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I. *On the most Effective Adjustment of an Induction-coil.*
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[Plate I.]

IN a recent paper † it was shown that the secondary potential and spark-length developed by an induction-coil, when a given current is interrupted in the primary circuit, are greatest under the following conditions:—

(1) The ratio n_2/n_1 of the two frequencies of electrical oscillation of the system has one of the values 3, 7, 11, ;

(2) $L_1C_1 = (1 - k^2)L_2C_2$ ‡.

The first is the condition that maxima of the two potential waves in the secondary circuit should occur simultaneously, the second that the sum of the amplitudes of the two waves should be a maximum for a given value of k .

It was further stated that the most effective of the ratios specified in (1) is $n_2/n_1 = 3$, and that in this case the conditions are satisfied by the adjustment $k = \cdot 756$, $L_1C_1 = \cdot 429 L_2C_2$.

* Communicated by the Author.

† Phil. Mag. xxvii. pp. 580-586, April 1914.

‡ L_1, L_2 are the self-inductances, C_1, C_2 the capacities, and k is the coupling coefficient of the primary and secondary circuits.

Phil. Mag. S. 6. Vol. 29. No. 169. Jan. 1915. B

The physical meaning of the above results may be further considered in the light of what goes on in the *primary* circuit after the interruption of the current. An expression for the potential-difference V_1 of the plates of the primary condenser, at any time t after the interruption of the current i_0 , has been given by Dibbern*. If the resistances of the circuits are neglected, Dibbern's expression becomes

$$V_1 = \frac{2\pi i_0 n_1 n_2^2}{C_1(n_2^2 - n_1^2)} \left(\frac{1}{4\pi^2 n_2^2} - L_1 C_1 \right) \sin 2\pi n_1 t \\ - \frac{2\pi i_0 n_1^2 n_2}{C_1(n_2^2 - n_1^2)} \left(\frac{1}{4\pi^2 n_1^2} - L_1 C_1 \right) \sin 2\pi n_2 t.$$

If n_2/n_1 has one of the values 3, 7, 11, . . . , then at the time $t = 1/4n_1$, $\sin 2\pi n_1 t = 1$, and $\sin 2\pi n_2 t = -1$. Consequently the value of V_1 becomes at this instant

$$\frac{i_0}{2\pi C_1} \cdot \frac{1}{n_2 - n_1} - 2\pi i_0 L_1 \cdot \frac{n_1 n_2}{n_2 - n_1}.$$

This may be expressed in terms of the ratio $L_2 C_2 / L_1 C_1$. Calling this m , and writing a for $\frac{1}{8\pi^2 L_2 C_2 (1 - k^2)}$, we have

$$n_1 n_2 = 2a \sqrt{m(1 - k^2)},$$

$$n_2 - n_1 = \sqrt{2a(m + 1) - 4a \sqrt{m(1 - k^2)}} \dagger.$$

$$\text{Hence } V_1 = \frac{i_0 \{ 1 - 8\pi^2 L_1 C_1 a \sqrt{m(1 - k^2)} \}}{2\pi C_1 \sqrt{2a(m + 1) - 4a \sqrt{m(1 - k^2)}}}.$$

Inserting the value of a and condition (2), *i. e.* $m = \frac{1}{1 - k^2}$, we find that this expression vanishes.

It appears, therefore, that if conditions (1) and (2) are satisfied, the amplitudes of the two potential waves in the primary circuit are equal, and that at the instant in question the potentials are at their maxima, but in opposite phase. The primary condenser is therefore uncharged, while at the same moment the secondary potential is at its maximum ‡, the two waves in the secondary circuit being then at their maxima and in the *same* phase. Further, since $dV_1/dt = 0$, and $dV_2/dt = 0$, there is no current in either circuit. The whole of the energy therefore exists at this moment in the

* E. Dibbern, *Ann. d. Physik*, xl. 4, p. 938 (1913); Inaug. Diss., Kiel.

† *L. c.* p. 581.

