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“ High-Speed Motors.”¹

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MANY members of The Institution of Civil Engineers may recollect the time when the science of thermo-dynamics was all but unknown, and when the principle of the conservation of energy, if it was accepted by men of science, was certainly not appreciated by the practical engineer.

In those days a steam-pressure of 5 to 7 lbs. above the atmosphere was considered high pressure; and one of the usual and necessary fittings of a steam-boiler was a vacuum-valve, or safety-valve, opening inwards, to prevent collapse of the boiler by excess of external pressure. In those days long connecting-rods were considered essential; it was believed by many that a short connecting-rod involved actual loss of power, not merely on account of friction due to lateral strain, but also on account of mere obliquity in the action of the rod. There was also a prejudice in favour of a very moderate speed of piston, which, it was thought, should not exceed a rate of 220 to 250 feet per minute. As the steam-engine, in its infancy, was chiefly employed for pumping, it was natural to make it with a beam, applying the piston at one end and the pump-bucket at the other. The same form was generally adopted in engines employed for giving rotation to a shaft.

Even in marine-engines a modified form of the beam-engine was universal, the beam being in this case divided into two side-levers, which were mounted at a low level in order to avoid top-hammer, and to communicate motion in a convenient manner to the paddle-shaft overhead. For a long period marine-engineers in America adhered to the early form of engine with its beam oscillating high above the vessels' deck. When the screw-propeller began to be adopted, the slowly moving side-lever engine was for some time

¹ The discussion upon this Paper was taken together with that upon the following one.

employed, and gearing was used to multiply the speed as required for the propeller-shaft.

Meanwhile, in America engineers had been gradually raising working-pressures, at the risk of explosions, which were frequent and serious. The Cornish engineers had also been moving in the same direction, and they almost startled the engineering world by the results which they obtained by the use of high initial pressures and large ranges of expansion.

In the locomotive, where simplicity and compactness are essential, the direct-acting form of engine was at once adopted and worked with considerable steam-pressure; and the invention of the tubular-boiler, furnishing a very large heating-surface in a compact and strong form, enabled engineers to command high pressures without great danger.

For a long time it was considered necessary to balance, or approximately balance, the moving parts; and indeed, when very low pressures were employed, approximate counterpoise of the moving weights was essential. By the use of horizontal-engines, as in the locomotive, this was rendered unnecessary, and this arrangement soon came to be adopted for marine-engines working screw-propellers. In these it was peculiarly applicable, as it kept the weights low, and brought the power to bear directly on the propeller-shaft at the required low level. It was also well adapted to the very short strokes employed to obtain considerable velocity of rotation without material increase of the piston-speed.

So far, in the history of steam-motors, there was comparatively little progress in respect of speed, though there was a great advance in the direction of high pressure and considerable expansion, as well as in the simplification of the mechanism, and in the removal of those racking strains which manifest themselves in the beam-engine. The marine engine-shaft seldom made more than 70 or 80 revolutions per minute, and a locomotive driving-wheel, 6 feet in diameter, had not to make 250 revolutions per minute for a speed of 50 miles an hour.

It was not until centrifugal-machines, torpedoes, and, most of all, dynamo-electric machines, had to be worked, that a demand arose for very highly-speeded motors suited for direct-driving, so as to avoid the noise and wear of toothed-gearing, or the slip and strain of driving-belts. Much has of late been done in the contrivance of highly-speeded prime-movers; but much still remains to be done, and it is worthy of consideration as to how invention should be best directed for providing high-speed motors, such as are now so largely required for many purposes.

At first, some form of rotary-engine would seem to commend itself as being peculiarly applicable where great speed of rotation is required. There is something captivating in the idea of obtaining rotation from the continuous uniform flow of a fluid under pressure. Accordingly many engineers have exercised a great amount of ingenuity in devising various forms of rotary-engines. Among others, the Author yielded to the fascination. When he had succeeded in devising a rotary-engine and getting it to work fairly, his former master, the late Mr. Joshua Field, Past-President Inst. C.E., pointed out to him that natural movements, such as walking, flying, and swimming, are reciprocatory, and that, though it may be convenient to apply power by rotation, as by the revolving shaft, the paddle or screw, it is on the whole better to obtain power by reciprocation, as in the action of the muscles of the animal frame.

