COMPARATIVE ECONOMY OF IRON AND STEEL RAILS.

BY ASHBEL WELCH, C. E.

THE unit of value used in the following investigation is that of a mile of rails; without regard to how it is made up of tons per mile or rate per ton.

We shall have occasion to speak of value in each of its three ordinary senses, always distinguishing which; otherwise our results will be conflicting. The greatest difficulty met with in the study of our subject, arose from the ambiguity of this term.

The *intrinsic value* of a mile of rails is measured by their capacity for usefulness, and that, mainly by their endurance; and that depends on their material, size, form and quality. It does not vary, and it does not distinguish between present and future usefulness.

The *exchangeable value* is measured by price, which depends not only on intrinsic value, but on many other things, and varies frequently.

The *economic value*, as we shall call it, is measured by the combined capacity and opportunity for usefulness, that is, the actual usefulness where used, under actual circumstances. It is the value to the user. Future usefulness and future expenses chargeable against it, must be reduced to present equivalents. It may vary by variation of traffic, that is of opportunity for usefulness. It is this kind of value with which we have principally to do. We shall use the word in this sense when not otherwise stated.

An engineer who speaks of value, is likely to mean *intrinsic value*; a merchant, *exchangeable*; a shareholder, or his representative, *economic value*.

Those who have ruined their employers by splendid engineering, have done so because they neglected the distinction between the intrinsic and economic values of their work. They have created capacity for usefulness entirely disproportionate to the opportunity for usefulness.

That is the best engineering which accomplishes the purpose most economically. The present enquiry is therefore a proper one for engineers. They err who look only at intrinsic values.

A heavy steel rail, laid in a car shed, has four or five times the intrinsic and exchangeable value of a light iron rail; but for use in that place, scarcely any more economic value.

These values, though widely different, are related. Economic

value depends partly on intrinsic, and so in that direction is indirectly related to exchangeable value, which is also partly dependent on intrinsic. Economic value is directly affected by future prices, which control the cost of renewal or replacement.

We shall speak of the duration of rails without distinguishing between the effects of endurance, amount of traffic, weight of machinery, or speed of trains. These and other things must be carefully considered in determining the data to be used in the calculations.

Except when otherwise stated, traffic, and therefore values as well as prices, will be considered constant.

The economic value of a short-lived superstructure is most affected by endurance; that of a long-lived one, by interest, on which depends the present values of future quantities. If one mile of rails will last one month, and another ten months, on the same road, and the cost of renewal in each case is equivalent to a total loss of the rails, then, for that road, the latter is worth nearly ten times as much as the former. But if one will last ten years, till money doubles, and the other ten times as long, then the latter, for that road, is not worth quite twice as much as the former.

The present value of the second decade of the life time of a rail, is only half that of the first; of the third decade, only a quarter, and of the tenth, less than a five hundredth part of that of the first.

On a road where iron rails will last six or eight years, it is therefore of comparatively little consequence whether steel will last half a dozen, or a dozen times as long.

As the value of the rails for a particular road depends not only on their endurance, but also on the amount of traffic they are to carry in some specified time, that is, on their opportunity for usefulness, it is important to estimate correctly, not only what they are able to carry, but what there will be for them to carry. An error in estimate of traffic on any road, is also an error in estimate of value of rails for that road.

It greatly facilitates the comparison of the values of rails, or other things of different duration, with constant traffic or constant tendency to deterioration, to compare both with those that under the same circumstances will last forever. The relations of the destructibles to the indestructible are simpler than their relations to each other.

The practical question has generally been, and often continues to be, not, which is most economical for perpetuation, but whether

iron should be used first, while traffic is light and money scarce, and steel dear, and then, when worn out, be replaced by steel.

Let I be the value of a mile of iron, or other inferior or light rails for perpetuation, and J for replacement by some superior rails, that in the place to be provided for will last through the time T ; s the value of steel, steel headed, or some superior or heavier rails that will last a longer time T' ; and v the value of a mile that under the same circumstances will last forever.

Let L and L' be the losses at the times T and T' respectively for renewals; including rerolling, transportation, relaying, interruptions, repairs and interest thereon up to the time of renewal, risks and all other expenses and inconveniences; and d the decrease in economic value of the superior rails at the time when the inferior are worn out.

