XXXVI.—*An Account of* CARNOT'S *Theory of the Motive Power of Heat;** *with Numerical Results deduced from* REGNAULT'S *Experiments on Steam.*† By WILLIAM THOMSON, Professor of Natural Philosophy in the University of Glasgow.

(Read January 2, 1849.)

1. The presence of heat may be recognised in every natural object; and there is scarcely an operation in nature which is not more or less affected by its all-pervading influence. An evolution and subsequent absorption of heat generally give rise to a variety of effects; among which may be enumerated, chemical combinations or decompositions; the fusion of solid substances; the vaporisation of solids or liquids; alterations in the dimensions of bodies, or in the statical pressure by which their dimensions may be modified; mechanical resistance overcome; electrical currents generated. In many of the actual phenomena of nature, several or all of these effects are produced together; and their complication will, if we attempt to trace the agency of heat in producing any individual effect, give rise to much perplexity. It will, therefore, be desirable, in laying the foundation of a physical theory of any of the effects of heat, to discover or to imagine phenomena free from all such complication, and depending on a definite thermal agency; in which the relation between the cause and effect, traced through the medium of certain simple operations, may be clearly appreciated. Thus it is that CARNOT, in accordance with the strictest principles of philosophy, enters upon the investigation of the theory of the motive power of heat.

2. The sole effect to be contemplated in investigating the motive power of heat is *resistance overcome*, or, as it is frequently called, "*work performed,*" or "*mechanical effect.*" The questions to be resolved by a complete theory of the subject are the following:

(1.) What is the precise nature of the thermal agency by means of which *mechanical effect* is to be produced, without effects of any other kind?

* Published in 1824, in a work entitled, "Réflexions sur la Puissance Motrice du Feu, et sur les Machines Propres à Déveloper cette Puissance. Par S. CARNOT." An account of CARNOT'S Theory is also published in the *Journal d'École Polytechnique*, vol. xiv., 1834, in a paper by Mons. CLAPEYRON.

† An account of the first part of a series of researches undertaken by Mons. REGNAULT, by order of the late French Government, for ascertaining the various physical data of importance in the theory of the steam-engine, has been recently published (under the title, "Relation des Expériences," &c.) in the *Mémoires de l'Institut*, of which it constitutes the twenty-first volume (1847). The second part of these researches has not yet been published.

(2.) How may the amount of this thermal agency necessary for performing a given quantity of work be estimated?

3. In the following paper I shall commence by giving a short abstract of the reasoning by which CARNOT is led to an answer to the first of these questions; I shall then explain the investigation by which, in accordance with his theory, the experimental elements necessary for answering the second question are indicated; and, in conclusion, I shall state the *data* supplied by REGNAULT's recent observations on steam, and apply them to obtain, as approximately as the present state of experimental science enables us to do, a complete solution of the question.

I. On the nature of Thermal agency, considered as a motive power.

4. There are [at present known] two, and only two, distinct ways in which mechanical effect can be obtained from heat. One of these is by means of the alterations of volume which bodies may experience through the action of heat; the other is through the medium of electric agency. SEEBECK's discovery of thermo-electric currents enables us at present to conceive of an electro-magnetic engine supplied from a thermal origin, being used as a motive power: but this discovery was not made until 1821, and the subject of thermo-electricity can only have been generally known in a few isolated facts, with reference to the electrical effects of heat upon certain crystals, at the time when CARNOT wrote. He makes no allusion to it, but confines himself to the method for rendering thermal agency available as a source of mechanical effect, by means of the expansions and contractions of bodies.

5. A body expanding or contracting under the action of force, may, in general, either produce mechanical effect by overcoming resistance, or receive mechanical effect by yielding to the action of force. The amount of mechanical effect thus developed will depend not only on the calorific agency concerned, but also on the alteration in the physical condition of the body. Hence, after allowing the volume and temperature of the body to change, we must restore it to its original temperature and volume; and then we may estimate the aggregate amount of mechanical effect developed as due solely to the thermal origin.