Very few engines are purely rotary; most of those that are so called involve intermittence or reciprocation in some of their movements. They usually have slides or flaps that move to and fro, so as to act either as abutments to the working pressure, or as pistons moving through part of a revolution in obedience to that pressure. In some cases these parts have their movements so arranged that one part acts for a time as an abutment, remaining stationary or nearly stationary, while another is propelled by the working fluid. Almost all such engines present great difficulties in their packings, and are subject to considerable inequalities in the wear of their rubbing surfaces.

Among engines of a purely rotary character may be mentioned the reaction-wheel, probably one of the earliest forms of apparatus in which an elastic fluid, such as steam, was used to produce continuous rotation. Where very great speed of rotation and extreme simplicity of structure are demanded without regard to economy of fuel, this form of engine may be serviceable. But, when it is considered that nothing short of infinite velocity of rotation suffices to bring out full efficiency, the reaction-wheel cannot be commended for many practical purposes.

The turbine is another kind of purely rotary-engine. But, in this apparatus, the extremely high velocity of rotation necessary to realize even moderate efficiency from a current of steam, forms an objection that cannot easily be got over. It would probably be possible to obtain considerable reduction of speed by employing steam of great density, and causing it to act on a series of turbine blades, giving out part of its velocity of flow to each blade, or set of blades, in succession. Here again, however, the engineer is met

by a very important element of loss, the friction of the fluid on the surfaces of the blades, and of the stationary guides that must be employed to give direction to the operating currents. This view seems to be borne out by the results of experiments made with a species of steam-turbine. The apparatus was constructed somewhat like the liquid brake-dynamometer, invented by the late Mr. W. Froude, M. Inst. C.E., but it was arranged to act in an inverse manner. In Mr. Froude's brake there is a wheel and a casing, both made with what may be termed ratchet-cells, in which liquid is put into rapid vortical movement by the revolution of the wheel, while the casing remains at rest. In the apparatus as modified to form a steam-turbine, jets of high-pressure steam were directed into some of the ratchet-cells, so that the steam moving vortically from cell to cell, gave a very rapid rotary motion to the wheel. With steam at a pressure of 30 lbs. per square inch above the atmospheric pressure, the highest efficiency was obtained when the wheel, which had a diameter of 6 inches, made 12,000 revolutions per minute. The quantity of steam used was ascertained by condensing the exhaust and measuring the water of condensation. It was nearly double the quantity required in an ordinary non-condensing engine to produce the same amount of power, a result by no means encouraging. Greater success is said to have attended the working of a multiple steam-turbine recently invented by the Hon. C. A. Parsons, Assoc. M. Inst. C.E.

In the reaction-wheel and the turbine, the action is due to the conversion of velocity into pressure. In a third form of purely rotary-engine, which acts tolerably well as a pump or blower, but which does not seem to have met with success as a steam-engine, the fluid acts by pressure simply. In this case, a pair of toothed-wheels, geared together, are fitted within a casing. Steam, admitted on one side of the casing, in its effort to pass to the other side, presses on the outer teeth of both wheels, but only on an inner tooth of one of them, as this tooth for a time shrouds a tooth of the other wheel. There is thus an unbalanced pressure on the area of the side of a tooth, causing the wheels to rotate. As the wheels must accurately fit the sides of the casing, and as these teeth must accurately fit each other, and also fit the ends of the casing, there is great difficulty in making or packing such an engine so as to prevent leakage, especially after it has been subject to a little wear. It presents, moreover, the great disadvantage that the steam does not act expansively, but flows through the casing at its full pressure, not expanding until after it has performed its work. Many different forms of gearing have

been tried in engines of this kind, but none of them appear to have been successful.

