{d\mu}{dl} = -\frac{C}{l^2} = -\frac{\mu^2}{C}. \quad \dots \quad (12)$$

Hence

$$\varrho = -\frac{r}{d\mu/dr} \cdot \frac{\mu^2}{C}. \quad \dots \quad (13)$$

But by (9) we have

$$\mu = 1 + Aq_0 e^{-\alpha(r-R)}. \quad \dots \quad (14)$$

Therefore

$$\frac{d\mu}{dr} = -\alpha Aq_0 e^{-\alpha(r-R)}. \quad \dots \quad (15)$$

Hence substituting (15) in (13) we have

$$\varrho = \frac{r e^{\alpha(r-R)}}{C \alpha A q_0} (1 + Aq_0 e^{-\alpha(r-R)})^2, \quad \dots \quad (16)$$

or by (9)

$$\varrho = \frac{r}{C \alpha} \frac{\mu^2}{\mu - 1} = \frac{r}{l \alpha} \frac{\mu}{\mu - 1}. \quad \dots \quad (17)$$

Now

$$\alpha = \frac{g}{GT} = \frac{g q_0}{p_0} = \frac{980 q_0}{10^6}.$$

Also q_0 the density of the air at the earth's surface under a pressure of 10^6 dynes per square centimetre is nearly equal to $1/800$. Hence, $\alpha = 98/(8 \times 10^7)$.

The dimensions of α are inversely as a length. Hence, if we take the centimetre as unit of length $1/\alpha = 8 \times 10^5$ nearly. If, however, we reckon lengths or heights in kilometres then the numerical value of $\alpha = 1/8$ nearly.

Accordingly the following simple expression gives the radius of curvature of the ray

$$\varrho = \frac{8r}{l} \frac{\mu}{\mu - 1}. \quad \dots \quad (18)$$

Consider, then, a ray starting horizontally from a point on the earth's surface. We have at that point $r=l=R$ =the earth's

radius, and $\mu=1.000294$. Hence, $\mu/(\mu-1)=10,000/3$ nearly, and the radius of curvature of the ray at starting $=80,000/3=26,666$ km., or 16,000 miles, or about four times the radius of curvature of the earth.

As the point considered is taken farther from the source the radius of curvature of the ray at that point increases. For we have

$$\varrho = \frac{r}{la} \frac{\mu}{\mu-1} = \frac{r\mu}{laAq}. \quad \dots \dots (19)$$

But μ is very nearly unity, and A and a are particular constants for each gas, whilst l is never very much greater than r , and the density q continually diminishes with increasing altitude.

Hence, ϱ rapidly increases with increase of distance of the point on the ray considered, as we take it farther from the source on the earth's surface.

Since a is equal to gq_0/p_0 , it is clear, if lengths are measured in kilometres, that $a=98q_0$. Also $\mu_0-1=Aq_0$. Accordingly, taking the source at the earth's surface, where l and r are equal, and considering the ray emitted tangentially by the earth's surface, we have

$$\varrho = \frac{\mu_0}{98A} \frac{1}{q_0^2}. \quad \dots \dots (19a)$$

But μ_0 for all gases is very nearly unity, and A has values for various gases as in Table III. below.

The constant A may be called the Gladstone and Dale constant, and q_0 is the density of the gas at 0°C . and 10^6 dynes per square centimetre pressure.

Gas.	TABLE III. Refractive index μ_0 at 0°C . and 10^6 dynes/cm. ²		$A = \frac{\mu_0 - 1}{q_0}$.
Hydrogen	1.000138		1.56
Helium.....	1.000035		0.197
Neon	1.0000687		0.0777
Nitrogen	1.000300		0.242
Oxygen	1.000272		0.193
Argon.....	1.000285		0.161
Krypton	1.000424		0.117
Xenon	1.000693		0.123
Air	1.000293		0.227

In the formula (19a), if we substitute the values for air, viz., $\mu_0=1.000293$, $A=0.227$ and $q_0=0.001293$ we find $\varrho=27,000$ km. as before, or 4.1 times the earth's radius.

Again, if we suppose the atmosphere consisted wholly of

hydrogen, then inserting in formula (19a) the values $\mu_0 = 1.000138$, $A = 1.56$ and $q_0 = 0.00008837$, we find $\rho = 880,000$ km. or 136 times the earth's radius.