6. Now the ordinarily-received, and almost universally-acknowledged, principles with reference to "quantities of caloric" and "latent heat," lead us to conceive that, at the end of a cycle of operations, when a body is left in precisely its primitive physical condition, if it has absorbed any heat during one part of the operations, it must have given out again exactly the same amount during the remainder of the cycle. The truth of this principle is considered as axiomatic by CARNOT, who admits it as the foundation of his theory; and expresses himself in the following terms regarding it, in a note on one of the passages of his treatise.*

* CARNOT, p. 37.

"In our demonstrations we tacitly assume that after a body has experienced a certain number of transformations, if it be brought identically to its primitive physical state as to density, temperature, and molecular constitution, it must contain the same quantity of heat as that which it initially possessed; or, in other words, we suppose that the quantities of heat lost by the body under one set of operations are precisely compensated by those which are absorbed in the others. This fact has never been doubted; it has at first been admitted without reflection, and afterwards verified, in many cases, by calorimetrical experiments. To deny it would be to overturn the whole theory of heat, in which it is the fundamental principle. It must be admitted, however, that the chief foundations on which the theory of heat rests, would require a most attentive examination. Several experimental facts appear nearly inexplicable in the actual state of this theory."

7. Since the time when CARNOT thus expressed himself, the necessity of a most careful examination of the entire experimental basis of the theory of heat has become more and more urgent. Especially all those assumptions depending on the idea that heat is a *substance*, invariable in quantity; not convertible into any other element, and incapable of being *generated* by any physical agency; in fact the acknowledged principles of latent heat; would require to be tested by a most searching investigation before they ought to be admitted, as they usually have been, by almost every one who has been engaged on the subject, whether in combining the results of experimental research, or in general theoretical investigations.

8. The extremely important discoveries recently made by Mr JOULE of Manchester, that heat is evolved in every part of a closed electric conductor, moving in the neighbourhood of a magnet,* and that heat is *generated* by the friction of fluids in motion, seem to overturn the opinion commonly held that heat cannot be *generated*, but only produced from a source, where it has previously existed either in a sensible or in a latent condition.

* The *evolution* of heat in a fixed conductor, through which a galvanic current is sent from any source whatever, has long been known to the scientific world; but it was pointed out by Mr JOULE that we cannot infer from any previously-published experimental researches, the actual *generation* of heat when the current originates in electro-magnetic induction; since the question occurs, *is the heat which is evolved in one part of the closed conductor merely transferred from those parts which are subject to the inducing influence?* Mr JOULE, after a most careful experimental investigation with reference to this question, finds that it must be answered in the negative.—(See a paper "*On the Calorific Effects of Magneto-Electricity, and on the Mechanical Value of Heat;* by J. P. JOULE, Esq." Read before the British Association at Cork in 1843, and subsequently communicated by the Author to the *Philosophical Magazine*, vol. xxiii., pp. 263, 347, 435.)

Before we can finally conclude that heat is absolutely generated in such operations, it would be necessary to prove that the inducing magnet does not become lower in temperature, and thus compensate for the heat evolved in the conductor. I am not aware that any examination with reference to the truth of this conjecture has been instituted; but, in the case where the inducing body is a pure electro-magnet (without any iron), the experiments actually performed by Mr JOULE render the conclusion probable that the heat evolved in the wire of the electro-magnet is not affected by the inductive action, otherwise than through the reflected influence which increases the strength of its own current.

In the present state of science, however, no operation is known by which heat can be absorbed into a body without either elevating its temperature, or becoming latent, and producing some alteration in its physical condition; and the fundamental axiom adopted by CARNOT may be considered as still the most probable basis for an investigation of the motive power of heat; although this, and with it every other branch of the theory of heat may ultimately require to be reconstructed upon another foundation, when our experimental data are more complete. On this understanding, and to avoid a repetition of doubts, I shall refer to CARNOT's fundamental principle, in all that follows, as if its truth were thoroughly established.

9. We are now led to the conclusion that the origin of motive power, developed by the alternate expansions and contractions of a body, must be found in the agency of heat entering the body and leaving it; since there cannot, at the end of a complete cycle, when the body is restored to its primitive physical condition, have been any absolute absorption of heat, and consequently no conversion of heat, or caloric, into mechanical effect; and it remains for us to trace the precise nature of the circumstances under which heat must enter the body, and afterwards leave it, so that mechanical effect may be produced. As an example, we may consider that machine for obtaining motive power from heat with which we are most familiar—the steam-engine.