Another rotary-engine, the Dudgeon-engine, also consists of a pair of toothed-wheels geared together; but in this case, the steam is admitted at the ends of the teeth, into the small clearance spaces that successively present themselves to the admission ports. These ports are so situated that the steam does not enter a clearance space until it has just passed the centre line joining the axes of the two wheels. The steam, expanding in this space, acts on the excess of area presented on the side of the more forward of the two teeth, over that presented on the side of the more backward tooth, and so gives propulsion to the wheels. The great difficulty in constructing such an engine, and maintaining it in efficient condition, lies in the accurate shaping of the teeth, for each tooth, although it touches the neighbouring tooth only along a line of contact, has to maintain that contact during engagement so completely as to prevent leakage of steam from the space in which it is expanding.

Almost all other so-called rotary-engines, such as the disk-engine, and those of similar type, involve dead-points, intermittencies, or reciprocations, which take away from their purely rotary character. In some, several cylinders are mounted on a revolving boss, having their pistons acting against an abutment eccentric, or inclined, to the axis around which the cylinders revolve. Such engines are really of a reciprocating kind, differing from ordinary reciprocating-engines only in the particular mechanism employed for converting the to-and-fro movements of their pistons into continuous rotation.

It may be said generally, that no good method has been discovered for utilizing the expansive force of an elastic fluid such as steam for the production of motive power, except by means of mechanism involving intermittence or reciprocation. This almost follows as a physical necessity; it is difficult to conceive how the expansive force of a fluid could be made to give movement to a solid body otherwise than by the dilatation of a cavity into which a charge of the fluid is admitted; this charge, by its effort to occupy an increased space, moving a boundary of the cavity containing it. Thus, in an ordinary steam-engine, the charge of steam admitted to a portion of the capacity of the cylinder, moves the piston enlarging that capacity. In like manner, in almost all known forms of rotary-engines, a rotating blade, or flap, performs the part of a piston, being moved onwards in a circular path by the pressure of the expanding charge. This kind of action necessarily

involves intermittence; in one phase of the movement there is admission of the charge and a cut-off; during another phase there is dilatation in an expanding space; and in a third phase there is discharge of the expanded fluid, the space that was occupied by it being again contracted to a condition ready for the reception of a succeeding charge. The alternate expansion and contraction of the space in which the fluid acts may be effected in either of two ways. In that which may be considered the rotary way, it is generally done by giving the space a form eccentric or oblique to the axis of rotation, so that alteration of its capacity is effected in a continuous manner, undergoing enlargement generally during one-half of each revolution, and becoming contracted during the other half-revolution. Or the alteration of space may be effected by reciprocation of a piston as in an ordinary steam-engine, this reciprocating-movement being by means of a crank, or some equivalent mechanism, converted into rotation.

In the class of reciprocating rotary-engines, there is one which has been carefully worked out by the Hons. R. C. and C. A. Parsons. In this engine two pairs of cylinders, having their axes at right-angles to each other, are fixed in a frame which revolves round a trunnion. Their piston-rods, which are jointed to opposite cranks on a shaft eccentric to the trunnion, cause this shaft to revolve at double the angular velocity of the cylinders themselves. This engine has met with considerable success when worked at high speed.

The subject of rotary-engines cannot be dismissed without reference to two ingenious forms of comparatively recent date.

In one of these, invented by Mr. Tower,¹ two revolving shafts are inclined to one another, and connected by a universal-joint. The gimbal-ring of the joint is filled up so as to form a complete disk or circular diaphragm, and the arms by which the shafts are jointed to this diaphragm are also filled up so as to form two semicircular blades. The central disk and the two blades are enclosed within a spherical shell, in which they can revolve. They divide the interior of the shell into four separate cavities, each of which, during one-half revolution of the shafts, expands from a minimum to a maximum of capacity, and during one-half revolution again becomes contracted to a minimum, thus operating as the equivalent of a cylinder and piston. By the revolution of ports opening into these four spaces, round stationary ports in the bosses of the casing, the admission and emission of steam are alter-

¹ Institution of Mechanical Engineers. Proceedings, 1885, p. 96.

nated. The difficulties of packing and lubricating the working parts are said to have been fairly overcome by the makers of these engines, which give good results in respect of efficiency. The four spaces within the shell act as four single-acting cylinders with pistons would act, one pair of them working a crank at right-angles to the crank worked by the other pair. Thus there is great evenness of movement, and within a small compass there is capacity for a considerable amount of power.

