JOURNAL
OF THE
FRANKLIN INSTITUTE.
OF THE STATE OF PENNSYLVANIA,
FOR THE PROMOTION OF THE MECHANIC ARTS.

VOL. CXI.

MAY, 1881.

No. 5.

THE Franklin Institute is not responsible for the statements and opinions advanced by contributors to the JOURNAL.

ON THE RATIO OF EXPANSION AT MAXIMUM
EFFICIENCY.

By R. H. THURSTON.

Read before the American Society of Mechanical Engineers, May, 1881.

In all heat-engines the method of transformation of heat-energy into useful mechanical work is the following:

A certain mass of the working fluid is heated from a temperature which is usually not far from that of the atmosphere up to some higher temperature, T . This is accompanied by a definite increase of volume, or of pressure, or of both, and in the case of liquids by a change of physical state after passing a certain point which is variable, but definite for each pressure; this latter temperature is the boiling point and the change is that known as vaporization. Evaporation being complete, the mass is expanded until it has attained a certain larger volume, v_2 , the magnitude of which is r times that of the initial volume, v_1 , with which expansion began. We thus have the "*rate of expansion*," $r = \frac{v_2}{v_1}$.

When expansion is complete, the whole volume, v_2 , of steam or gas at the pressure p_2 is rejected from the cylinder into a condenser or into the atmosphere, and the piston which it has impelled through the

WHOLE No. VOL. CXI.—(THIRD SERIES, Vol. lxxxi.) 21

total volume, v_2 , returns to the starting point, resisted by the "back-pressure," p_3 , of the condenser or of the atmosphere. During the latter operation all heat which has not been transformed into work is rejected, and an additional amount is expended, which is equivalent to the work done by the piston upon the fluid during its expulsion. This operation is also similar in result to the following: Instead of exhausting the working fluid as described, abstract heat from it at the maximum volume, v_2 , until its pressure becomes equal to the back-pressure; then compress at the constant pressure p_3 until the fluid is restored to its original volume at the pressure p_3 , which volume, in the case of the steam engine, may be neglected. In the case of the gas engine or hot air engine this volume is that of the working fluid at initial temperature and pressure.

This process is thus graphically represented: In Figure 1, the fluid, initially in the state measured by the pressure a E or a' E' and volume O a or O a' , is heated, sometimes at constant volume, as O a , and sometimes with compression, as from O a' to a higher temperature, the pressure and volume varying as shown by E A or by E' A . Heated next at constant pressure or at constant temperature, the mass expands, doing work, to B or to B' . At this point, v_1 , p_1 , the supply of heat ceases and the fluid expands "adiabatically," transforming into mechanical energy all the heat demanded as equivalent to the work measured by the area b B c C , and drawing upon its own stock of heat to supply this demand. At the end of this stage the fluid has a lower temperature and a pressure and a volume, c C , O c (p_2 , v_2), determined by that temperature and the value of $r = \frac{v_2}{v_1}$, and which are indicated by the location of the point C .

Fig. 1.

Rejecting heat at constant volume, v_2 , pressure falls to D , p_3 , and then rejection of heat continuing at constant pressure, p_3 , the volume is reduced to that with which it started.

The *total* or *gross* work done is, in gas engines, measured by the area A B C c a A , in steam and vapor engines by this area increased by a very considerable amount—the measure of internal, of molecular, work which cannot appear on the indicator diagram.

The net work done is measured by the area included in the indicator diagram $A B C D E A$. This work is the equivalent of all heat transformed into mechanical work or energy. *The efficiency of the fluid* is the ratio of *net work* done to total heat received by the fluid, and is a maximum when the area $A B C D E$ is a maximum, assuming the *rate of expansion* alone to vary. It is evident that this maximum is determined, therefore, by the conditions which make the area $b B c C$ a maximum, which conditions are very simple in the hot air engine, and are easily expressed, while in the steam and in vapor engines they are very difficult of determination and expression in consequence of their extreme variability.

But the efficiency of the fluid is but one factor in the determination of the rate of expansion for maximum economy. The heat in the fluid is compelled to do its work, not simply through that fluid as a transmitting mechanism, but also through a machine which, as an apparatus intended to imprison and direct so subtle and elusive a form of energy as heat, is extremely imperfect, and which has the additional and very serious defect of being itself cumbersome and difficult to start and to keep in motion without considerable loss of power within itself.

The useful work of the machine is that which it transmits beyond its own boundaries to other mechanisms, and this is a maximum at that rate of expansion which gives energy to the machinery of transmission beyond the engine at least cost in heat expended. This *efficiency of the system* is therefore the product of two factors, the *efficiency of the fluid* and the *efficiency of the engine* considered as a piece of mechanism.

(1) Taking first the purely ideal case in which the mechanism is assumed to be perfect and the rate of expansion the only variable element, we may by examining Figure 2 see at once what should be the value of that rate.

