Review

Author(s): W. P. Workman

Review by: W. P. Workman

Source: *The Mathematical Gazette*, Vol. 1, No. 20 (Mar., 1900), pp. 333-336

Published by: [Mathematical Association](#)

Stable URL: <http://www.jstor.org/stable/3605005>

Accessed: 22-02-2016 14:13 UTC

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.

Mathematical Association is collaborating with JSTOR to digitize, preserve and extend access to *The Mathematical Gazette*.

<http://www.jstor.org>

Hence the locus is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{9} \left(\frac{b_1^2}{b^2} - \frac{a_1^2}{a^2} \right)^2$,

an ellipse similar to the outer of the given ellipses.

Another application will be to find the point of intersection of normals at α, β, γ . It may readily be shown that this is in general given by

$$\frac{2ax_0}{a^2 - b^2} = C_1 - C_3 = \left(\frac{b_1^2}{b^2} - \frac{a_1^2}{a^2} - 1 \right) C_3 \text{ here,}$$

$$\frac{2by_0}{a^2 - b^2} = -S_1 - S_3 = \left(\frac{a_1^2}{a^2} - \frac{b_1^2}{b^2} - 1 \right) S_3 \text{ here.}$$

These can be further reduced if we suppose the porismatic condition to be $\frac{a_1}{a} + \frac{b_1}{b} = 1$, as is usually the case. Then we find

$$x_0 = -\frac{a_1}{a^2} (a^2 - b^2) C_3,$$

$$y_0 = -\frac{b_1}{b^2} (a^2 - b^2) S_3.$$

It may now be verified that with this combination of signs $\left(\frac{a_1}{a} + \frac{b_1}{b} = 1 \right)$ the side β, γ is

$$\frac{x}{a_1} \cos \alpha + \frac{y}{b_1} \sin \alpha + 1 = 0,$$

or is the polar of the vertex a with respect to

$$\frac{x^2}{aa_1} + \frac{y^2}{bb_1} + 1 = 0.$$

Hence all the poristic triangles are self-polar with respect to this conic.

T. J. I'A BROMWICH.

REVIEWS.

Exercices d'Arithmétique. Énoncés et Solutions. Par J. FITZPATRICK, Ancien Professeur de Mathématiques, et GEORGES CHEVREL. 2me édition. Paris, Hermann, 1900; 8vo, pp. 680.

In England Arithmetic has always been the Cinderella of the Sciences. It has been her portion to do the marketing, to sweep up after Mensuration, to set the rooms of Physics in order, and to be a nursemaid to Algebra. She has never had a room of her own. It is only in France that she may put on gala-dress and appear as a princess in her own right. In the Germany where Gauss wrote she is almost forgotten, and it is doubtful whether she was ever known in England. Nor must English readers of the delightful volume named above expect to find a quite familiar figure therein. It is Arithmetic indeed, but most of it would receive short shrift at the hands of those terrible English ex-

aminers who head their papers, "No question will receive any credit where other than Arithmetical Methods are employed." It is Arithmetic as Gauss knew it, but it is not one of the three R's; local examiners know it not, and it is a stranger unto the Department.

It is, to say the least, doubtful whether one English teacher in ten knows what Arithmetic is, and where Arithmetic stops and Algebra begins. To take a concrete example. Your problem is, "An article bought for a certain sum is sold at 27s., which is one-eighth as much again as the cost-price. What was the cost-price?" If you solve this thus :

$$"x + \frac{x}{8} = 27, \quad \therefore \frac{9x}{8} = 27, \quad \therefore x = 24,"$$

your solution is ruthlessly crossed out as "Algebra." Write it

$$\begin{aligned} & \text{"Cost-price} + \frac{1}{8}(\text{cost-price}) = 27, \\ & \therefore \frac{9}{8} \text{cost-price} = 27, \quad \therefore \text{cost-price} = 24," \end{aligned}$$

and you will probably get full marks for your arithmetical ability. This means that if you denote the unknown by one letter (x) it is Algebra, if by nine letters (cost-price) it is Arithmetic. Could anything be more childish? Or, again, it is Arithmetic, say text-books and examiners, to write

$$\text{"Simple interest} = \frac{\text{Principal} \times \text{rate} \times \text{number of years}}{100},"$$

but it is Algebra to abbreviate it into $\frac{Prn}{100}$. So that it is clear that to

most people Arithmetic is Algebra writ large, and the best arithmetician is the most garrulous algebraist. The real distinction between Arithmetic and Algebra, which appears to us to lie in the fact that the second is the science of continuously varying quantity, of number regarded as capable of changing from $+\infty$ to $-\infty$, is altogether overlooked, and stress is laid upon the accident of denotation of a number by a particular kind of shorthand. The necessary result has followed, and Arithmetic has become a farrago of rules whose value is measured by their commercial utility, while Elementary Algebra has, in our judgment, entirely lost its educational value.

The present book is a protest against such a lowering of ideals. Arithmetic to our authors is the Science of Numbers, and includes the greater part of the Elementary Theory of Numbers. It is true that in the present edition they have bowed the knee to Baal, and "added a large number of exercises of commercial arithmetic"; but as the answers to these are not given, nobody will be likely to use this part of their book, and there is no harm done. The rest of the book is full of interest. It often reminds one strongly of the brilliant work of M. Edouard Lucas; and whole pages are quoted from the *Récréations Mathématiques*, the *Théorie des Nombres*, and that fascinating fragment *L'Arithmétique Amusante*; but it is no mere copy and here and there is to be found a suggestive novelty. The authors, it must be said, lack the firm touch of M. Lucas. One can never feel quite safe in their hands. We rubbed our eyes, uncertain whether we saw aright, when on page 129 we read,

"In order that an odd number p should be prime, it is necessary and sufficient that the square of the number $\frac{p-1}{2}$ added to p should give a perfect square."