In the other engine to which reference is now made, the invention of Mr. Fielding, there are also two revolving shafts inclined to one another, and connected by a universal-joint. The gimbal-ring of this joint carries on each side of it a pair of pistons, which work in cavities that are not properly cylinders, but are segments of bent pipe, each being, as it were, a cylinder formed round an axis, which, instead of being straight, is a circular arc struck from the intersection of the axes of the two shafts as a centre. Each of the four pistons moves to-and-fro in this circular path. Here there are the equivalents of four single-acting cylinders and pistons, acting on cranks at right-angles to each other, and so producing evenness of action. The packing in this engine has the merit of simplicity, consisting of ordinary spring-rings held in grooves in the pistons. Both the Tower and the Fielding engines, owing to their compactness and the even character of their action, seem well adapted to operate as high-speed motors for driving electric-machines and for suchlike work.

After this necessarily brief review of several of the principal types of rotary-engines, it may not be unprofitable to consider in some detail, certain conditions inherent to reciprocating-engines, which at first would seem to affect them unfavourably, especially when worked at high speeds.

One rather formidable objection to a reciprocating-engine, when it is driven at great velocity, consists in the shock and noise resulting from the change, twice in every revolution, of the strain on the working joints. These joints, by which the connecting-rod is linked to the piston and to the crank-pin, and also the bearings of the crank-shaft, cannot be absolute fits, but must have a certain amount of slackness, and the sudden change at each half-revolution from a pull to a push, and from a push to a pull, causes a blow which is both disagreeable and detrimental. Mr. Brotherhood was one of the first to direct particular attention to this, and to provide against the evil in his three-cylinder engine by making each cylinder single-acting. The joints of each connecting-rod

are thus always subjected to strain in only one direction, that of thrust. In the forward-stroke of the piston it is pressed by the steam, and in the backward-stroke it is forced back by the crank in opposition to its friction, and to such back-pressure as the exhaust may exert upon it. These engines have succeeded well as high-speed motors, and have been extensively used for driving the propellers of torpedoes as compressed-air engines, and for driving electric-machines as steam-engines. Of late, Mr. Brotherhood and other makers have been constructing three-cylinder engines with duplicate cylinders to act as compound-engines.

Another condition inherent to reciprocating-engines, which would appear to affect them unfavourably, might under proper adjustment be made to contribute largely to that evenness of action which is especially desirable in high-speed motors. This condition has been touched upon in some treatises, but it has not received that degree of attention which its importance demands. It is therefore proposed to consider it in some detail, as its discussion will probably suggest to engineers a line of thought that may be useful, especially when they are engaged in designing engines suited for high speeds.

In a reciprocating-engine, a certain mass of matter, consisting of the piston and its rod, the cross-head or slipper, and the connecting-rod, has to be put in motion at the beginning of every stroke; it has next to be accelerated until nearly the middle of the stroke, and it has then to be retarded until it is finally brought to rest at the end of the stroke. On the principle of the conservation of energy, this alternate acceleration and retardation can neither add to, nor subtract from, the total power developed during the stroke. The effect, however, at different parts of the stroke is of a very important character, especially when high initial pressure and a large range of expansion are adopted. At first, when the steam-pressure is high, a certain amount of its propelling effect is taken up in mere acceleration of the reciprocating-mass. At last, when the steam-pressure is greatly reduced by expansion, its propelling effect is largely added to by retardation of the reciprocating-mass. Thus the inertia of this mass operates as a very effective fly-wheel or regulator, subtracting from the excess of power at one part of the stroke, and adding to the defect of power at another part of the stroke. An analytical investigation of this action is given in an Appendix to the Paper. Here it may suffice to indicate results with reference to a few diagrams which show these in graphic form.