Fig. 2.

It is obvious that the rate of expansion simply determines how far the transformation of *stored*

heat-energy existing at B shall be continued by the development of work during the expansion of the working fluid. It is equally obvious that this expansion shall continue the gain of work by further expansion is more than balanced by is avoidable by termination of that process.

First: Where the only loss is due to a fixed back-pressure, $FD=p_3$, it is seen that, were expansion to cease at C , the work which would have been done had the expansion line BC extended to the right, beyond C is lost, and that the counterwork of back pressure beyond that point is gained; but the former exceeds the latter and the net result is a loss by incomplete expansion. On the other hand, were the rate of expansion increased so that the expansion line becomes $B'E$, the back-pressure line is reached at D' ; beyond this point, we note a gain of work done usefully, which is measured by the area $D'E F F' D'$, while a loss accrues by back-pressure measured by $D'D F F' D'$. We thus again meet with a net loss which is represented by $D'D E D'$, and expansion has evidently been carried too far. Making the value of $r = \frac{v_2}{v_1}$ such that expansion reaches the back-pressure line at D

and p_2 becomes equal to p_3 , we meet with neither kind of loss, and it follows that expansion should in this ideal case be continued until the expansion line meets the back-pressure line.

This may be readily shown by other methods. It was shown, nearly two generations ago, by Sadi Carnot, that maximum efficiency of *fluid* is attained when expanding between the widest possible limits of temperature.

It is now well known, and it is shown by every elementary treatise on physics, or mechanics, or thermodynamics, and on heat-engines, that the efficiency of the *fluid* in any heat engines is measured by the expression $\frac{T_1 - T_2}{T_1}$ in which T_1 and T_2 are the temperatures of recep-

tion and rejection of heat measured from the "absolute" zero. But this maximum range of temperature corresponds to the maximum attainable range of pressure, and, the upper limit being fixed, this range is determined by the value of r and is a maximum when $p_2 = p_3$ and expansion continues to the back-pressure line.

A general analytical demonstration is obtained in the following manner: Given p_1, v_1, v_2, p_3 to find the value of *rate of expansion* r

to make the net work done a maximum. This work, $A B C D E$, Fig. 1, is measured by

$$W_a = p_1 v_1 + \int_{v_1}^{v_2} p dv - p_3 v_2; \quad (1)$$

it is a maximum when the variable part $\int_{v_1}^{v_2} p dv - p_3 v_2$ is a maximum.

The method of variation of p with variation of v is determined by various conditions which need not be discussed here and which do not affect our analysis. Let this relation be such that we may write, as experiment indicates that we may do with practically close approximation,

$$p_1 v_1^n = p_2 v_2^n.$$

Thus we have

$$\begin{aligned} W_a &= p_1 v_1 + \int_{v_1}^{v_2} p dv - p_3 v_2; \\ &= p_1 v_1 + \frac{p_1 v_1 - p_2 v_2}{n-1} - p_3 v_2; \end{aligned} \quad (2)$$

or, where $n = 1$,

$$W_a = p_1 v_1 (1 + \log_e r) - p_3 v_2 \quad (3)$$

Determining the maximum for the first and usual case, we get

$$\frac{dW_a}{dr} = d \left(p_1 v_1 + \frac{p_1 v_1 - p_1 v_1 r^{1-n}}{n-1} - p_3 r v_1 \right) \frac{1}{dr} = 0;$$

$$\text{whence} \quad r = \left(\frac{p_1 v_1}{p_3 v_1} \right)^{\frac{1}{n}} = \left(\frac{p_1 v_1}{p_2 v_1} \right)^{\frac{1}{n}} = \frac{v_2}{v_1} \quad (4)$$

Hence $p_3 = p_2$ and the rate of expansion for maximum efficiency is that which makes the terminal direct pressure equal to the pressure resisting the motion of the piston.

This analysis must be modified when the expansion line is an equilateral hyperbola, in which case we have $n = 1$ and $p_1 v_1 = p_2 v_2$. This case is often assumed in the theory of gas and air engines, as it is that of isothermal expansion; but it is probably rarely observed in actual practice, and perhaps never occurs in steam and vapor engines. In simple calculations of work, however, the assumption does not usually lead to serious error,* and, so expanding the working

* See a neat demonstration of this case by Professor Marks in the *JOUR. FRANKLIN INSTITUTE*, June, 1880.

fluid, the energy exerted by it up to the point of cut-off is equal to the lost work due to back pressure; the net work done is measured by the total area under the expansion line of the indicator diagram, and the efficiency is proportional to $\log_e r$.

Thus we have

$$\begin{aligned} W_1 &= p_1 v_1 (1 + \log_e r) - p_3 r v_1 \\ \frac{d W_1}{d r} &= 0 = \frac{p_1 v_1}{r} - p_3 v_1 \\ r &= \frac{p_1}{p_3} = \frac{v_2}{v_1} = \frac{p_1}{p_2}; \end{aligned} \quad (5)$$

whence we again find $p_2 = p_3$.