10. Here, we observe, that heat enters the machine from the furnace, through the sides of the boiler, and that heat is continually abstracted by the water employed for keeping the condenser cool. According to CARNOT's fundamental principle, the quantity of heat thus discharged, during a complete revolution (or double stroke) of the engine must be precisely equal to that which enters the water of the boiler;* provided the total mass of water and steam be invariable, and be restored to its primitive physical condition (which will be the case rigorously, if the condenser be kept cool by the external application of cold water, instead of by injection, as is more usual in practice), and if the condensed water be restored to the boiler at the end of each complete revolution. Thus, we perceive, that a certain quantity of heat is *let down* from a hot body, the metal of the boiler, to another body at a lower temperature, the metal of the condenser; and that there results from this transference of heat, a certain development of mechanical effect.

11. If we examine any other case in which mechanical effect is obtained from a thermal origin, by means of the alternate expansions and contractions of any substance whatever, instead of the water of a steam-engine, we find that a similar transference of heat is effected, and we may therefore answer the first question proposed, in the following manner:—

The thermal agency by which mechanical effect may be obtained, is the transference of heat from one body to another at a lower temperature.

* So generally is CARNOT's principle tacitly admitted as an axiom, that its application in this case has never, so far as I am aware, been questioned by practical engineers.

II. On the measurement of Thermal Agency, considered with reference to its equivalent of mechanical affect.

12. A *perfect* thermo-dynamic engine of any kind, is a machine by means of which the greatest possible amount of mechanical effect can be obtained from a given thermal agency; and, therefore, if in any manner we can construct or imagine a perfect engine which may be applied for the transference of a given quantity of heat from a body at any given temperature, to another body, at a lower given temperature, and if we can evaluate the mechanical effect thus obtained, we shall be able to answer the question at present under consideration, and so to complete the theory of the motive power of heat. But whatever kind of engine we may consider with this view, it will be necessary for us to prove that it is a perfect engine; since the transference of the heat from one body to the other may be wholly, or partially, effected by conduction through a solid,* without the development of mechanical effect; and, consequently, engines may be constructed in which the whole, or any portion of the thermal agency is wasted. Hence it is of primary importance to discover the criterion of a perfect engine. This has been done by CARNOT, who proves the following proposition :—

13. *A perfect thermo-dynamic engine is such that, whatever amount of mechanical effect it can derive from a certain thermal agency; if an equal amount be spent in working it backwards, an equal reverse thermal effect will be produced.*†

14. This proposition will be made clearer by the applications of it which are given below (§ 29), in the cases of the air-engine and the steam-engine, than it could be by any general explanation; and it will also appear, from the nature of the operations described in those cases, and the principles of CARNOT's reasoning, that a perfect engine may be constructed with any substance of an indestructible texture as the alternately expanding and contracting medium. Thus we might conceive thermo-dynamic engines founded upon the expansions

* When "thermal agency" is thus spent in conducting heat through a solid, what becomes of the mechanical effect which it might produce? Nothing can be lost in the operations of nature—no energy can be destroyed. What effect then is produced in place of the mechanical effect which is lost? A perfect theory of heat imperatively demands an answer to this question; yet no answer can be given in the present state of science. A few years ago, a similar confession must have been made with reference to the mechanical effect lost in a fluid set in motion in the interior of a rigid closed vessel, and allowed to come to rest by its own internal friction; but in this case, the foundation of a solution of the difficulty has been actually found, in Mr JOULE's discovery of the generation of heat, by the internal friction of a fluid in motion. Encouraged by this example, we may hope that the very perplexing question in the theory of heat, by which we are at present arrested, will, before long, be cleared up.