In order to simplify the inquiry, the case will first be considered
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of a connecting-rod of infinite length always moving parallel to itself, or, what amounts to the same thing, where the crank-pin is engaged in a slot of a cross-head perpendicular to the line of the stroke. In Fig. 1, CP being the position of the crank when the piston has made the part AR of its stroke, let an ordinate QR be taken to represent the pressure per square inch of the piston-area, which is then required to effect acceleration of the reciprocating-mass. It will be found that the summits of all ordinates such as RQ are in a straight line DCE , passing through the crank-centre C , and inclined to the line of stroke AB at an angle depending on the weight and velocity of the reciprocating-mass, and the length of the crank-radius CP . The accelerating pressure AD is greatest at the beginning of the stroke; it gradually diminishes until the

FIG. 1.

middle of the stroke, when the piston has attained its maximum velocity, at which point the pressure becomes nil. The retarding pressure then gradually increases until it attains its maximum BE at the end of the stroke. All pressures acting from A to C may be considered positive, the ordinates representing them being drawn above the centre-line to indicate positive inertia or accelerating force absorbed. All pressures from C to B may be considered negative, the ordinates representing them being drawn below the centre-line to indicate negative inertia, or retarding force given back. The area of the triangle DAC above the centre-line represents the amount of work done by the engine in effecting the acceleration of the reciprocating-mass. The area of the triangle EBC below the centre-line represents the amount of work given back to the engine by the retardation of the reciprocating-mass.

cating-mass. These two areas are obviously equal, that is to say, there is no addition to nor subtraction from the total of the engine-work throughout the stroke caused by reciprocation; but there is a very sensible absorption of work during the first half of the stroke, and an equivalent return during the last half of the stroke. The effect of this absorption and return of work, in assisting to give evenness of action to the engine, may be best seen by drawing on the base-line AB the outline of a theoretical indicator-diagram $F G H$, showing a cut-off at G , and expansion from G to H . It is then observed that the virtual base is not the horizontal line AB , but the inclined line DE . At the beginning of the stroke, although AF represents the actual pressure on the piston, yet, since AD is that part of it which is absorbed in accelerating the reciprocating-mass, the difference DF is the pressure effective for external work. Again, at the end of the stroke, where the steam-pressure is only BH , since an additional pressure BE is given out in retarding the reciprocating-mass, the effective pressure for external work is EH , the sum of the two ordinates. And, in like manner, throughout the stroke, the pressure available for doing the external work of the engine is measured at any point by the height of the indicator-ordinate above the virtual base-line DE .

A line $K M L$ may be drawn parallel to DE , and averaging the indicator-area. The area of the inclined parallelogram $D K L E$, which is the same as the figure $D F G H E$, or as the indicator-figure $A F G H B$, represents the total external work; but the parallelogram $D K L E$ represents its distribution throughout the stroke when the inertia of the reciprocating-parts is taken into account. Obviously the inequalities that are left for the fly-wheel to subdue are the parts immediately above and below the inclined line $K M L$. The area $K F G M$ above the line is a measure of the amount of impulse given to the fly-wheel by excess of pressure over that which is required for the engine and its work. The equal area $H M L$ below the line $K L$ is a measure of the amount of impulse received back from the fly-wheel, to make up for defect of effective pressure available for moving the engine and performing its work.

When there is a connecting-rod of finite length, such, for instance, as three times the crank-radius, the inertia-diagram is no longer bounded by a straight line such as DE in Fig. 1; it is a curve of a somewhat complex character, as to which the Appendix is referred to. Fig. 2 illustrates the form of the diagram when the connecting-rod is thrice the length of the crank. In this case the line, instead of passing through the centre C , is a curve cutting

the base-line at a point N corresponding with that part of the stroke where the velocity of the piston is at its maximum, the connecting-rod being then nearly tangential to the crank-circle. The ordinates are figured corresponding to every 30° of the half-revolution of the crank, in terms of lbs. per square inch, it being assumed that the crank-radius is 1 foot, that the reciprocating-parts have a weight equivalent to 1 lb. on every square inch of the piston-area, and that the speed of the engine is at the rate of 1 revolution per second. In order to suit different conditions as to the crank-radius and the weight and speed of the reciprocating-parts, so long as the ratio of the connecting-rod to the crank remains the same, 3 to 1, the Diagram Fig. 2 has only to be drawn to a different vertical scale, all the ordinates being increased or diminished in the same proportion.¹ This proportion is

FIG. 2.

determined by a simple rule, which may be thus stated: Multiply together the crank-radius in feet, the weight of the reciprocating-mass estimated in lbs. per square inch of piston-area, and the square of the number of revolutions per second. The product is a coefficient by which each of the ordinates of the inertia-diagram has to be multiplied in each particular case.