(2) But in all real engines we have a resistance to the motion produced by the expanding fluid, which is composed of two parts, an actual back pressure on the piston $p_b = p_3$, as in the ideal case above, and a resistance due to friction of engine, including its pumps and attachments. It is evident that, as the latter, p_f , like the back pressure, p_b , is a constant source of lost work, we must terminate the expansion as soon as this source of loss produces a greater loss of power or of work than is gained by further expansion. In fact, given a certain value for the sum of these resistances, $R = p_b + p_f$, it is obvious that we may consider the whole as back pressure, if we choose, and that it is a matter of indifference, so far as the determination of the rate of expansion is concerned, what are their individual magnitudes.

To determine $R = p_b + p_f$ the sum of resistances due to back pressure, p_b , and to the frictional and other resistances—as of pumps, etc.—denoted by p_f , take an indicator card from the engine unloaded. Its mean pressure measures the friction, p_f , of the unloaded engine, and this, increased by a fraction of the pressure added by the load,* is the value of p_b . Or, still better, determine the indicated and the dynamometric power of the engine simultaneously; their difference is lost work, and the value of p_b corresponding to that work, is that required.

Hence, for actual engines, where no other cause of loss exists to any appreciable extent, as in some types of air engines, we may write

$$W_b = p_1 v_1 + \frac{p_1 v_1 - p_2 v_2}{n-1} - (p_3 + p_f) v_2 \quad (5)$$

* Experiments made at the Novelty Works for the Navy Department, 1865-67, and others made later, do not exhibit this increase.

and by the process already outlined we obtain a maximum and deduce

$$p_2 = p_3 + p_t \quad (6)$$

Hence: *Where the only variable loss is due to back pressure and to friction of the engine, the rate of expansion should be such as to cause expansion to the mean pressure line of the engine diagram taken without load;* the useful work is, as before, the gross work done during expansion, and, thus adjusted, the net useful work and the efficiency are nearly proportional to $\log. r$.

These conclusions may be taken practically without qualification for all actual engines in which the working fluid is gaseous and subject to little direct loss of heat.

(3) For steam and other vapor engines still further and still greater modification is necessary, since in such engines the departure from the ideal conditions first assumed is not only greater than in gas and air engines, but is so great as, in most cases, to lead to radically different rates of expansion. Even in the gas engines, the action of the working fluid, as assumed above, is very greatly modified by such variations from the ideal conditions as are here referred to.

These engines—steam and vapor—are impelled by a fluid, which is a vastly better receiver and transmitter of heat than the permanent gases. Steam takes up and loses heat, in the processes of formation and of condensation, with extreme rapidity. The working fluid, in all steam engines, is readily condensible, and exchanges heat with the metallic surfaces of the working cylinder with the greatest freedom. It is usually more or less wet, and its humidity is subject to rapid and extreme variation in the course of the movement of the piston.

Explosive and other gas engines are impelled by a mixture of hot gaseous and vaporous products of combustion, of which the latter portion is, like the working fluid in the steam and other vapor engines, subject to rapid and considerable changes of thermal state. Enclosed, usually, in a chamber the sides of which are kept cool by a water-jacket, enormous quantities of heat are lost as expansion proceeds, and the efficiency of the machine is correspondingly diminished, and the economical rate of expansion is altered by the increased losses which accompany the higher rates. It is easily seen that, should these losses increase in a high ratio, with large rates of expansion, a point will be reached, and may be reached quickly, at which any greater expansion will result in a loss exceeding in amount the work gained by the extension of the expansion line. This point may be reached, and pro-

bably often is reached, long before attaining the limit set by the value of the resistances already studied. In such a case, we may call this limit that of the *virtual back pressure due to condensation*, and may designate it as p_c .

(4) In the steam engine a still more complicated set of phenomena is to be met with, and the result of their action is similar to that just described.

Suppose steam to enter the steam cylinder perfectly dry, and to expand *adiabatically*.* As expansion progresses, after the closing of the steam valve by the expansion gear, the work done by the working fluid results in the transformation of so much heat into mechanical energy—which heat can now only be obtained by drawing upon the stock contained in the steam itself—that a part of the steam becomes liquefied.

This fact was shown by Rankine and by Clausius, by the study of the thermodynamics of the case; but it can easily and satisfactorily be shown by any student or engineer who will take the trouble to calculate the “total heats” of steam at the pressures found at the beginning and at the end of the expansion line. It will be found that the work done during the expansion is greater than the mechanical equivalent of this difference, and it follows that a part of it must have been done at the expense of the heat of evaporation of the expanding mass. This must be the case up to a limit at which the sensible heat of steam is just equal to the latent heat of the mass when indefinitely expanded at the absolute zero, and is about $\frac{1103550}{772} = 1437^\circ \pm$ absolute, a temperature only attained at the red heat.