‡ *L. c.* p. 581.

electrostatic form in the secondary circuit, and the secondary potential must in these circumstances have its greatest possible value for a given initial energy-supply $\frac{1}{2}L_1i_0^2$.

It was shown in the previous paper to which reference has been made * that the value of this maximum secondary potential, still on the assumption that the resistances are negligible, is

$$V_{2m} = i_0 \cdot \frac{L_{21}}{\sqrt{L_2 C_2}} \cdot \frac{1}{k} \quad \dots \quad (3 a)$$

$$= i_0 \sqrt{\frac{L_{21}}{L_{12}}} \cdot \sqrt{\frac{L_1}{C_2}} \quad \dots \quad (3 b)$$

The energy equation is therefore

$$\frac{1}{2}L_1i_0^2 = \frac{1}{2}C_2V_{2m}^2 \cdot \frac{L_{12}}{L_{21}}.$$

The factor L_{12}/L_{21} arises from the manner in which the capacity C_2 of the secondary circuit is defined. It is the charge on one-half of the secondary coil (and the bodies connected to its terminal), divided by the difference of potential of the terminals. The charge on this portion of the secondary circuit is C_2V_{2m} , but as some of this charge is at a lower potential than that of the terminals, the energy must be less than $\frac{1}{2}C_2V_{2m}^2$. The correcting factor is L_{12}/L_{21} .

There are many values of k and L_1C_1/L_2C_2 which satisfy the above conditions. The first four, probably the only ones having any practical importance, are given in Table I.

TABLE I.

$n_2/n_1.$	$k.$	$\frac{L_1C_1}{L_2C_2} = 1 - k^2.$
3	·756	·429
7	·914	·164
11	·950	·098
15	·965	·070

If we define the efficiency of an induction-coil as the ratio of the maximum electrostatic energy in the secondary to the electrokinetic energy in the primary circuit just before the interruption of the current (*i. e.*, $\frac{1}{2}L_1i_0^2$), then the efficiency

* *L. c.* pp. 581, 584.

is unity in each of the adjustments specified in Table I. if the resistances of the circuits are negligible. But it is easy to see that of these adjustments the first (*i. e.*, $n_2/n_1=3$, $L_1C_1=.429 L_2C_2$) is the one which gives the longest spark for a given primary current. For let us suppose that an induction-coil is so constructed, and the primary condenser so chosen, that $k=.914$ and $L_1C_1=.164 L_2C_2$. This is the second adjustment of Table I., and it allows the whole of the primary energy $\frac{1}{2}L_1i_0^2$ to be converted at a certain moment after the interruption into electrostatic energy in the secondary circuit. Now by inserting coils of suitable self-inductance, say l_1 , in the primary circuit (coils which do not act inductively on the induction-coil) we can reduce the coupling coefficient to .756; and if the primary condenser is chosen so that $(L_1+l_1)C_1=.429 L_2C_2$, we then have the first adjustment, with $n_2/n_1=3$. The system has again unit efficiency, but the maximum electrostatic energy in the secondary is now $\frac{1}{2}(L_1+l_1)i_0^2$, and is greater than the former value in the ratio of L_1+l_1 to L_1 . The capacity C_2 of the secondary coil being unaltered, the secondary potential is increased in the ratio $\sqrt{L_1+l_1}$ to $\sqrt{L_1}$. The 3/1 adjustment is thus more effective (though not more efficient) than the others because it allows a greater quantity of initial energy to be converted into electrostatic energy in the secondary circuit, in which therefore the charge and potential developed are greater than in the other cases.

There are other ways in which the coupling coefficient could be reduced (*e. g.*, from .914 to .756), but none of them are so effective as the plan of introducing external inductance into the primary circuit. For instance*, external inductance may be introduced in the *secondary* circuit. This increases C_2 , and to some extent L_{12} , without altering L_1 and L_{21} , and therefore, by (3*b*), lowers the secondary potential. Again, k may be reduced by removing a part of the iron core. This diminishes L_1 without causing any increase in L_{21}/L_{12} , or any change in C_2 , and therefore, by (3*b*), reduces V_{2m} . The reduction of the coupling may also be effected by withdrawing the primary coil with the core to a suitable distance along the axis of the secondary. This process reduces L_{12} , L_{21} , and L_2 , without changing L_1 , and should cause no change in the secondary potential except such as may be due to a slight diminution of C_2 or of the ratio L_{21}/L_{12} .