Let r be the rate of interest for the unit of time; $a = (1 + r)^T - 1$ the rate of accumulated interest for the time T , and a' for the time T' ; and a'' the rate for a time equal to $T' - T$.

Then supposing traffic and cost of renewals constant,

$$a v = a I + L \text{ and } a' v = a' s + L' \quad \text{Hence}$$

$$(I.) \quad v = I + \frac{L}{a} = s + \frac{L'}{a'}$$

$$(II.) \quad I = v - \frac{L}{a} = s + \frac{L'}{a'} - \frac{L}{a}$$

$$(III.) \quad s = v - \frac{L'}{a'} = I + \frac{L}{a} - \frac{L'}{a'}$$

The whole current cost for each unit of time $= r v$; which consists of two parts, the interest $r I$ or $r s$, which is paid as it accrues, and the economic depreciation $\frac{r L}{a}$ or $\frac{r L'}{a'}$ the payment of which, with the interest on it, is postponed till the rails are worn out.

The whole current cost that accrues during the lifetime of the rails is, $a v$ or $a' v$, consisting of interest $a I$ or $a' s$, and losses on renewal L or L' .

The present value of the loss L is $\frac{L}{a+1}$:

(IV.) The present value of L and its successors forever $= \frac{L}{a}$

The present value of the accumulated interest and loss at the time T is,

(V.) $\frac{aI + L}{a + 1}$, which is sometimes a convenient measure of the expensiveness of the rail.

The *physical* depreciation and consequent decrease in intrinsic value of a steel rail, when the iron rail is worn out, is,

(VI.) $\frac{T L'}{T'}$; but the *economic* depreciation and decrease in economic value, d is

$$(VII.) d = \frac{a L'}{a'} = a v - a s, \text{ supposing traffic constant.}$$

If an iron rail will last till money doubles, and a steel rail four times as long, the physical depreciation of the steel, when the iron is worn out, is one-fourth of the cost of renewal; but the economic depreciation, the decrease in value to the owner, is only one-fifteenth of that cost. For, supposing the present value of the first decade to be 8; that of the second will be 4; of the third 2; and of the fourth 1; and the sum of present values of the first three decades will be 14; that of all four decades only 15; so that the value to the owner, is reduced only one-fifteenth by a reduction of one-fourth of the duration of the rail.

If the loss on renewal is a percentage m on I or m' on s , then

$$(VIII.) I = \frac{a v}{a + m} = \frac{a s (a' + m')}{a' (a + m)} = \frac{a s}{a + m} + \frac{a m' s}{a' (a + m)}$$

Steel partly worn, that will still last just as long as new iron, is not necessarily worth the same as new iron, because worn out steel is probably worth more than worn out iron. So partly worn heavy rails that will last just as long as light new rails are worth more, because the worn out heavy are worth more than the worn out light rails.

The value of the partly worn steel is $s - d$, and the current cost for their residuary lifetime is,

$$(IX.) a'' s + L' - (a'' + 1) d.$$

If the price p of the iron is not the same as its economic value I , as compared with steel, the advantage or disadvantage of the iron is of course the difference between p and I ; and the current cost for the unit of time becomes $r p + \frac{r L}{a}$.

But p in that case cannot be substituted for 1 in the foregoing equations.

Any future changes in the prices of rails to be perpetuated are most conveniently provided for by increasing or decreasing the values of L and L' .

In comparing steel with iron to be replaced by steel, it is necessary to take the changes in price into the calculation. Let q be the expected price of steel at the time of replacement; and let b = balance of value of the worn out iron rails over the cost of removal, and of laying down the steel to replace them, together with all the attendant risks, inconveniences and expenses. Then to make iron for replacement equally advantageous with steel

$$(X.) J = s + \frac{b + d - q}{a + 1}. \text{ Or, if the price of steel is constant,}$$

$$J = \frac{a v + b}{a + 1} = \frac{a s + b + d}{a + 1}$$

If the traffic is expected to vary, the time must be cut up into periods coextensive with the lifetimes of the iron rails, throughout any one of which the traffic must be considered uniform, and the depreciations and values at the end of each period found by the preceding rules.