Steam, therefore, condenses in the steam cylinder unless, by superheating or by the use of an efficient jacket, considerable heat is supplied it during expansion or before. This amount is, however, insignificant in comparison with direct losses of heat; it can probably never approach ten per cent. of the heat supplied, and is more likely, usually, to be a very much smaller figure, perhaps two or two and a half per cent. for average cases.

Initial condensation and later re-evaporation of steam in the steam engine, and initial condensation without subsequent re-evaporation, in gas engines, give rise to losses that are both absolutely and relatively very great wherever the range of temperature during expansion is very considerable, and especially with low back pressure.

* Without receiving or losing heat by exchange with surrounding surfaces.

The steam passing out of the exhaust ports to the condenser or into the atmosphere is moist and heavy with the water of condensation, and is a good conductor of heat as well as a very greedy absorbent. It sweeps out of the cylinder large quantities of heat abstracted from the inner surfaces of the cylinder, leaving those surfaces comparatively cold and wet with a chilling dew. The entering steam meets these cold metallic and liquid masses and is condensed in sufficient quantity to reheat them to the temperature of prime steam. As the piston moves forward it uncovers new surfaces, and condensation continues until, sometimes, a large fraction of the steam supplied lies in the cylinder or floats in the uncondensed steam as water and mist. Toward the end of expansion, and especially during exhaust, re-evaporation occurs at lower pressures and to a similarly serious extent. Thus heat is constantly transferred from the steam to the exhaust side, and, doing almost no work, is wasted, and the efficiency of the engine and the cost of fuel are greatly affected. The heat thus lost frequently amounts to 25 per cent. of the total supplied, and has been known not infrequently to amount to 50 per cent. In one case noted by the writer initial condensation was as high, *at least*, as 80 per cent.*

Since loss from this cause has been found to be so great, and to increase so rapidly with increased expansion, that it practically often sets an early limit to the economical increase of the rate of expansion, it is evident that we may determine a point such that, expansion being carried beyond it, the losses due this cause will exceed the gain of work done by the expanding fluid, precisely as in the cases already cited. Measuring the loss at such a point, we may determine the equivalent in foot-pounds of work, and thence deduce the magnitude of a new equivalent "*virtual back pressure*," $p_v = p_c$, which, if actually existing as such pressure, would similarly limit expansion.

* The extent of the loss from this cause is very seldom realized by engineers, and still less by unpractical writers on the theory of the steam engine. Even Rankine, the greatest of all known writers, seems to have failed to detect an initial condensation of 26 per cent, shown apparently by the value obtained for the index in his formula for the adiabatic expansion of steam ($p v^{\frac{10}{9}} = \text{constant}$). He seems to check the discrepancy introduced into his analysis, amounting to a loss of about one-fourth of all heat supplied, by under-estimating the efficiency of the boilers, crediting them with but 0.54, where a low estimate, in the opinion of the writer, would be 0.65, and a fair figure would be 0.68 or 0.70 for the cases taken. The writer has never known the efficiency given in those estimates to be attained with boilers of such low value.

This value inserted in the formula representing the work of the steam would give a measure of the rate of expansion of maximum efficiency.

We should have, assuming no other losses :

$$W_e = p_1 v_1 + \frac{p_1 v_1 - p_2 v_2}{n-1} - p_e v_2 \quad (7)$$

and should expand until $p_2 = p_e$, and make

$$r = \frac{v_2}{v_1} = \left(\frac{p_1}{p_e} \right)^{\frac{1}{n}}.$$

Thus, for all cases, whether of expansion of steam, of air, or of the products of combustion in explosive gas engines, we may determine for each case a certain "*virtual back pressure*," which we may call p_v , by which to identify a point beyond which continued expansion leads to a loss of heat, or of work, or of both combined, that is greater than the gain by work done* in that additional expansion, and may write generally :

$$W_v = p_1 v_1 + \frac{p_1 v_1 - p_1 v_1 r^{1-n}}{n-1} - p_1 v_1 r^{1-n} \quad (8)$$

$$r = \left(\frac{p_1}{p_v} \right)^{\frac{1}{n}}.$$

Where, as in the case in which air expands isothermally, the expansion line is an equilateral hyperbola, it is seen that the loss by the virtual back pressure is always equal to the work done in the engine up to the point of cut-off; for $p_1 v_1 = p_v v_2$.