It should be clearly understood that these reductions of the coupling are supposed to bring the system from

* In each of these supposed modifications the primary capacity is to be adjusted so that $L_1C_1=(1-k^2)L_2C_2$.

one state of unit efficiency to another, *e. g.* from the second to the first adjustment of Table I., for it is only in these states that equations (3 *a*), (3 *b*) hold. If, on the other hand, k lies between two of the unit-efficiency values, the spark-length for a given current may be increased by withdrawing the primary coil and core to a suitable distance along the axis of the secondary. For example, the coil with which I have experimented has a coupling coefficient of $\cdot 876$, and the longest spark occurs when the primary capacity is about $\cdot 06$ microfarad*. A marked increase is, however, produced in the spark-length by withdrawing the primary coil and core to a distance of 36.5 cm. from its nearly symmetrical position in the secondary, and increasing the capacity to $\cdot 15$ microfarad.

We may conclude that any induction-coil in which the coupling is greater than $\cdot 756$ can be improved in spark-length by connecting external series inductance in the primary circuit so as to reduce the coupling to this value. An induction-coil should not be constructed so that the coupling is less than $\cdot 756$, for there is no convenient way of increasing the coupling to this value. Nor should the coupling coefficient of the primary and secondary coils have precisely this value, though if no external coils are used this is the best arrangement. Better effects are obtained by coupling the coils more closely than this, and adding series inductance to the primary circuit so as to reduce k to this value. The ratio L_1/C_2 is thereby increased, and it is upon this quantity, when the adjustments (1) and (2) are effected, that the maximum secondary potential for a given primary current chiefly depends.

In fig. 1 (A) are shown curves representing the two potential waves in the primary circuit of an induction-coil in which the above adjustment has been effected, and in which the damping of the oscillations is negligible. The amplitude of each wave is taken as 5000 volts, and the periods are $\cdot 0024$ sec. and $\cdot 0008$ sec. These values are chosen so as to correspond approximately (for $i_0 = 10$ amperes) with those found in an actual case, as described below. Fig. 1 (B) shows the result of superposing the two curves of (A). The equation to curve (B), the ordinate of which represents the potential of the primary condenser t seconds after the interruption, is

$$V_1 = -5000 \sin 150,000 t - 5000 \sin 450,000 t,$$

the angles being given in degrees.

* It was explained in the previous paper (*l. c.* pp. 584, 585) how the most effective primary capacity in any such case can be calculated.

Fig. 1.

Primary potential.

The curves in fig. 1 cover one half-period of the longer wave ; their continuation in the second half-period is merely a repetition with the ordinates reversed in sign.

Fig. 2.

Secondary potential.

Fig. 2 (A) shows the corresponding potential waves in the secondary circuit ; fig. 2 (B) the effect of their superposition. The amplitudes here are taken as 600,000 and 200,000 volts

respectively. It will be seen that the two positive maxima coincide at $\cdot 0006$ sec. after the interruption, giving the maximum potential of 800,000 volts at the secondary terminals. At this moment the potential of the primary condenser is zero. Fig. 2 (C) shows (for comparison with the oscillograph curves described below) the curve obtained by squaring the ordinates of fig. 2 (B).

The above theoretical conclusions apply strictly only to the case of an ideal induction-coil, in which the resistances of the circuits and other causes of damping of the oscillations are negligible. Some applications of the theory were, however, given, in the former paper referred to*, in which it was shown that the results of the theory agree closely with experiment in so far as they concern the conditions under which the greatest spark-length is given by an induction-coil. The value of the secondary potential is of course greatly modified by the damping.