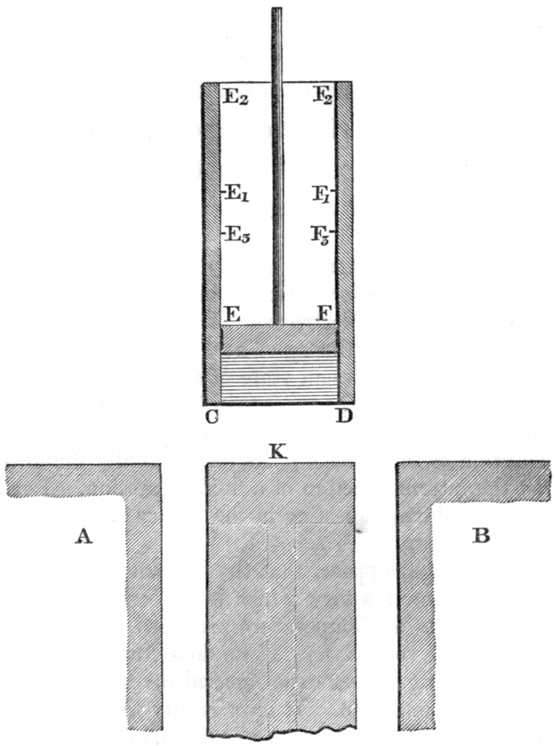
It might appear, that the difficulty would be entirely avoided, by abandoning CARNOT's fundamental axiom; a view which is strongly urged by Mr JOULE (at the conclusion of his paper "On the Changes of Temperature produced by the Rarefaction and Condensation of Air." *Phil. Mag.*, May 1845, vol. xxvi.) If we do so, however, we meet with innumerable other difficulties—insuperable without farther experimental investigation, and an entire reconstruction of the theory of heat, from its foundation. It is in reality to experiment that we must look—either for a verification of CARNOT's axiom, and an explanation of the difficulty we have been considering; or for an entirely new basis of the Theory of Heat.

† For a demonstration, see § 29, below.

and contractions of a perfectly elastic solid, or of a liquid; or upon the alterations of volume experienced by substances, in passing from the liquid to the solid state,* each of which being perfect, would produce the same amount of mechanical effect from a given thermal agency; but there are two cases which CARNOT has selected as most worthy of minute attention, because of their peculiar appropriateness for illustrating the general principles of his theory, no less than on account of their very great practical importance; the steam-engine, in which the substance employed as the transferring medium is water, alternately in the liquid state, and in the state of vapour; and the air-engine, in which the transference is effected by means of the alternate expansions and contractions of a medium, always in the gaseous state. The details of an actually practicable engine of either kind are not contemplated by CARNOT, in his general theoretical reasonings, but he confines himself to the ideal construction, in the simplest possible way in each case, of an engine in which the economy is perfect. He thus determines the degree of perfectibility which cannot be surpassed; and, by describing a conceivable method of attaining to this perfection by an air-engine or a steam-engine, he points out the proper objects to be kept in view in the practical construction and working of such machines. I now proceed to give an outline of these investigations.

CARNOT'S *Theory of the Steam-Engine.*

15. Let $C D F_2 E_2$ be a cylinder, of which the curved surface is perfectly impermeable to heat, with a piston also impermeable to heat, fitted in it; while the fixed bottom C D, itself with no capacity for heat, is possessed of perfect conducting power. Let K be an impermeable stand, such that when the cylinder is placed upon it, the contents below the piston can neither gain nor lose heat. Let A and B be two bodies permanently retained at constant temperatures, S^0 and T^0, respectively, of which the former is higher than the latter. Let the cylinder, placed on the impermeable stand, K, be partially filled with water, at the temperature S, of the body A, and (there being no air below it) let the piston be placed in a position E F, near the surface of the water. The



* A case minutely examined in another paper, to be laid before the Society at the present meeting.

pressure of the vapour above the water will tend to push up the piston, and must be resisted by a force applied to the piston,* till the commencement of the operations, which are conducted in the following manner.

(1.) The cylinder being placed on the body A, so that the water and vapour may be retained at the temperature S, *let the piston rise any convenient height* $E\,E_1$, *to a position* $E_1\,F_1$, *performing work by the pressure of the vapour below it during its ascent.*

[During this operation a certain quantity, H, of heat, the amount of latent heat in the fresh vapour which is formed, is abstracted from the body A.]

(2.) The cylinder being removed, and placed on the impermeable stand K, *let the piston rise gradually, till, when it reaches a position* $E_2\,F_2$, *the temperature of the water and vapour is* T, *the same as that of the body* B.

[During this operation the fresh vapour continually formed requires heat to become latent; and, therefore, as the contents of the cylinder are protected from any accession of heat, their temperature sinks.]