If, however, the atmosphere consisted wholly of krypton, for which $\mu_0 = 1.000424$, $A = 0.117$ and $q_0 = 0.0036125$, we find $\rho = 6,682$ km., or about the same as the earth's mean radius.

Finally, if the atmosphere consisted wholly of xenon, for which $\mu_0 = 1.000693$, $A = 0.123$, $q_0 = 0.005643$, then we find $\rho = 2,768$ km. which is much less than the earth's radius.

On the assumption, therefore, above made as to uniform temperature we find the following remarkable result.

If the earth's atmosphere consisted wholly of krypton a ray of light sent out tangentially to the earth's surface would be refracted round the earth parallel to the surface, and never escape at all from the atmosphere. If, therefore, the atmosphere consisted entirely of krypton, wireless telegraphy right round the earth might be easily possible in consequence of this circular refraction of a tangentially emitted ray.

This curious result can be confirmed in another way. If we consider a plane wave travelling round the earth with wave front always vertical or in a radial direction then the condition for such a wave progress is that the velocity of the wave v at a height h above the earth's surface must be to its velocity v_0 at the surface as $h+R$ is to R , where R is the earth's mean radius. If μ and μ_0 are the refractive indices at these places then for ray transmission round the earth parallel to the surface we must have

$$\frac{h+R}{R} = \frac{v}{v_0} = \frac{\mu_0}{\mu} \quad \dots \dots \dots (20)$$

But $\mu - 1 = Aq$, hence

$$\frac{h}{R} + 1 = \frac{Aq_0 + 1}{Aq + 1} \quad \dots \dots \dots (21)$$

or

$$\frac{h}{R} = \frac{A(q_0 - q)}{Aq + 1} \quad \dots \dots \dots (22)$$

Differentiate this last equation with regard to h , and we have

$$Aq + 1 + Ah \frac{dq}{dh} = -AR \frac{dq}{dh} \quad \dots \dots \dots (23)$$

or

$$-\frac{dq}{dh} = \frac{q + A^{-1}}{h + R} \quad \dots \dots \dots (24)$$

Hence at the surface of the earth where $h=0$ and $q=q_0$ we have

$$-\frac{dq}{dh} = \frac{q+A^{-1}}{R}. \quad (25)$$

But now by equation (4) we have

$$q = q_0 e^{-\frac{g}{GT}h}. \quad (26)$$

Hence, differentiating (26) with regard to h , we have

$$-\frac{dq}{dh} = \frac{gq}{GT}. \quad (27)$$

Therefore equating (25) and (27)

$$\frac{q+A^{-1}}{R} = \frac{gq}{GT}. \quad (28)$$

Hence

$$GTq + \frac{GT}{A} = gRq. \quad (29)$$

But

$$\mu - 1 = Aq.$$

Therefore,

$$\mu - 1 = \frac{1}{\frac{Rg}{GT} - 1}. \quad (30)$$

But $g/GT = \alpha = 98A$, as already shown, and since Rg/GT is always very much greater than unity, the formula (30) is equivalent to

$$R = \frac{1}{98Aq^2}. \quad (31)$$

This is practically identical with formula (19a), when $\rho = R$. Hence (31) gives us the value of q required for circular refraction. Accordingly if the terrestrial atmosphere consisted wholly of a gas having a density about twice that of air, but the same Gladstone and Dale constant A , then the radius of curvature at starting of a ray of light sent out tangentially to the earth's surface would be identical with the curvature of the earth, and the ray would follow round the earth and never leave it.

It remains to consider the modification which will be introduced into the expression (19a) for the radius of curvature of the ray if we take into account the temperature gradient.

If in place of the expression (4) for the density at various

heights, which neglects temperature gradient, we use the expression (7) which assumes a uniform temperature gradient, and employ (7) in place of (4) in obtaining the formula for the radius of the curvature of the ray it is quite easy to show that we obtain the result

$$\varrho = \frac{r}{\alpha} \frac{\mu}{\mu - 1} \left(1 - \frac{\beta}{T_0} r - R \right), \quad . \quad . \quad . \quad (32)$$

where the letters have the same signification as in equations (8) to (19). Hence when $h = r - R = 0$ the last expression becomes identical with (19a) and also with (31).