The net work done is always, for such expansion,

$$W_n = p_1 v_1 \log_e r = p_1 v_1 \log_e \frac{p_1}{p_v}.$$

In general, for the net work shown on the card as $p_2 = p_v = p_1 r^{-n}$; $p_v v_v = p_1 v_1 r^{1-n}$; $p_v r^n = p_1$, we get for net work :

$$W_n = \frac{n}{n-1} p_1 v_1 \frac{r^{n-1} - 1}{r^{n-1}} = \frac{n}{n-1} p_1 v_1 \left[1 - \left(\frac{p_1}{p_v} \right)^{\frac{1-n}{n}} \right]$$

$$= \frac{n}{n-1} p_1 v_1 \left(1 - \frac{1}{r^{n-1}} \right). \quad (9)$$

* See "The Limitations of the Steam Engine," by Prof. Marks; JOURNAL FRANKLIN INSTITUTE, August, 1880. The use of the term "virtual back-pressure" is not logically correct, as the two methods of loss are quite different; but the writer has not yet found a more satisfactory term to take its place.

In the effort to determine the value of $p_v = p_e$ for this last method of loss of efficiency, we meet with great difficulties. The loss from initial condensation and later re-evaporation is the most serious of all those losses which in expansive engines are in any degree due to defects of the machine as a machine, and they are among those which are controllable to a considerable extent by the engineer. No two engines, however, ever exhibit them in the same degree, and modifying conditions are so numerous and so potent that the result of the most painstaking efforts to classify and to formulate them are likely to be exceedingly unsatisfactory.

This loss is proportionally greater as the range of temperature during expansion is greater; it is increased by slow speed of engine,* by reduction of the real back pressure, by increase in size of engine for a given amount of work done, by increase in conductivity of the surfaces of the working cylinder, and especially by wetness of steam. It is reduced by low rates of expansion, by increasing back pressures by reducing initial pressures, by increasing speed of engine and by special expedients, as steam-jacketing, superheating and the division of the expansion between two or more cylinders, as in "compound" or double-cylinder engines. Even increasing compression may reduce this loss and thus give a higher steam-line and an altered expansion-line.

The waste becomes the less, when the sides of cylinders only are jacketed, the smaller their diameter; it is lessened, when both heads and sides are jacketed, by increasing diameters, volumes being in both cases equal. With superheated steam, and where there is little initial condensation to be anticipated, the shape of cylinder is determined by the minimum ratio of volume to *internal* superficies, *i. e.*, $\frac{\text{diam.}}{\text{length}} = \frac{1}{2}$,

except—as is often, if not usually, the case—when it is controlled by commercial considerations. The surfaces of the piston must evidently be included, since the principal losses—those due to initial condensation and to re-evaporation—occur upon those surfaces.

* With speeds so low that the range of temperature of cylinder surfaces is not restricted, the total weight of steam condensed is probably constant, and the loss becomes inversely as the speed of piston, or as the weight of steam passed through the engine. For fairly high and for very high speeds the writer takes this loss proportion to the reciprocal of the square root of the speed. See records of U. S. expansion experiments.

In general, we may say that the efficiency of an engine is some function of Δt , V , P and A ; but the difference of temperature, Δt , is a function of pressures and time of exposure; the speed, V , determines time and exposure, and the area of surface exposed, A , is a function of volume, per unit of weight of steam, and of shape of cylinder. All of the conditions are so involved and interdependent that the simple approximate expressions to be presently given may be found preferable to any exact formula, even were it possible to devise them satisfactorily, and as these simple expressions yield, all things considered, very fair results, we may be fully justified in their use until extended and exact investigations shall yield better.

In gas engines the waste is decreased in those in which the working fluid meets only non-conducting surfaces, while it amounts as a minimum to 60 or 70 per cent. in some slow-running water-jacketed cylinders.

Again, we find some interesting compensations. The difference in back pressure between non-condensing and condensing engines is productive of such a wide difference in the range of temperatures worked through in usual cases, that the writer has been accustomed to consider the compensation so complete as to justify the assumption that the value of this "virtual back pressure" may be assumed to be independent of the magnitude of the actual back pressure, and to be determined solely by other conditions above noted. In steam-jacketed engines the efficiency of the steam-jacket is reduced by high speed, while the losses that it is designed to check are rendered less by the reduced effect of other causes of variation of the amount of initial condensation, and, while this compensation is by no means complete, the error introduced by the assumption that it is so may perhaps be neglected in presence of so many other and such complicated causes of irregularity of action. Our approximation must be anything but close at best. The exact expression would probably involve the Newton law of cooling and values of differences of temperatures, deduced from Rankine's formula: $\log. p = A - \frac{B}{T} - \frac{C}{T^2}$.

The best that the writer has been able to do in this direction, as yet, is to make simple and roughly approximate expressions for values of $p_i = p_r$, the proper terminal pressure, which, while widely departed from in many cases, may fairly represent average practice, and serve

as a guide to the designer and engineer until something better can be done, thus:

For the common unjacketed steam engine take the limit for p_v that determined by back pressure solely up to pressures of 6 or perhaps 7 atmospheres.