(3.) The cylinder being removed from K, and placed on B, *let the piston be pushed down, till, when it reaches the position* $E_3\,F_3$, *the quantity of heat evolved and abstracted by* B *amounts to that which, during the first operation, was taken from* A.

[During this operation the temperature of the contents of the cylinder is retained constantly at T^0, and all the latent heat of the vapour which is condensed into water at the same temperature, is given out to B.]

(4.) The cylinder being removed from B, and placed on the impermeable stand, *let the piston be pushed down from* $E_3\,F_3$ *to its original position* E F.

[During this operation, the impermeable stand preventing any loss of heat, the temperature of the water and air must rise continually, till (since the quantity of heat evolved during the third operation was precisely equal to that which was previously absorbed), at the conclusion it reaches its primitive value, S, in virtue of Carnot's fundamental axiom.]

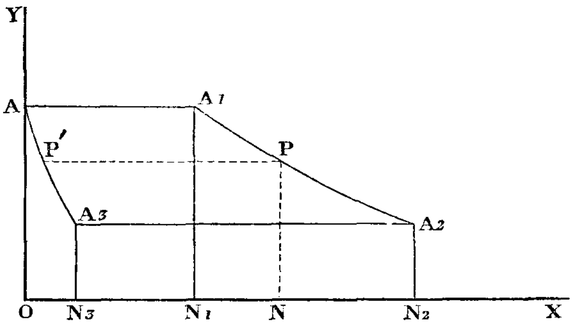
16. At the conclusion of this cycle of operations† the total thermal agency has been the *letting down* of H units of heat from the body A, at the temperature S, to B, at the lower temperature T; and the aggregate of the mechanical effect has been a certain amount of *work produced*, since during the ascent of the piston in the first and second operations, the temperature of the water and vapour, and therefore the pressure of the vapour on the piston, was on the whole higher than during the descent, in the third and fourth operations. It remains for us actually to evaluate this aggregate amount of work performed; and for this purpose the

* In all that follows, the pressure of the atmosphere on the upper side of the piston will be included in the applied forces, which, in the successive operations described, are sometimes overcome by the upward motion, and sometimes yielded to in the motion downwards. It will be unnecessary, in reckoning at the end of a cycle of operations, to take into account the work thus spent upon the atmosphere, and the restitution which has been made, since these precisely compensate for one another.

† In CARNOT's work some perplexity is introduced with reference to the temperature of the water, which, in the operations he describes, is not brought back exactly to what it was at the commencement; but the difficulty which arises is explained by the author. No such difficulty occurs with reference to the cycle of operations described in the text, for which I am indebted to Mons. CLAPEYRON.

following graphical method of representing the mechanical effect developed in the several operations, taken from Mons. CLAPEYRON's paper, is extremely convenient.

17. Let O X and O Y be two lines at right angles to one another. Along O X measure off distances O N_1, N N_2, $N_2 N_3$, N_3 O, respectively proportional to the spaces described by the piston during the four successive operations described above; and, with reference to these four operations respectively, let the following constructions be made :—



(1.) Along O Y measure a length O A, to represent the pressure of the saturated vapour at the temperature S; and draw A A_1 parallel to O X, and let it meet an ordinate through N_1, in A_2.

(2.) Draw a curve A_1 P A such that, if O N represent, at any instant during the second operation, the distance of the piston from its primitive position, N P shall represent the pressure of the vapour at the same instant.

(3.) Through A_2 draw $A_2 A_3$ parallel to O X, and let it meet an ordinate through N_3 in A_3.

(4.) Draw the curve A_3 A such that the abscissa and ordinate of any point in it may represent respectively the distances of the piston from its primitive position, and the pressure of the vapour, at some instant during the fourth operation. The last point of this curve must, according to Carnot's fundamental principle, coincide with A, since the piston is, at the end of the cycle of operations, again in its primitive position, and the pressure of the vapour is the same as it was at the beginning.