When the traffic varies, the depreciation in value of the steel has no relation to interest past, but only to future interest. It must be found by comparing the remaining duration of the partly worn steel under the expected circumstances, with that of new steel. Then a'' being the rate of accrued interest for a time equal to the difference between the residuary lifetime of the partly worn steel and new steel, and that of new steel put down at the same time being a' , the depreciation of the partly worn steel is

$$(XI.) d = \frac{L'}{1 + a''} - \frac{a'' L'}{a' (1 + a'')}.$$

The loss from the value of new steel on renewal being L' , that from the value of partly worn steel will be $L' - d$.

When the traffic varies, the value of v varies of course with each change.

Simple and obvious when stated, as the foregoing views and rules are, they were arrived at through calculations much more complex, and sometimes apparently conflicting.

These rules, and the tables deduced from them, do not aid in determining data, nor in any degree supersede experience and sound judgment. But as there are two chances of error, one of judgment and another of calculation, convenient formulæ reduce such chances to one; or at least, diminish the second.

In the following tables some illustrations of the applications of the formulæ, some results and some numbers useful in finding other results are given.

TABLE 1.

Quantities frequently represented in the foregoing formulæ. Steel supposed to last eight times as long as Iron. Traffic constant.						Quantities frequently represented in the foregoing formulæ. Steel supposed to last eight times as long as Iron. Traffic constant.					
T Dura'n of Iron Rails.	a or a' Rate of accrued interest.	$L = \frac{4000}{a}$ Present value of loss on renewals.	$L' = \frac{a}{5000}$ Present value of loss on renewals.	T' Corresponding dura'n of Steel Rails.	$d = \frac{a L'}{a'}$ Depreciation of Steel.	T Dura'n of Iron Rails.	a or a' Rate of accrued interest.	$L = \frac{4000}{a}$ Present value of loss on renewals.	$L' = \frac{a'}{5000}$ Present value of loss on renewals.	T' Corresponding dura'n of Steel Rails.	$d = \frac{a L'}{a'}$ Depreciation of Steel.
1	0.071+	56,330	70,425	8	484	54	40.07	100	125		
2	0.147+	27,210	34,012	16	366	56	46.13	87	109		
3	0.229+	17,470	21,837	24	271	60	61.06	66	82		
4	0.317—	12,620	15,775	32	197	64	80.72	50	62		
5	0.411+	9,732	12,165	40	140	66	92.77	43	54		
6	0.511+	7,828	9,785	48	98	70	122.5	33	41		
7	0.619—	6,462	8,077	56	67	72	140.7	28	35		
8	0.734	5,450	6,812	64	45	78	213.1	19	24		
9	0.858	4,662	5,827	72	30	80	244.7	16	20		
10	0.990	4,041	5,051	80	20	88	425.0	9	11		
11	1.132	3,534	4,417	88	12	90	487.9	8	10		
12	1.283	3,118	3,897	96	8	96	737.7	5	6		
14	1.620	2,469	3,080			100	971.8	4	5		
16	2.007	1,993	2,491								
18	2.450	1,633	2,041								
20	2.959	1,389	1,736								
24	4.214	949	1,186								
30	6.878	581	726								
32	8.040	497	621								
36	10.90	367	459								
40	14.68	272	340								
42	16.99	235	294								
48	26.18	153	191								
50	30.19	132	165								

TABLE 2.

Values of one mile of Rails to be equally economical.					
L assumed at 4000, and					
L' " " 5000.					
T Dura'n of Iron.	I = value of Iron for perpetuation. $I = \frac{v-L}{a}$	J = value of Iron for replacement. $b = 2500$ $J = \frac{a v + b}{a + 1}$	T' Dura'n of Steel.	s Value of Steel.	V = value of Indestructible Rails.
1	—39,528	3449	8	10,000	16,812
2	—14,719	3780	16	10,000	12,491
3	— 6,284	4126	24	10,000	11,186
4	— 1,999	4462	32	10,000	10,621
5	,608	4784	40	10,000	10,340
6	2,368	5101	48	10,000	10,191
7	3,647	5409	56	10,000	10,109
8	4,612	5701	64	10,000	10,062
9	5,373	5980	72	10,000	10,035
10	5,979	6241	80	10,000	10,020
11	6,477	6488	88	10,000	10,011
12	6,888	6718	96	10,000	10,006

TABLE 3.