The same expression is therefore obtained in three different ways for the radius of curvature of a horizontally emitted ray at the point of emission.

There appears to be a considerable amount of evidence that many of the spectral lines in the aurora spectrum are due to the more volatile constituents in the air. The question is discussed at length in the Presidential Address and lecture of Sir James Dewar, already mentioned. If any auroral lines due to electric discharges at a height above 40 km. or 50 km. are lines of krypton or xenon then the question presents itself: How do these dense rare gases rise to such heights? A very similar anomaly is seen in the solar atmosphere. The outer portions of the sun's atmosphere, like that of the earth, are composed principally of hydrogen and helium. But calcium, which has an atomic weight 40 is also present in the chromosphere, and no answer has yet been given to the question, by what actions an element of this molecular weight can rise so high. If one may be so bold as to hazard a rash suggestion, perhaps the neon and krypton are manufactured *in situ* by atmospheric electric discharges passing through the rarefied hydrogen, in the manner suggested by recent researches of Profs. Collie and Patterson.

Another question which suggests itself for consideration is, whether the ionisation of the upper regions of the atmosphere, upon which the actual bending of radio-telegraphic waves partly depends, is more easily effected because of the presence there of these non-valent gases neon, helium, &c. The high conductivity and small dielectric strength of these gases must be connected with ease of ionisation. It might be worth while to test the facility for ionisation of different gases under ultra-violet light.

In conclusion, the effect of a variation in the earth's diameter

may be pointed out. Referring to equation (30) since Rg/GT is always large compared with unity, the equation can be written

$$Rg = \frac{GT}{\mu_0 - 1} = \frac{GT}{Ag_0} \quad \dots \dots \dots (33)$$

But at 0°C ., $GT = 10^6/q_0$. Hence for the same temperature and gas the product Rgq_0^2 is constant.

Now gravity at the surface of a planet varies as the radius, if the mean density of the planet remains constant. Hence for the same chemical constitution and the same pressure and temperature of the atmosphere the conditions for the circular refraction of a tangential ray or its following closely round the surface will be complied with if the radius of the planet varies inversely as the surface density of the atmosphere.

If the Gladstone and Dale constant A had a constant value of 0.25 then on our earth, it would be necessary for the atmosphere to have double its present density to fulfil the condition for circular refraction. Therefore, for the same density as at present it would be sufficient if the earth had double its present radius also to fulfil it.

This shows that on a large planet such as Jupiter, having also a dense atmosphere, no ray of light emitted at or near the horizontal direction at any point could escape from the atmosphere. Also it follows that, except in so far as distant vision was hindered by absorption of light, it would be possible to see objects on the surface at a much greater angular distance than is possible on our earth.

These conclusions have merely a theoretical interest, but they show how closely the possible range of long-distance radiotelegraphy is connected both with the size of the planet and the nature of the atmosphere in which it is conducted.

Having regard to the great variations which exist in planetary atmospheres it is quite possible our earth is unique in this respect, as in many others, in being perhaps the only planet on which long-distance wireless telegraphy is possible.

ABSTRACT.

In this Paper the author considers the conditions under which true atmospheric refraction would be sufficient to carry a ray of light or electromagnetic radiation sent out horizontally from any point on the earth's surface round the earth parallel to its surface. It is now generally agreed that pure diffraction is insufficient to account for all the phenomena of long-distance wireless telegraphy, but that we

have to postulate some action of the atmosphere which tends to curve the radiation round the earth. The theory of ionic refraction has been put forward, which is based on the theoretical conclusion that in ionised air the velocity of long electric waves is increased. We know as a matter of fact that the atmosphere decreases in density as we rise upwards, and this alone produces a decrease of refractive index and an increase in velocity.