If, for example, the weight of the piston, piston-rod, slipper or cross-head and connecting-rod be such as to amount to 2 lbs. per square inch of piston-area; if the crank-radius be 9 inches or $\frac{3}{4}$ foot, and if the engine makes 4 revolutions per second, then $\frac{3}{4} \times 2 \times 4^2 = 24$ is the coefficient by which, in this particular case, each of the ordinates of the curve in Fig. 2 has to be multiplied in order

¹ The left-hand side of Fig. 2 has been foreshortened to admit of the Fig. coming within the width of the page.

to give the correct form of the inertia-diagram. Fig. 3 shows the curve thus computed applied to an indicator-diagram produced by steam at considerable initial pressure, cut-off at about one-third of the stroke, and expanding during the remainder of the stroke. The shaded portion of the figure represents the distribution of pressure throughout the stroke, after subtraction and addition of the pressures due to inertia; and thus it appears that the effective pressure available for external work is not very greatly varied throughout the stroke, notwithstanding the great alteration of the steam-pressure on the piston.

This illustration will suffice to show how, by a proper adjustment of the reciprocating-mass, the crank-radius and the speed, to the initial pressure, the cut-off and the expansion of the steam, the

FIG. 3.

inertia of the moving parts may be utilized in equalizing the action of a rapidly-moving engine. Thus it appears that reciprocation, so far from being objectionable, is of great and positive advantage, the alternate acceleration and retardation of the reciprocating-mass fitting admirably in with, and to a great extent compensating, the variations of pressure shown on the indicator-diagram.

The inertia-curve, illustrated by Figs. 2 and 3, is, as stated above, that which results from the use of a connecting-rod three times the length of the crank-radius. Where the connecting-rod is longer, the curve approaches more nearly to the straight line D C E in Fig. 1. With a shorter connecting-rod the curve departs farther from the straight line, and might be made to correspond more nearly with the indicator-curve than that which appears in

Figs. 2 and 3. If, for instance, the proportion of the connecting-rod to the crank were 2 to 1 instead of 3 to 1, the initial ordinate A D would be increased by about one-eighth in height, and the final ordinate B E would be reduced by about one-fourth in depth. Generally if c express the ratio of the connecting-rod to the crank, the initial ordinate A D is proportional to $(1 + \frac{1}{c})$, and the final ordinate B E is proportional to $(-1 + \frac{1}{c})$. Thus, for different proportions of connecting-rods to cranks, the initial and final ordinates are as follow :—

c , Ratio of connecting-rod to crank	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	∞
Initial ordinate A D . .	1.500	1.400	1.333	1.286	1.250	1.00
Final „ B E . .	-0.500	-0.600	-0.667	-0.714	-0.750	-1.00

A very short connecting-rod is objectionable on account of the great lateral strain thrown by it on the slipper or crosshead-guides; but even this disadvantage may be more than made up in high-speed motors by the great equalization of the effective pressure throughout the stroke, resulting from the parallelism of the inertia-curve to that drawn by the indicator, this parallelism being increased by the use of a short connecting-rod.

In what has preceded on the subject of reciprocation, it has been assumed that the reciprocating-mass moves horizontally, or that, if it moves vertically, it is balanced. Should it not be balanced, the weight, when it is acting favourably, adds to the area of the diagram by transferring the inertia-curve that forms its base a little lower down, and when it is acting unfavourably it reduces the area by transferring the base a little upwards. When the moving mass is light as compared with the acting pressures, the variation of the up- and the down-strokes, due to weight, may be disregarded. But when, in order to take full advantage of inertia as an equalizing agent, the weight of the reciprocating-mass is largely increased, the alternation of upward and downward movement causes an inequality, which may be serious in a motor applied to such a purpose as that of driving electric-machinery, where great uniformity of action is desirable and even essential.