Then the terminal pressure should be, nearly, for non-condensing engines $p_t = p_v = p_b + p_r = 14.7 + 3.3 + 2 = 20$ lbs. per sq. inch (1.4 kilos. per sq. centimetre = 1.14 atmos.),* and we should expand to that pressure.

For condensing engines and for non-condensing engines under very high pressures, the limit is fixed by the extent of the losses of heat just described. For the common unjacketed engine at moderate speed the writer has been accustomed to assume for the rate of expansion giving maximum economy in the common single cylindered unjacketed engine, working at fair speed, $r_e < \frac{1}{2} \sqrt{P}$; for high-speed engines of best proportions and for compound engines, $r_e < \frac{3}{4} \sqrt{P}$ and $r_e < \sqrt{P}$ respectively. Where more variable conditions must be considered, he has written, for unjacketed cylinders or for ineffectively jacketed engines,

$$r_e = \frac{1}{p_v} = \frac{1}{p_c} = 0.02 \sqrt{VNP_1} = 0.13 \sqrt{V_m NP_m}, \text{ nearly; } \dagger$$

$$p_t = p_v = p_c = \frac{50}{\sqrt{VNP_1}} = \frac{8}{\sqrt{V_m NP_m}}, \text{ nearly;}$$

where S is the speed of piston, N the number of cylinders when the engine is "compound," P_1 and P_m are the initial pressures per square inch and per square centimetre, and p_t is the terminal pressure.

For well-jacketed engines we may take, roughly,

$$r_e = \frac{1}{p_v} = \frac{1}{p_c} = 0.5 \sqrt{NP_1} = 1.75 \sqrt{NP_m}, \text{ nearly.}$$

$$p_t = p_v = p_c = \frac{2}{\sqrt{NP_1}} = \frac{0.6}{\sqrt{NP_m}}, \text{ nearly.}$$

* With large ports and dry exhaust steam this figure should be reduced 10 per cent.

† Emery has proposed, as fair values, $r_e = \frac{P+37}{22}$, where P is the gauge pressure,

and considers the values thus obtained as large for ordinary single cylinder engines, and small for the best compound engines.

In determining the values of the index n in the assumed expression $pv^n = \text{constant}$, for the equation of the expansion line, new difficulties arise. This index is itself variable in each case as expansion progresses, and no two cases give the same mean value. Rankine takes $n = \frac{10}{9}$ for expansion in non-conducting cylinders, assuming that in those engines on which he experimented the conditions were practically such as to give this value. The writer has no doubt that the steam supplied them was practically dry, but Zeuner has shown that the value of n is pretty nearly

$$n = 1.035 + \frac{x}{10}$$

where x measures the proportion of steam present or the "dryness fraction." For $x = 1$, $n = 1.135$, as tabulated, and for $n = 1.111$, as taken by Rankine, we have

$$x = 10 (1.111 - 1.035) = 0.76,$$

showing that initial condensation must have produced 24 per cent. water, a fact which introduces an error in his estimates of heat supplied to the engine from the boiler.

In none of the values above given is the fact exhibited that the effect of the phenomenon here studied at some length is to cause a rapid fall of the expansion line at the start and a considerable rise at the end, thus causing the line to depart from the curve represented by $pv^n = C$ to an extent that cannot be definitely stated. The value of the index is rendered by this cause, also, not only very variable for different cases, but it is probably usually varied constantly along the expansion line in each case by these new variable conditions. So great are these departures from any laws yet expressed by formulas that we may be justified often in taking advantage of the fact that the curve as often approaches the equilateral hyperbola as any other regular curve.

Where steam-jacketing is so efficient as to prevent condensation during expansion, and where, assuming it possible, superheating can be made so effective as to prevent initial condensation, the transfer of heat from steam to exhaust without transformation into work, in the manner here considered, would be greatly reduced, and perhaps so far as to make the limit $p_v = p_c + p_b$ as in non-condensing engines with low steam.

The compound engine, with receiver, offers peculiar opportunities to secure these conditions by superheating between the two cylinders, as has been done by Corliss, Leavitt and others.

The last column of the large table exhibits this case, assuming $p_r = p_t = p_b + p_t = 7\frac{1}{2}$ lbs. The value of r is increased for a given value of p_r , often to a considerable extent in actual engines, by re-evaporation.

The tabulated values of r_e may be taken as maxima for low pressures and minima for high steam, and in the latter case considerable departures from them in the direction of larger values have often produced but little difference of efficiency.

The following are values of n for various cases commonly found in real work, or taken in theoretical discussions:

VALUES of n IN $pv^n = \text{CONSTANT}$.