The first part of the Paper is concerned with the deduction of formulæ expressing this variation of density with heights taking into account as far as possible the known temperature variation with increase of height. It is shown that at a height of 100 km. the terrestrial atmosphere must consist substantially of hydrogen and helium. An expression is then obtained for the radius of curvature at any point of a ray of light sent out horizontally from the earth's surface, and it is shown that this radius at the starting point is given by the formula $\rho = \mu_0 / (98Aq_0^2)$, where μ_0 and q_0 are the refractive index and density at the surface of the earth, and A is the Gladstone and Dale constant for the gas which forms the atmosphere. From known values for various gases it is shown that for air ρ is four times the earth's radius, for hydrogen 136 times, and for krypton equal to the earth's radius. Accordingly if the terrestrial atmosphere consisted wholly of krypton a ray sent out horizontally would be refracted round the earth, and in such an atmosphere wireless telegraphy to the Antipodes would be possible. The above formulæ are deduced in three ways. It is also shown that for the same atmospheric density and constant A this circular refraction would result if the earth were twice its present diameter.

The question of atmospheric composition is then considered in the light of what is known about the auroral spectrum. The suggestion is made that perhaps the non-valent gases neon and krypton are manufactured at great atmospheric heights by electric discharges occurring in the rarefied hydrogen atmosphere. Also that by their ease of ionisation they contribute to produce the ionised layer demanded by the theories of Heaviside and Eccles to account for the actual achievements of long-distance wireless telegraphy.

Finally, it is suggested that our earth is perhaps unique in being the only planet on which such long-distance radiotelegraphy is possible.

DISCUSSION.

Mr. DUDDELL considered it was very difficult to follow what was going on in long-distance transmission. One difficulty was that it seemed probable that the heavier gases were absent from the upper atmosphere, yet we had to assume their presence either to get the refraction effect or the Eccles effect.

Dr. C. CHREE thought Prof. Fleming's Paper emphasised the importance of the field common to meteorology and wireless telegraphy. With regard to the relations between temperature, pressure and altitude in the atmosphere, aqueous vapour was an element which meteorologists had to take serious account of; its variability exercised a great influence on the results within 2 km. or 3 km. of the ground. With respect to the constitution of the upper atmosphere, more or less successful attempts had been made of late years to obtain samples by means of apparatus sent up by pilot balloons. On the theoretical side, there were Papers more

recent than the pioneer one by Sir James Dewar quoted by Dr. Fleming. In particular he would call attention to two Papers by Wegener in the "Physikalische Zeitschrift" for 1911. A fundamental point was whether the auroral spectrum did or did not connote a gas different from all hitherto isolated at the earth's surface. Wegener thought it did, and believed the unknown gas thus indicated to be a very light one, which at great heights was even more important than hydrogen. The fact that Prof. Störmer's recent photographic determination of auroral heights had in some cases supplied heights well over 300 km. was evidence that an atmosphere of some kind extended to a very great height. The fact emphasised by Dr. Fleming that wireless results by day and night differed markedly certainly seemed to support strongly his contention that the upper atmosphere played a most important part in the phenomenon. A similar conclusion had been drawn in the case of the ordinary diurnal variation of the elements of terrestrial magnetism, where normally changes were much larger by day than by night, and much larger in summer than winter. It had been found, however, that while the difference between midsummer and midwinter existed in high latitudes, the difference between night and day was there much reduced, even at the equinoxes. Systematic wireless observations in the Arctic or Antarctic might throw a great deal of light upon the whole question.

Prof. G. W. O. HOWE congratulated Prof. Fleming on his interesting and suggestive Paper, and pointed out that although the upper atmosphere undoubtedly had a profound effect on the transmission of electromagnetic waves over long distances, they were still uncertain to what extent long-distance radiotelegraphy would be possible without any assistance from the upper atmosphere. As successive mathematicians attacked the problem errors were discovered in the previous work and up to the present the effect of the correction seemed to have been in every case to increase the amount of energy diffracted around the globe.

Prof. C. H. LEES mentioned that in the formula $(\mu-1)/q=A$, the value assumed for μ was for sodium light, whereas the value required was for $\lambda=500$ metres.

Prof. MARCHANT mentioned that in some work recently published it was shown that the difference between the carrying power of signals by night and by day was very much greater in summer than in winter. He asked within what limits of accuracy Gladstone and Dale's law held.

Prof. FLEMING, in reply, said that Dr. Chree's remarks were of importance, and the questions raised required to be cleared up. In reply to Dr. Lees, he said that for all gases the square root of the dielectric constant for steady voltage practically agreed with the refractive index for sodium light. It would be useful to have the known facts as to the nature of the atmosphere collected for the use of wireless investigators.