Had the problem of the ages, we wondered, been solved at last, and was the solution so childishly simple? We were disillusioned on page 501, where the authors confess their blunder and delete the word "sufficient." But the blunder never ought to have been made. Even less happy is the foolish Question 403: "Prove that, if $\phi(n)$ is a multiple of $n-1$, the number n is prime," where $\phi(n)$ is the number of numbers less than n and prime to it.

Occasionally, too, the solutions when correct are far from elegant. We may instance Question 209: "Prove that there exists no whole number which, substituted for n in $\frac{n-6}{15}$ and $\frac{n-5}{24}$, makes both fractions integral." The authors fail to notice that this problem is equivalent to the obvious "prove that $n-6$ and $n-5$ cannot both be divisible by 3." Moreover, we question whether the widest interpretation of the term Arithmetic can include the theory of the balance; and when our familiar tradesman with his fraudulent balance enters in Question 260 we think that occasion has been given to the adversary, even though the question be elegant: "Prove that a tradesman who uses the pans of his balance alternately for weighing his goods is bound to lose."

But with all its defects, and they are not few, the book is suggestive and of sterling value. It is just the kind of work to stimulate original thought in a boy with the elements of Algebra at command, and the not uncommon conviction that he "knows Arithmetic." It will show him refracted vistas beyond his horizon, confusedly perhaps, but none the less temptingly. The present writer may be pardoned a personal reminiscence. He regards the most important stimulus to his own mathematical development to have been the action of a master who put into his hands Whitworth's *Choice and Chance* when he was just beginning Algebra. Such a question as "Show that in taking a handful of shot from a bag it is more probable that an odd number will be drawn than an even number," is eminently provocative of thought, despite its faulty English. There are many such questions in the book before us. For instance, there is an elegant application of the binary scale of notation in a method of multiplication, said to be actually employed by Russian peasants, which it will interest our readers to see.

"Russian peasants, when they have to make a multiplication, usually proceed as follows: Take for example the multiplication of 35 by 42. They divide the multiplicand by 2 and at the same time double the multiplier, and, if the multiplicand is odd, they mark the multiplier by the sign +. Thus, in the given example, they would write:

35	42 +
17	84 +
8	168
4	336
2	672
1	1344 +

Adding together the numbers marked +, they obtain the product 1470."

Or, again, there are the frequent hints, always so fascinating to a boy, that in certain provinces of Arithmetic we are literally at the outposts of human reason, and that until the great army has pushed its conquests further we must stand "with a wild surmise Silent upon a peak in Darien." Such are the references to Fermat's work, and many of the quotations from Lucas. For instance, on page 158: "Father J. Per-vouchine, of Perm, has verified that the number $2^{2^{23}}$ composed of 2525223 figures is divisible by the prime number 167772161." Or, on the same page, the collection of formulae, from Lucas, representing prime numbers, though, of course, not exclusively:

$$x^2 + x + 41, \quad x^2 + x + 17, \quad 2x^2 + 29 \text{ (Euler).}$$

Starting from $x=0$, these formulae give prime numbers for all values of x up to 40, 16, and 28 respectively. As our readers will be aware, no one has yet disproved Lucas' conjecture that the numbers of the series

$$2 + 1, \quad 2^2 + 1, \quad 2^{2^2} + 1, \quad 2^{2^{2^2}} + 1, \dots \text{ are primes.}$$

On page 366 we find, quoted from Lucas' *Récréations*, the remarkable result of M. Le Lasseur concerning the factorisation of $a^{1260} - b^{1260}$, where a and b are the roots of $x^2 = x - 2$. The last five of the factors given, all of which are asserted to be prime, are 125541359, 25215201901, 34449677641, 153790567559, 733268745721. Or, once more, on page 8 we find the solution of a pretty question which our readers may care to try for themselves:

"The numbers 123456789101112 ... being written continuously without any stops between them, what is the n th digit of the series?"

We commend the book to all who have the true interests of Arithmetic at heart, and we cannot do better than quote the suggestive words from the preface contributed by M. Jules Tannery: "The problems of Arithmetic and of Elementary Geometry have a special value for the formation of the mind and the development of initiative. Each of them is something fresh. Ingenuity is needed to discover in the new problem attacked any feature of resemblance to those already known, and it is in the art of recognising these hidden resemblances, these distinct affinities, that mathematical talent lies." Only be it understood that by "the problems of Arithmetic" M. Tannery does not primarily mean the proceedings of stockbrokers. W. P. WORKMAN.

MATHEMATICAL NOTES.

79. Note on the Trilinear Coordinates of the Focoids.

[Read before the Mathematical Association, Saturday, January 27th, 1900.]

If ABC be the triangle of reference, the trilinear coordinates of the focoids (imaginary circular points at infinity) are such that for one

$$-\beta/\gamma = e^{iA}; \quad -\gamma/\alpha = e^{iB}; \quad -\alpha/\beta = e^{iC};$$

and for the other

$$-\gamma/\beta = e^{iA}; \quad -\alpha/\gamma = e^{iB}; \quad -\beta/\alpha = e^{iC}.$$

[NOTE. $-1 = \cos \pi + i \sin \pi$.]

The above coordinates satisfy the equation of the line at infinity

$$\sum \alpha \cdot \sin A = 0,$$