Air, isothermal expansion,	.	.	1.0
" adiabatic	"	.	1.4
" wet and adiabatic,	.	.	1.2
Gases generally, isothermal,	.	.	1.0
" " adiabatic,	.	.	1.4
" in explosive gas engines,	.	.	1.6
Steam, dry and saturated,	.	.	1.046
" adiabatic,	.	.	1.135
Steam, 0.76; water, 0.24,	.	.	1.111
" superheated,	.	.	1.333
Steam and water generally,	$n = 1.035 + \frac{x}{10}$		

The absolute values of the weights of steam used in engines under the conditions above considered cannot be predicted with any greater accuracy than the proper rates of expansion. The expenditure of heat in this method of waste increases in some undetermined ratio with the increase of the rate of expansion, and the writer has usually anticipated a loss at least proportional to the square root of that rate, and would add a percentage equal usually to at least $h_e = 0.1 \sqrt{r_e}$, and often to $h_e = .25 \sqrt{r_e}$, to the amount calculated ordinarily by Rankine's methods, and would expect the weight of steam used to reach $W = \frac{200}{\sqrt{P_1}}$; $W = \frac{24}{\sqrt{P_m}}$; nearly, in general practice and with good engines

Probable Terminal Pressures and Rates of Expansion at Maximum Efficiency.

Initial Pressures. Absolute.		Speed of Piston. V ft. m.		SINGLE CYLINDERS.						COMPOUND ($N=2$) CONDENSING.					
				Non-condensing.			Condensing.			Sides jacketed.			Heads and sides jacketed.		Heads and sides jacketed with efficient super-heating.
				Unjacketed.		Jacketed.	Unjacketed.		Jacketed.	$p_e = p_v = p_c$		r_e	$p_e = p_v = p_c$		r_e
				$p_e = p_v$	r_e		$p_e = p_v = p_c$	r_e		Lbs. on sq. in.	Kilos on sq. cm.		Lbs. on sq. in.	Kilos on sq. cm.	
P^1	P^w	V	V	Lbs. on sq. in.	Kilos on sq. cm.	Lbs. on sq. in.	Kilos on sq. cm.	Lbs. on sq. in.	Kilos on sq. cm.	Lbs. on sq. in.	Kilos on sq. cm.	Lbs. on sq. in.	Kilos on sq. cm.	Lbs. on sq. in.	Kilos on sq. cm.
40	2.8	400	122	20	1.4	20	1.4	16	1.1	2.5	13	.9	9	.6	4.5
		625	185	20	1.4	20	1.4	13	.9	3.0	13	.9	9	.6	4.5
60	4.2	400	122	20	1.4	30	2.0	20	1.4	3.0	15	1.1	11	.8	5.5
		625	185	20	1.4	30	2.0	15	1.1	4.0	15	1.1	11	.8	5.5
80	5.6	400	122	23	1.6	3.5	2.3	23	1.6	3.5	18	1.3	13	.9	6.5
		625	185	20	1.4	4.0	2.8	18	1.3	4.5	18	1.3	13	.9	6.5
100	7.0	400	122	25	1.8	4.0	2.8	25	1.8	4	20	1.4	14	1.1	7.0
		625	185	20	1.4	5.0	3.5	20	1.4	5.0	20	1.4	14	1.1	7.0
120	8.4	400	122	27	1.9	4.5	3.2	27	1.9	4.5	22	1.5	16	1.1	7.5
		625	185	22	1.5	5.5	3.9	22	1.5	5.5	22	1.5	16	1.1	7.5
150	10.5	400	122	30	2.1	5.0	3.5	30	2.1	5.0	25	1.8	18	1.3	8.5
		625	185	25	1.8	6.0	4.2	25	1.8	6.0	25	1.8	18	1.3	8.5
200	14.1	400	122	36	2.6	5.5	4.8	36	2.5	5.5	29	2.1	21	1.4	10.0
		625	185	29	2.1	7.0	5.0	29	2.1	7.0	29	2.1	21	1.4	10.0

$p_e = p_v$ is taken as equal to p_c except where $p_v = p_b + p_t$ exceeds p_c , when this greater value is adopted; e, g , in non-condensing engines with low steam, and in highly superheated steam engines.

Deduct 14.7 lbs. per sq. in. $\approx$ 1 kg. per sq. cm. to obtain gauge pressures. Hyperbolic expansion is assumed.

In the general expression for loss by initial condensation, $h_0 = a \sqrt{r_e}$, the writer has used the following values for the coefficient a :

Unjacketed cylinders,	$\begin{cases} 1600 \div Vd \\ 1200 \div V_m d_m \end{cases}$
Cylinders with sides jacketed,	0.10
Cylinders with sides and heads jacketed,	0.07

in which d and d_m are diameters of cylinders in inches and centimetres and V and V_m are velocities of piston in feet and metres.