In some reciprocating-engines, which are said to have succeeded well as electric-motors, there are two vertical single-acting cylinders having their pistons connected to opposite cranks on a shaft below

them. In this case the reciprocating-weights are completely balanced, and the strain at the joints is always in one direction, that of thrust. Moreover, as only one of the pistons acts at one time, it has to deal with the inertia, not only of itself and its connections, but also with that of the other piston and its connections, so that practically the reciprocating-mass acting by inertia is always doubled. The principal objection to this form of engine is that there is nothing except the fly-wheel to get over the dead-points that occur twice in every revolution. In order to avoid the inequalities of movement resulting from the recurrence of these dead-points, probably the best form of engine would be an arrangement of three vertical single-acting cylinders having their pistons connected to a three-throw crank below. This arrangement, while it would provide well for continuous action, and would also satisfy the conditions of balancing the moving parts and of maintaining strain in one direction at the joints, would not utilize to the full extent the equalizing effect of inertia, as may readily be seen by constructing a diagram showing the simultaneous action of the three sets of reciprocating-parts.

If, in what has preceded, the equalizing effect of inertia has been correctly traced, a question may arise as to whether there is any advantage in working engines in the compound way. It is undoubtedly the fact that, when high-pressure steam is admitted in small quantity into a cylinder of large area, and then expanded throughout the greater part of the stroke, the variation of pressure, if there were nothing to compensate it, would be very objectionable. If the moving parts were light, there would at first be a severe shock, and at last a considerable drag. It has, therefore, generally been preferred to employ two cylinders of different dimensions, directing the high-pressure steam first on a piston of small area, and then, after a certain amount of expansion, directing the steam at reduced pressure on a piston of large area. This arrangement involves almost double work in respect of cylinders slides, pistons and their connections, and necessarily adds to the amount of cooling-surface and to the risks of leakage. If, instead of providing cylinders in duplicate, a considerable addition were made to the weight of the reciprocating-parts for one cylinder, the inertia of the increased mass would to a great extent neutralize the inequality of the acting pressure. The best way of accomplishing this would be to increase the mass of the piston, being the part that is directly and immediately acted on by the steam-pressure. With a very heavy piston to be put in motion, accelerated, retarded and brought to rest in each stroke, no shock

would be occasioned by the impetus of the high-pressure steam, and no drag from the great fall of pressure. The inertia of the piston itself might be made sufficient to produce complete equalization of effective pressure throughout the stroke, by first absorbing the excess of pressure for its acceleration, and afterwards restoring the equivalent by its retardation.

In the form of compound-engine largely used for marine propulsion, where the high-pressure and the low-pressure pistons act on cranks at right-angles to one another, there is considerable equalization of acting-pressure throughout each revolution, but the inertia has comparatively little equalizing influence. It may be open to discussion whether, by the use of two equal cylinders of mean area, each provided with a very heavy piston, so that full advantage might be taken of inertia as an equalizing agent, results as good as those obtained from the compound-engine might not follow.

Within the limits of a Paper like the present, it has not been possible to do more than take a cursory view of the subject of high-speed motors, which is a large one, involving many questions that ought to be dealt with in detail. The intention of the Author has been chiefly to direct attention to a few leading points, which seemed to him to demand more consideration than has generally been given to them. On these points certain conclusions present themselves, which may be thus summarised :—

1. For high-speed motors, engines of the rotary kind present no special advantages as compared with direct-acting reciprocating-engines.

2. Reciprocation, instead of being an evil, may, by proper adjustment of mass and speed, be made a very effective equalizing agent, particularly when there is great variation in the acting pressure.

3. Single action is to be preferred in high-speed motors in order to avoid the shocks resulting from alternations of strain at the working joints.

The Paper is accompanied by three diagrams, from which the Figs. in the text have been engraved.

APPENDIX.

NOTE AS TO INERTIA OF THE RECIPROCATING-MASS IN A RECIPROCATING-ENGINE.

Referring to Fig. 1:—

Let $r =$ crank-radius C P;

a = uniform angular velocity of crank;

t = time from beginning of stroke at A until the crank-pin reaches P;

$s = A R$, the portion of stroke performed in time t ;

$i = Q R$, the ordinate representing accelerating-pressure at time t .

Then $at = AP$, the arc traversed by the crank in time t .