Annual current cost of one mile of rails. Rails of equal economy.						
Traffic constant.						
L = 4000. L' = 5000.						
T	T'	r v	r I	$\frac{r L}{a}$	r s	$\frac{r L'}{a'}$
1	8	1194			710	484
2	16	887			710	177
3	24	794			710	84
4	32	754			710	44
5	40	734	48	691	710	24
6	48	724	168	556	710	14
7	56	718	259	459	710	8
8	64	714	327	387	710	4
9	72	712	382	330	710	2
10	80	711	425	286	710	1
11	88	710	460	250	710	0
12	96	710	489	221	710	0

TABLE 4.

Comparative economy of Steel Rails and Iron Rails for replacement, taking into account changes in price from 1867 to 1870.

J , value of iron; s , value of steel; q , price of steel when iron replaced; b , value of old rails over expenses; d , depreciation of steel; a , accrued interest; p , price of iron.

$$J = s + \frac{d + b - q}{a + 1}$$

In 1867, steel cost.....	\$15,000	Iron,.....	\$8,500
“ 1868, “ “	14,000	“ \$7,600, $b =$	3,500
“ 1869, “ “	12,000	“ 7,600, $b =$	3,500
“ 1870, “ “	10,000	“ 7,400, $b =$	3,000
After 1870, “ “	9,000	 $b =$	2,500

1867			1869		
Dura'n of Iron.	$J =$ value of iron for replacement.	$p - J =$ Advantage of steel.	Dura'n of Iron.	$J =$ value of Iron for replacement.	$p - J =$ Advantage of Steel.
1	5650	2850	1	5917	1683
2	7912	588	2	6655	945
3	9526	—1026	3	6983	667
4	10213	—1713	4	7213	387
5	10491	—1991	5	7491	109
6	10763	—2263	6	7763	—163
8	11277	—2777	8	8277	—677
10	11743	—3243	10	8743	—1143
12	12157	—3657	12	9157	—1557

1868			1870		
Dura'n of Iron.	$J =$ value of iron for replacement.	$p - J =$ Advantage of steel.	Dura'n of Iron.	$J =$ value of Iron for replacement.	$p - J =$ Advantage of Steel.
1	6517	1083	1	4384	3016
2	8219	—619	2	4655	2745
3	8933	—1333	3	4933	2467
4	9213	—1613	4	5213	2187
5	9491	—1891	5	5491	1909
6	9763	—2163	6	5763	1637
8	10277	—2677	8	6277	1123
10	10743	—3143	10	6743	657
12	11157	—3557	12	7157	243

The values of iron for replacement, in the foregoing table, are different from those in Table 2, Col. 3, for the reason that in the latter the price of steel is considered constant, while in this table it is constantly declining.

In these tables the losses preceding and attendant on renewal of iron rails, are assumed at \$4000. With light rails and not very heavy traffic this is ample to cover all inconveniences. With heavy rails and frequent trains to be interrupted, the cost and inconveni-

ence will be much more. The loss on renewal of steel is assumed to be \$5000. That will depend partly on the cost of new steel, and that on the import duties, which are not yet settled. We have assumed that it will be \$9000 per mile, which contemplates an advance in duties sufficient to counterbalance the decline in premium on gold. The value of worn out steel is as yet quite uncertain. These, like all other data, must be ascertained for each case, and corrected on new information.

The percentage of difference between \$4000 and any other value of L , will also be the percentage of difference between the numbers in third column of table I., and the numbers deduced from such other value.

It is obvious from inspection of table IV., that iron rails to be replaced by steel have heretofore been found more economical than steel; provided, that those laid in 1867 would last two or three years, those laid in 1868, a year or two, and those laid in 1869 half a dozen years.

But in 1870 it is equally obvious that steel is more economical than iron that will last even ten or a dozen years, if our data for the future are at all correct.

As a good iron rail will last several times as long as a poor one, and there is no standard of quality, it is impossible to say how much longer steel will last than iron. It will last more than twenty times as long as much of the iron laid during the past ten years. In the tables it is assumed to last eight times as long. On most roads, for the reasons already given, this question is of but little importance.

If the cost of renewal or replacement is a fixed sum, then the difference between the values of iron and steel is also a fixed sum; and there should always be the same difference between the prices. If the cost of renewal is a percentage, the values do and the prices should differ by a percentage.