Another experiment will now be described, from the results of which some further information may be gathered as to the working of an induction-coil when in its most effective adjustment.

The coil experimented upon was the one employed in the previous experiments, viz.: an 18-inch coil in which the coefficient of coupling of the primary and secondary coils (the secondary terminals being connected to the oscillograph and to a variable spark-gap) is $\cdot 876$. The primary circuit was fed by a number of storage-cells, and included an amperemeter, a rheostat, and a slow interruptor which broke the current about once per second. The condenser was connected directly across the interruptor.

The experiment consisted in increasing the self-inductance of the primary circuit by the addition of air-core coils, and varying the capacity of the condenser, until the longest spark was produced by a given primary current. The curve of secondary potential was then photographed, and the constants of the circuits determined. The best effect was found to be obtained with four extra coils in the primary circuit, and with a condenser of $\cdot 2$ microfarad capacity. It was not very easy to decide with great exactness upon the best self-inductance and capacity, owing to the fact that the spark-length varies slowly with these quantities in the neighbourhood of the maximum. A difference of $\cdot 025$ mfd. either way in C_1 caused no appreciable change in the spark-length. From the practical point of view this slow variation near the maximum is in itself no disadvantage, since very exact

* *L. c.* pp. 582-586.

adjustment is not necessary in order to produce a spark-length practically equal to the maximum.

The constants of the circuits were measured by methods which were fully described in the previous paper, and found to be as follows :—

$$\frac{1}{L_1 C_1} = 19.57 \cdot 10^6 \text{ c.g.s.},$$

$$\frac{1}{L_2 C_2} = 8.763 \cdot 10^6 \quad ,,$$

$$k = .748,$$

$$L_{21} = 20.4 \text{ henries},$$

$$\frac{R_1}{L_1} = 680,$$

$$\frac{R_2}{L_2} = 825.$$

R_0 , the resistance of the primary circuit for steady currents, was 8 ohms.

Hence

$$\theta_1 = \frac{1}{2} R_1 C_1 = 1.738 \cdot 10^{-5} \text{ c.g.s.}$$

$$\theta_2 = \frac{1}{2} R_2 C_2 = 4.709 \cdot 10^{-5} \quad ,,$$

$$\theta_0 = \frac{1}{2} R_0 C_1 = 8 \cdot 10^{-7} \quad ,,$$

The quantities R_1/L_1 , R_2/L_2 , were determined from the decay factors of the circuits when oscillating separately. The effective resistances R_1 , R_2 , are much greater than the steady-current values ; they depend also upon losses due to leakage, hysteresis, and absorption. A considerable part of R_2 is probably due to leakage through the electrometer. The decay factors are not constant ; the values of R_1/L_1 , R_2/L_2 given above prevail during oscillations of fairly large amplitude in the primary and secondary circuits respectively.

The calculated frequencies are :

$$n_1 = 413.9,$$

$$n_2 = 1208,$$

giving $n_2/n_1 = 2.919.$

Also $L_1 C_1 / L_2 C_2 = .448,$

while $1 - k^2 = .440.$

The differences between these values and those of the ideal case ($n_2/n_1 = 3$, $k = .756$, $L_1 C_1 / L_2 C_2 = .429$) are within the limits of experimental error, and may be due to the above-mentioned difficulty of adjusting L_1 and C_1 accurately

to give the best effect. If L_1 and C_1 were reduced, by amounts which would cause no appreciable change in the spark-length, the differences would disappear.