Similarly rough statements of probable weight of steam per horsepower and per hour for a wider range of *average* good conditions, as used by the writer, have been (minima)

$$W = \frac{40}{\log. P_1}; W_m = \frac{7.7}{\log. P_m}, \text{ for fast or jacketed engines;}$$

$$W = \frac{100}{\log. (P_1 V)}; W_m = \frac{20}{\log. (P_m V_m)}, \text{ for unjacketed cylinders.}$$

Where the ratios of expansion have been made those of maximum efficiency, the closest approximation has been attained by taking

$$W = \frac{15}{\log. r_e} (1 + .1 \sqrt{r_e}); W_m = \frac{7}{\log. r_e} (1 + .1 \sqrt{r_e}),$$

as the expenditure will in such cases approximate most nearly to a direct ratio with the net energy obtained per diagram. These values are adopted in the following table. It is evident that the higher the value of r_e , the better the type of engine, and that we are here given a good gauge by which to make comparisons of the efficiency of different kinds of engines.

These values accord moderately well with the observation and experience of the writer where engines of good design have been compared, and may possibly prove useful to others in designing or in drawing up specifications. Like the values of p_v or of r , they can only be taken as probable means, and adopted provisionally, until better and more accurate values have been determined for a wider range of conditions.

Thus, we have the following probable values of weight of steam demanded where the ratio of expansion is correctly adjusted:

Probable Minimum Weights of Steam per Hour per Horse-power.

r_e	W Pounds.	W_m Kilos.	r_e	W Pounds.	W_m Kilos.	r_e	W Pounds.	W_m Kilos.
3	32	15	8	20	9	13	19	8
4	27	12	9	19	9	14	16	7
5	25	11	10	19	9	16	16	7
6	22	11	11	18	9	20	15	7
7	20	9	12	17	8	25	15	7

Taking the probable minimum expenditure of coal per hour and per horse-power at *one-ninth* the weight of steam demanded, we get

$$W' = \frac{1.7}{\log. r_e} (1 + 0.1\sqrt{r_e}), \quad W'_m = \frac{.8}{\log. r_e} (1 + 0.1\sqrt{r_e}).$$

and thus, assuming, as before, the best probable conditions and the ratio of expansion giving a minimum cost of steam, we obtain the following:

Probable Minimum Weights of Coal per Horse-power per Hour.

r_e	W' Pounds.	W'_m Kilos.	r_e	W' Pounds.	W'_m Kilos.	r_e	W' Pounds.	W'_m Kilos.
3	3.5	1.6	8	2.2	1.0	13	1.8	0.9
4	3.0	1.4	9	2.1	1.0	14	1.8	0.8
5	2.8	1.3	10	2.1	1.0	16	1.8	0.8
6	2.3	1.1	11	2.0	0.9	20	1.7	0.8
7	2.2	1.0	12	1.9	0.9	25	1.7	0.8

For cases in which the boiler gives an evaporation of ten pounds of water per pound of coal we may get ten per cent. better figures.*

* A private letter, lying on the table of the writer, giving results corresponding with the case assumed as giving $r_e = 20$, states the coal consumption at 1.5 pounds. This is obtained by one of the oldest and most distinguished engineers in the United States. The boiler has about this maximum efficiency.

So far as the experience of the writer and comparisons made by him with data given by the best experiments have extended, these figures have proven so far accordant that he does not hesitate to use them in estimating probable results. Adding, say, 20 per cent. will give figures on which to base a guarantee in making up contracts, for skillfully designed engines.

The introduction of this element practically completes the theory of the steam engine. Every practicing engineer will look with interest for experimentally derived data and exact expressions that may replace approximate formulas which as here provisionally used are purely empirical and have no scientific value.

How far the high efficiencies here seen to be probably attainable are worth paying for is a commercial question of great importance, but is quite distinct from that here considered.

REFERENCES.—The quantities and the empirical formulas and rules given by the writer as deduced from experience and observation may be compared with the following, which comprise nearly all that he has been able to find bearing upon the subject with any degree of definiteness: D. K. Clarke, "Manual for Mechanical Engineers," pp. 888, 890; "Northcott on the Steam Engine," pp. 157, 158; Isherwood's "Engineering Researches"; "Cotterill on the Steam Engine," chap. xi, especially pp. 294 to 296; R. H. Buel's "Addenda," to "Du Bois' Weisbach," vol. ii, § 512; Rankine's "Papers"; Rankine's "Steam Engine," § 282, 289; "Porter on the Richards Indicator," London, 1874, sect. iii.

New Actinic Phenomenon.—M. Phipson prepares a zinc white by precipitating a sulphate of zinc by means of a more or less complete solution of barium sulphide. The precipitate is placed under a hydraulic press and then heated to redness in a furnace, care being taken to prevent too great oxidation. When the experiment is properly conducted and the product is exposed to sunlight the snowy white gives place gradually to a brownish tint, which becomes of a slate color in about twenty minutes. If placed again in the darkness the dark shade is gradually lost, and in the course of five or six hours it becomes as white as snow. The experiment may be repeated with the same specimen as often as may be wished.—*Les Mondes*. C.