[illegible]

[illegible]

$$\frac{d^2 s}{dt^2} = a^2 r \cos at = \text{accelerating force} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

As the mass accelerated is constant, the pressure i producing acceleration is proportional to the accelerating-force ;

hence $i : r \cos at (4)$

$$\text{or } QR : RC \dots\dots\dots (5)$$

Hence all the ordinates such as Q R are bounded by an inclined straight line D C E.

All the ordinates on the one side of the centre, which are proportional to cosines of arcs from 0° to 90° are positive, indicating acceleration. The ordinates on the other side of the centre, which are proportional to cosines of arcs from 90° to 180° are negative, indicating retardation.

Referring to Fig. 2 and retaining the above notation—

Let $c =$ the ratio of the connecting-rod to the crank-radius,

so that $cr = \text{length of the connecting-rod,}$

then $TV = TA + AC - CX - VX$,

or

$$\begin{aligned} s &= cr + r - r \cos at - \sqrt{c^2 r^2 - r^2 \sin^2 at} \\ &= r \{ c + 1 - \cos at - \sqrt{c^2 - \sin^2 at} \}. \end{aligned}$$

$$\frac{ds}{dt} = ar \left\{ \sin at + \frac{\sin at \cos at}{\sqrt{c^2 - \sin^2 at}} \right\} \dots \dots \dots (7)$$

$$\begin{aligned} \frac{d^2 s}{dt^2} &= a^2 r \left\{ \cos at + \frac{\cos^2 at - \sin^2 at}{\sqrt{c^2 - \sin^2 at}} + \frac{\sin^2 at \cos^2 at}{(c^2 - \sin^2 at)^{\frac{3}{2}}} \right. \\ &= a^2 r \left\{ \cos at + \frac{c^2 (\cos^2 at - \sin^2 at) + \sin^4 at}{(c^2 - \sin^2 at)^{\frac{3}{2}}} \right\} \dots \dots (8) \end{aligned}$$

In the case where c is infinite, the fraction in (8) vanishes, and the equation becomes simply $\frac{d^2 s}{dt^2} = \alpha^2 r \cos at$, as already appears in (3).

When $at = 0^\circ$, in which case $\sin at = 0$ and $\cos at = 1$, the quantity within brackets is $1 + \frac{1}{c}$, which, multiplied by a proper coefficient (as hereafter computed), gives the length of the initial ordinate of the inertia-curve.

When $at = 180^\circ$, in which case $\sin at = 0$, and $\cos at = -1$, the quantity within brackets is $-1 + \frac{1}{c}$, which gives the length of the final ordinate of the curve.

For determining the coefficient, to give pressure—

Let n = the number of revolutions per second,

$$\therefore a = 2\pi n.$$

Let w = weight (lbs.) of the reciprocating parts per square inch of piston-area;

g = force of gravity (32 feet per second); then r being measured in feet, the accelerating-pressure per square inch of piston-area is as

$$\frac{w}{g} (2\pi n)^2 r, \text{ or as } \frac{(2\pi)^2}{g} = 1.23 \text{ nearly, the coefficient is } 1.23 w r n^2. \quad (9)$$

In the case where $w = 1$ lb., $r = 1$ foot, and $n = 1$ per second, the coefficient is simply 1.23, and from (8)—

$$i = 1.23 \left\{ \cos at + \frac{c^2 (\cos^2 at - \sin^2 at) + \sin^4 at}{(c^2 - \sin^2 at)^{\frac{3}{2}}} \right\} \quad (10)$$

By giving at a number of successive values $0^\circ, 30^\circ, 60^\circ$ up to 180° , the corresponding values of i can be computed, and the inertia-curve can be drawn.

These values are—

$at =$	0°	30°	60°	90°	120°	150°	180°
$i =$	1.640	1.282	0.411	-0.435	-0.819	-0.848	-0.820

As these ordinates may be drawn to any scale, still retaining their relative values, it is only necessary, for any weight w , radius r and speed n , to select a scale such as is equivalent to the multiplication of each ordinate by the coefficient $w r n^2$, the product of the weight in lbs., the crank-radius in feet, and the square of the number of revolutions per second.