From the above data the secondary potential can now be calculated*, and the result, expressed in volts for $i_0=10$ amperes, is

$$V_2 = 601300 \cdot e^{-266t} \sin(149000t - 1.95^\circ) \\ - 206900 \cdot e^{-1440t} \sin(435000t - 5.7^\circ).$$

From this expression values of V_2 , the difference of potential at the secondary terminals t seconds after the interruption of the primary current, were calculated for

Fig. 3.

various values of t up to $\cdot 003$ second, a time covering rather more than one whole period of the longer wave. The results are exhibited in fig. 3, in which (A) shows the two oscillations

* The general expression for V_2 is given in the previous paper, *l. c.* pp. 565, 574. The statement in the footnote on p. 574 requires correction. When the condenser is connected directly across the interruptor, so that the battery (e.m.f. E) is included in the primary oscillating circuit, we have initially $V_1=0$, but finally $V_1=-E$. The results as given on pp. 574, 575, apply to this case provided V_1 is taken as E —the potential-difference of the primary condenser.

of the secondary circuit separately, and (B) the result of their superposition. It will be seen that two positive maxima in the oscillations agree at about $t=0.0006$ sec., giving rise to the maximum secondary potential (596900 volts) at this time. Fig. 4 shows the square of the secondary

Fig. 4.

Square of secondary potential.

potential plotted against the time. This curve shows clearly the peaked maxima and the flattened zeroes which are characteristic of the 3/1 ratio.

In Plate I. figs. 6 and 7 are shown photographic records of the secondary potential wave obtained with the oscillograph. In these curves the ordinates are proportional to the square of the secondary potential, owing to the idiosyncratic connexion of the instrument, and these curves are therefore directly comparable with fig. 4. The currents interrupted when these photographs were taken were 1.5 and 2.0 amperes respectively*. It will be seen that the greatest ordinates of the curves are proportional to the squares of the currents. In period †, rate of decay, and form the calculated curve of fig. 4 agrees well with the photographs. The photographic curves are perhaps rather more

* The greatest spark-length for 2 amperes, between spark-balls 2 cm. in diameter, was 18.2 cm. The external inductance being removed, and the primary coil being still in its symmetrical position within the secondary, the greatest spark-length at 2 amperes ($C_1=0.6$ mfd.) was 12.8 cm. No sparks passed when the photographs shown were taken.

† The time curve shown on the photographs has a period of $1/768$ sec.

peaked at the summits, and more flattened at the zeroes, than the calculated curve, which indicates that the calculated damping factor of the shorter wave is rather too great in comparison with that of the slower oscillation. This may possibly be due to the existence of an appreciable difference (arising from various causes) between the values of the effective resistances of the circuits when oscillating separately, and their values when the circuits are oscillating as a coupled system.

Turning now to the primary circuit, Dibbern's formula allows the potential wave in the primary condenser to be calculated*. Using the values given above for the constants of the circuits, and taking i_0 , the primary current interrupted, as 10 amperes, Dibbern's expression becomes in the present case

$$V_1 \dagger = -5020 e^{-266 t} \sin (149000 t + 8.02)^\circ \\ - 4940 e^{-1440 t} \sin (435000 t - 9.07)^\circ, \quad . \quad . \quad (4)$$

where V_1 is the primary potential in volts.

The amplitudes of the two oscillations in the primary circuit are thus nearly equal, as required by condition (2).

The two oscillations represented by (4) are shown in fig. 5 (A), the result of their superposition in fig. 5 (B). It will be seen that the negative potential‡ of the primary condenser reaches a maximum of 6800 volts at about .00025 sec., and a minimum of 2250 volts at about .00065 sec. after the interruption. Thus, at the moment at which the secondary potential reaches its greatest value the primary condenser, instead of being uncharged as would be the case in an ideal induction-coil, is still charged to about 2250 volts, and this is due almost entirely to the difference in the damping factors of the two oscillations.

The effective resistances of the circuits therefore act in two ways in reducing the efficiency of the arrangement. First, they give rise to dissipation of energy and consequent decay of the amplitudes of both oscillations. Second, owing to the difference between the damping factors of the two oscillations, there is some energy stored in the primary condenser at the moment when the secondary potential is at its maximum.

* The existence of the two oscillations in the primary circuit after the interruption is well shown by a current oscillogram taken by Wertheim-Salomonson, *Physik. Zeitschr.* xi. p. 539, fig. 1 (1910).