These formulæ may be used in comparing the ultimate economy of bridges, buildings, vessels, and other things of different cost and durability used for the same purposes.

If single headed rails cost \$7500 and last five years, and double headed, with the chairs they set on, \$10,000, and last twice as long, and if the expenses of renewals are \$4000 and \$5000 respectively, then $p = 7500$. $s = 10000$. $L = 4000$. $L' = 5000$. $\alpha = 0.411$ and

$a' = 0.99$ and (neglecting the cost of reversal and constant cost of wedging the double headed).

$$I = s + \frac{L'}{a'} - \frac{L}{a} = 10000 + \frac{5000}{0.99} - \frac{4000}{0.411} = 10000 + 5051 - 9732 = 5319$$

So that the double headed has the advantage of $p - I = 7500 - 5319 = 2181$ over the single headed for perpetuation.

Suppose that a wooden bridge will cost \$10,000, and last sixteen years; and that the repairs, insurance, watching, and all expenses on it, with the interest on those expenses up to the time of renewal, and the cost of removal and the inconveniences, all together, amount to \$5000, over the value of the old materials; and that a permanent bridge can then be built at the same cost as at first. Then $I = 10000$. $L = 10000 + 5000 = 15000$, and $a = 2$. Substituting these in the equation $v = I + \frac{L}{a}$, we have $10000 + \frac{15000}{2} =$

17500, equal to the value of a bridge that will last forever.

When the business is very heavy, the inconvenience of renewals, and the risks from fire or accident of temporary structures, are of course controlling considerations. If the interruption caused by the burning of the wooden bridge would involve a loss of \$50,000, and the annual chance of burning is one in a hundred, then the annual risk of \$500 capitalized, adds \$7042 to the difference between the values, making that of the indestructible bridge \$24,542.

As traffic generally increases faster than is expected, (though profits do not) steel rails and permanent structures become more advantageous than the calculations. They also have the advantage of safely allowing increased weights and speeds, which may become important.

On the other hand, where safety is not involved, and interruption would not cause serious loss, and especially where, (as happens so often in this country,) changes of route or plan are liable to be made, and where calculation makes the ultimate economy nearly equal, it is best to adopt the cheaper rail or structure. This is especially the case in station buildings and shops. Dead capital and outlay for a future generation better able to help itself than we are to help it, are thus avoided.

Our calculations are made on interest at 7 per cent., compounded semi-annually. Few railroad companies borrow at lower rates, many at much higher. Of course the calculations for each road must be based on the actual rate.

The writer having had much occasion to deal with these questions, has arranged the foregoing rules and tables for his own convenience. He presents them for publication, hoping they may sometimes abridge the labor of others who have to deal with the same questions.

Comparisons such as that of the ultimate economy of the wooden and stone bridges are often judged of in this country, though perhaps not often formally made. But an eminent European Engineer told the writer, some years ago, that the idea of such a comparison was entirely new to him.

Printers' Copying Ink.—The want of a method by which the press-copying of printing could be as easily and efficiently accomplished as that of written matter has been so universally felt, that the advent of any process claiming to perform the work will be looked upon with interest by all who make constant use of printed blanks, of which they wish to retain a press copy.

Mr. McIlvain's Printers' Copying Ink, which, as the name indicates, is intended to accomplish this result, consists of a number of soluble colors, prepared to be used in printing, by a process upon which he has obtained a patent. If we may judge from press copies in our possession, the process of Mr. McIlvain leaves little or nothing more to be desired in this direction. The colors are in some variety (black, purple, red, green and blue), and with each of them a satisfactory transfer can be made.

The fact that the copy is a complete duplicate of the original, in printed as well as in written matter, infinitely increases the value of the former, and is sufficient to set aside any doubts which might otherwise arise concerning the authenticity or exactness of the transaction involved.

We understand that Mr. McIlvain's invention has already been considerably and favorably introduced. The Pennsylvania Railroad Company, The Empire Transportation Company, M. Baird, and others having adopted it to print their forms, &c.

Lime Lights in the Caisson of the East River Bridge.—We learn from Mr. C. H. Stoddard, Secretary of the New York Oxygen Gas Manufacturing Co., that they have contracted to furnish oxygen for six limelights to be burned day and night in the six compartments of the huge caisson whose structure and launching we described on p. 223 of our last volume.