18. Let us now suppose that the lengths, O N_1, $N_1 N_2$, $N_2 N_3$, and N_3 O, *represent numerically* the volumes of the spaces moved through by the piston during the successive operations. It follows that the mechanical effect obtained during the first operation will be *numerically represented* by the area A A_1 N_1 O; that is, the number of superficial units in this area will be equal to the number of "foot-pounds" of work performed by the ascending piston during the first operation. The work performed by the piston during the second operation will be similarly represented by the area $A_1 A_2 N_2 N_1$. Again, during the third operation a certain amount of work is spent on the piston, which will be represented by the area $A_2 A_3 N_3 N_2$; and lastly, during the fourth operation, work is spent in pushing the piston to an amount represented by the area A_3 A O N_3.

19. Hence the mechanical effect (represented by the area O A $A_1 A_2 N_2$) which was obtained during the first and second operations, exceeds the work (represented by $N_2 A_2 A_3$ A O) spent during the third and fourth, by an amount represented by the area of the quadrilateral figure A$A_1 A_2 A_3$; and, consequently, it

only remains for us to evaluate this area, that may determine the total mechanical effect gained in a complete cycle of operations. Now, from experimental data, at present nearly complete, as will be explained below, we may determine the length of the line $A A_1$ for the given temperature S, and a given absorption H, of heat, during the first operation; and the length of $A_2 A_3$ for the given lower temperature T, and the evolution of the same quantity of heat during the fourth operation: and the curves $A_1 P A_2$, $A_3 P' A$ may be drawn as graphical representations of actual observations.* The figure being thus constructed, its area may be measured, and we are, therefore, in possession of a graphical method of determining the amount of mechanical effect to be obtained from any given thermal agency. As, however, it is merely the area of the figure which it is required to determine, it will not be necessary to be able to describe each of the curves $A_1 P A_2 A_3 P' A$, but it will be sufficient to know the difference of the abscissas corresponding to any equal ordinates in the two; and the following analytical method of completing the problem is the most convenient for leading to the actual numerical results.

20. Draw any line P P′ parallel to O X, meeting the curvilineal sides of the quadrilateral in P and P′. Let ξ denote the length of this line, and p its distance from O X. The area of the figure, according to the integral calculus, will be denoted by the expression

$$\int_{p_3}^{p_1} \xi \, dp,$$

where p_1, and p_3 (the limits of integration indicated according to FOURIER's notation) denote the lines O A, and $N_3 A_3$, which represent respectively the pressures during the first and third operations. Now, by referring to the construction described above, we see that ξ is the difference of the volumes below the piston at corresponding instants of the second and fourth operations, or instants at which the saturated steam and the water in the cylinder have the same pressure p, and, consequently, the same temperature which we may denote by t. Again, throughout the second operation the entire contents of the cylinder possess a greater amount of heat by H units than during the fourth; and, therefore, at any instant of the second operation there is as much more steam as contains H units of latent heat, than at the corresponding instant of the fourth operation. Hen ce, if k denote the latent heat in a unit of saturated steam at the temperature t, the volume of the steam at the two corresponding instants must differ by $\frac{H}{k}$. Now, if σ denote the ratio of the density of the steam to that of the water, the volume $\frac{H}{k}$ of steam will be formed from the volume $\sigma \frac{H}{k}$ of water; and, consequently, we have

* See Note at the end of this Paper.

for the difference of volumes of the entire contents at the corresponding instants,

$$\xi=(1-\sigma)\frac{H}{k}.$$

Hence the expression for the area of the quadrilateral figure becomes

$$\int_{p_3}^{p_1}(1-\sigma)\frac{H}{k}\,dp.$$

Now, σ, k, and p, being quantities which depend upon the temperature, may be considered as functions of t; and it will be convenient to modify the integral so as to make t the independent variable. The limits will be from $t=T$ to $t=S$, and, if we denote by M the value of the integral, we have the expression

$$M=H\int_T^S(1-\sigma)\frac{\frac{dp}{dt}}{k}\,dt \quad . \quad . \quad . \quad . \quad . \quad (1)$$

for the total amount of mechanical effect gained by the operations described above.

21. If the interval of temperatures be extremely small; so small that $(1-\sigma)\frac{\frac{dp}{dt}}{k}$ will not sensibly vary for values of t between T and S, the preceding expresssion becomes simply

$$M=(1-\sigma)\frac{\frac{dp}{dt}}{k}.\,H\,(S-T) \quad . \quad . \quad . \quad . \quad (2).$$

This might, of course, have been obtained at once, by supposing the breadth of the quadrilateral figure $A\,A_1\,A_2\,A$ to be extremely small compared with its length, and then taking for its area, as an approximate value, the product of the breadth into the line $A\,A_1$, or the line $A_3\,A_2$, or any line of intermediate magnitude.