† As explained in the footnote on p. 9 above, V_1 here means $E +$ the potential of the condenser. E is here 80 volts.

‡ The potential of the primary condenser is taken negative when it opposes the battery E.M.F.

The efficiency is easily calculated in the present case. The maximum secondary potential, for $i_0 = 10$ amperes, is 596900 volts. If the resistances were negligible, and if the system

Fig. 5.

were exactly in the first unit-efficiency adjustment, the maximum secondary potential would be, by (3a), 798700 volts. Consequently the efficiency is $\left(\frac{5969}{7987}\right)^2 = .559$.

The energy originally in the system is $\frac{1}{2}L_1i_0^2$, *i. e.* since L_1 is about .255 henry, 12.75 joules. The maximum electrostatic energy in the secondary circuit is therefore $.559 \times 12.75$, or about 7.1 joules. The energy then stored in the primary condenser (capacity .2 mfd.) is $\frac{1}{2} \cdot .2 \cdot 2250^2 \cdot 10$ ergs, or about .5 joule.

Consequently of the original 12.75 joules rather over 5 are dissipated, half a joule is stored in the primary condenser, and the remaining 7 joules are available to provide for the secondary discharge at the moment of maximum potential.

The charge then on the secondary is C_2V_2 and may be calculated when the capacity C_2 is known. If we knew the value of the ratio L_{21}/L_{12} the capacity could be at once

determined from the known values of k^2 , L_1 , L_{21} , and L_2C_2 , since

$$C_2 = \frac{L_{21}}{L_{12}} \cdot \frac{L_1}{L_{21}^2} \cdot k^2 \cdot L_2 C_2. \quad . \quad . \quad . \quad (5)$$

With regard to the ratio L_{21}/L_{12} , this differs from unity because the current in the secondary coil during the oscillations is not uniformly distributed, but is greatest at the central winding and nearly zero at the ends. If all the windings of the secondary had equal inductive effects on the primary, when reckoned per unit current, and if the current in a turn of the secondary at a distance z from the central winding were proportional to $\cos \frac{\pi z}{h}$, where h is the length of the secondary coil, it is easily seen that L_{21}/L_{12} would be equal to $\pi/2$. In the actual case, however, the inductive effect of the secondary windings (per unit current) diminishes from the centre towards each end. This was tested by ballistic galvanometer experiments in which the mutual inductance of the primary and a single turn of wire, wound on the secondary (or primary) in various positions, was compared with its value for the central position. From the results of these measurements it was found that this mutual inductance could be represented approximately by the expression $a - bz^2 - cz^4$ *. Consequently L_{12} is proportional to

$$\int_{-h/2}^{+h/2} (a - bz^2 - cz^4) \cos \frac{\pi z}{h} dz,$$

while L_{21} is proportional to

$$\int_{-h/2}^{+h/2} (a - bz^2 - cz^4) dz,$$

since the current in the primary coil is uniformly distributed. The value of L_{21}/L_{12} is thereby reduced from $\pi/2$, and becomes in the present case $\cdot 95 \pi/2$.

Another correction is necessary if, as in the present experiments, the secondary terminals are connected to a capacity which is not negligible in comparison with that of the coil. In this case the secondary current is not quite zero at the ends of the coil, but should be represented as proportional to $\cos \frac{\pi z}{h'}$, where h' is greater than h . The value

* The dimensions of the coils are :—primary, length 90 cm., mean diameter 6.9 cm.; secondary, length (between the terminals) 51 cm., diameter (outside) 24 cm.