The expression (2) is rigorously correct for any interval $S-T$, if the mean value of $(1-\sigma)\frac{\frac{dp}{dt}}{k}$ for that interval be employed as the coefficient of $H\,(S-T)$.

CARNOT'S *Theory of the Air-Engine.*

22. In the ideal air-engine imagined by CARNOT four operations performed upon a mass of air or gas enclosed in a closed vessel of variable volume, constitute a complete cycle, at the end of which the medium is left in its primitive physical condition; the construction being the same as that which was described above for the steam-engine, a body A, permanently retained at the temperature S, and B at the temperature T; an impermeable stand K; and a cylinder and piston, which, in this case, contains a mass of air at the temperature S, instead of

water in the liquid state, at the beginning and end of a cycle of operations. The four successive operations are conducted in the following manner:—

(1.) The cylinder is laid on the body A, so that the air in it is kept at the temperature S; and the piston is allowed to rise, performing work.

(2.) The cylinder is placed on the impermeable stand K, so that its contents can neither gain nor lose heat, and the piston is allowed to rise farther, still performing work, till the temperature of the air sinks to T.

(3.) The cylinder is placed on B, so that the air is retained at the temperature T, and the piston is pushed down till the air gives out to the body B as much heat as it had taken in from A, during the first operation.

(4.) The cylinder is placed on K, so that no more heat can be taken in or given out, and the piston is pushed down to its primitive position.

23. *At the end of the fourth operation the temperature must have reached its primitive value* S, *in virtue of* CARNOT'S *axiom.*

24. Here, again, as in the former case, we observe that work is performed by the piston during the first two operations; and, during the third and fourth, work is spent upon it, but to a less amount, since the pressure is on the whole less during the third and fourth operations than during the first and second, on account of the temperature being lower. Thus, at the end of a complete cycle of operations, mechanical effect has been obtained; and the thermal agency from which it is drawn is the taking of a certain quantity of heat from A, and *letting it down*, through the medium of the engine, to the body B at a lower temperature.

25. To estimate the actual amount of effect thus obtained, it will be convenient to consider the alterations of volume of the mass of air in the several operations as extremely small. We may afterwards pass by the integral calculus, or, practically, by summation, to determine the mechanical effect whatever be the amplitudes of the different motions of the piston.

26. Let dq be the quantity of heat absorbed during the first operation, which is evolved again during the third; and let dv be the corresponding augmentation of volume which takes places while the temperature remains constant, as it does during the first operation.* The diminution of volume in the third operation must be also equal to dv, or only differ from it by an infinitely small

* Thus, $\frac{dq}{dv}$ will be the partial differential coefficient, with respect to v of that function of v and t, which expresses the quantity of heat that must be added to a mass of air when in a "standard" state (such as at the temperature zero, and under the atmospheric pressure), to bring it to the temperature t, and the volume v. That there is such a function, of two independent variables v and t, is merely an analytical expression of CARNOT'S fundamental axiom, as applied to a mass of air. The general principle may be analytically stated in the following terms:—If $M\,dv$ denote the accession of heat received by a mass of any kind, not possessing a destructible texture, when the volume is increased by dv, the temperature being kept constant, and if $N\,dt$ denote the amount of heat which must be supplied to raise the temperature by dt, without any alteration of volume; then $M\,dv + N\,dt$ must be the differential of a function of v and t.