14 *Most Effective Adjustment of an Induction-coil.*

of h' may be estimated if we know the ratio of the external capacity C_e to the total capacity C_2 . If C_e is small in comparison with C_2 the approximate value of the latter (obtained from equation (5) by neglecting the present correction) may be used here. If the current in the secondary windings varies as $\cos \frac{\pi z}{h'}$, the charge per unit length will be proportional to $\sin \frac{\pi z}{h'}$. Hence the ratio of C to C_2 is equal to the ratio of

$$\int_{-h/2}^{+h/2} \sin \frac{\pi z}{h'} dz \quad \text{to} \quad \int_0^{h/2} \sin \frac{\pi z}{h'} dz,$$

$$i. e. \quad \frac{C_e}{C_2} = \cos \frac{\pi h}{2h'}.$$

This determines h'/h , and we then have

$$\frac{L_{12}}{L_{21}} = \frac{1}{h} \int_{-h/2}^{+h/2} \cos \frac{\pi z}{h'} dz$$

$$= \frac{2}{\pi} \cdot \frac{h'}{h} \cdot \sin \frac{\pi h}{2h'}.$$

In the present experiments C_e is the capacity of the electrometer and the spark-gap terminals, and this is about one-sixth of the total secondary capacity C_2 , the value of which is already known approximately. Hence $\cos \frac{\pi h}{2h'} = \frac{1}{6}$, and $\frac{h'}{h} = 1.12$, from which $\frac{L_{12}}{L_{21}} = 1.10 \cdot \frac{2}{\pi}$. The effect of this correction is therefore further to reduce L_{21}/L_{12} by about 10 per cent.

Taking both corrections into account we have approximately

$$L_{21}/L_{12} = .85\pi/2 = 1.335,$$

from which by (5)

$$C_2 = .000052 \text{ microfarad.}$$

The charge of the secondary circuit at the moment of maximum potential is therefore $5.2 \times 5.969 \times 10^{-7}$ c.g.s., or 31.10^{-6} coulomb. If the whole of this charge escaped from the terminals, in the form of a spark or other discharge, the discharge current would be, at n interruptions per second, nC_2V_2 . Further experiments are, however, required in order to decide whether this complete discharge takes place, or whether some of the electricity does not return through the secondary coil and continue to oscillate in it.

The conditions for maximum potential are the same when a rapid interruptor is employed as for the slow break used in the above experiments. They also hold whether the primary current is supplied by a small storage-battery, a 100-volt battery, or the 200-volt mains. Plate I. fig. 8 shows the course of the secondary potential at two successive "breaks" effected by a motor mercury interruptor. The effect of the small potential at the "make" is also noticeable. On this occasion the 100-volt battery was used to supply the primary current, the total (steady-current) resistance in the primary circuit being 11 ohms. The greatest spark-length was 18.4 cm., and the mean primary current, as indicated by an amperemeter in the primary circuit, was .3 ampere. The current immediately before interruption would, however, be over 2 amperes.

Bangor, November 1914.

II. *On Motion in a Periodic Field of Force.*

By C. V. RAMAN, M.A.*

[Plate II.]

Vibrations maintained by a Periodic Field of Force.

THE experimental study of the motion of a dynamical system in a periodic field of force leads to results of quite exceptional interest. One aspect of the problem, *i. e.* the oscillatory motion of the system about a position of equilibrium in the field, has some affinities to the case of vibrations maintained by a variable spring which I have dealt with in my previously published work, but the two classes of investigations lead to results which differ from one another, yet are related in a most remarkable way. By experimenting on stretched strings subjected to a variable tension, I showed that a normal variation of spring will enable the oscillations of the system to be maintained, when the frequency of these oscillations is sufficiently nearly equal to $\frac{1}{2}$ of, or $\frac{2}{3}$ times, or $\frac{3}{2}$ times, or $\frac{4}{3}$ times, &c. the frequency of variation of the spring, these ratios forming an ascending series †. By experiments on the vibrations of a body about a position of equilibrium in a periodic field of force (to be described below), I have shown that the frequency of the oscillations maintained may be equal to, or half of, or one-third, or one-fourth, &c. of the frequency of the field; in other words, it

* Communicated by the Author.

† See Phil. Mag. Oct. 1912, "The Maintenance of Forced Oscillations of a New Type."

FIG. 6.

Primary current 1.5 amperes.

FIG. 7.

Primary current 2 amperes.

FIG. 8.

Two successive breaks by motor interruptor.