quantity of the second order. During the second operation we may suppose the volume to be increased by an infinitely small quantity φ; which will occasion a diminution of pressure, and a diminution of temperature, denoted re-respectively by ω and τ. During the fourth operation there will be a diminution of volume, and an increase of pressure and temperature, which can only differ, by infinitely small quantities of the second order, from the changes in the other direction, which took place in the second operation, and they also may, therefore, be denoted by φ, ω, and τ, respectively. The alteration of pressure, during the first and third operations, may at once be determined by means of MARIOTTE's law, since, in them, the temperature remains constant. Thus, if, at the commencement of the cycle, the volume and pressure be v and p, they will have become $v+dv$ and $p\frac{v}{v+dv}$ at the end of the first operation. Hence the diminution of pressure, during the first operation, is $p-p\frac{v}{v+dv}$ or $p\frac{dv}{v+dv}$; and, therefore, if we neglect infinitely small quantities of the second order, we have $p\frac{dv}{v}$ for the diminution of pressure during the first operation; which, to the same degree of approximation, will be equal to the increase of pressure during the third. If $t+\tau$ and t be taken to denote the superior and inferior limits of temperature, we shall thus have for the volume, the temperature, and the pressure at the commencements of the four successive operations, and at the end of the cycle, the following values respectively:—

(1.)	v,	$t+\tau$,	p;
(2.)	$v+dv$,	$t+\tau$,	$p\left(1-\frac{dv}{v}\right)$;
(3.)	$v+dv+\varphi$,	t,	$p\left(1-\frac{dv}{v}\right)-\omega$;
(4.)	$v+\varphi$,	t,	$p-\omega$;
(5.)	v,	$t+\tau$,	p.

Taking the mean of the pressures at the beginning and end of each operation, we find

(1.) $p\left(1-\frac{1}{2}\frac{dv}{v}\right)$

(2.) $p\left(1-\frac{dv}{v}\right)-\frac{1}{2}\omega$

(3.) $p\left(1-\frac{1}{2}\frac{dv}{v}\right)-\omega$

(4.) $p-\frac{1}{2}\omega$,

which, as we are neglecting infinitely small quantities of the second order, will be the expressions for the mean pressures during the four successive operations. Now, the mechanical effect gained or spent, during any of the operations, will be found by multiplying the mean pressure by the increase or diminution of volume which takes places; and we thus find

$$(1.)\qquad p\left(1-\tfrac{1}{2}\frac{d\,v}{v}\right)d\,v$$

$$(2.)\qquad \left\{p\left(1-\frac{d\,v}{v}\right)-\tfrac{1}{2}\,\omega\right\}\varphi$$

$$(3.)\qquad \left\{p\left(1-\tfrac{1}{2}\frac{d\,v}{v}\right)-\omega\right\}d\,v$$

$$(4.)\qquad (p-\tfrac{1}{2}\,\omega)\,\varphi$$

for the amounts gained during the first and second, and spent during the third and fourth operations; and hence, by addition and subtraction, we find

$$\omega\,d\,v-p\,\varphi\,\frac{d\,v}{v},\text{ or }(v\,\omega-p\,\varphi)\,\frac{d\,v}{v},$$

for the aggregate amount of mechanical effect gained during the cycle of operations. It only remains for us to express this result in terms of $d\,q$ and τ, on which the given thermal agency depends. For this purpose, we remark that φ and ω are alterations of volume and pressure which take place along with a change of temperature τ, and hence, by the laws of compressibility and expansion, we may establish a relation* between them in the following manner.

Let p_0 be the pressure of the mass of air when reduced to the temperature zero, and confined in a volume v_0; then, whatever be v_0, the product $p_0\,v_0$ will, by the law of compressibility, remain constant; and, if the temperature be elevated from 0 to $t+\tau$, and the gas be allowed to expand freely without any change of pressure, its volume will be increased in the ratio of 1 to $1+\mathrm{E}\,(t+\tau)$, where E is very nearly equal to ·00366 (the centigrade scale of the air-thermometer being referred to), whatever be the gas employed, according to the researches of REGNAULT and of MAGNUS on the expansion of gases by heat. If, now, the volume be altered arbitrarily with the temperature continually at $t+\tau$, the product of the pressure and volume will remain constant; and, therefore, we have

$$p\,v=p_0\,v_0\,\{1+\mathrm{E}\,(t+\tau)\}.$$

Similarly
$$(p-\omega)\,(v+\varphi)=p_0\,v_0\,\{1+\mathrm{E}\,t\}.$$

Hence, by subtraction, we have

$$v\,\omega-p\,\varphi+\omega\,\varphi=p_0\,v_0\,\mathrm{E}\,\tau,$$

or, neglecting the product $\omega\,\varphi$,

$$v\,\omega-p\,\varphi=p_0\,v_0\,\mathrm{E}\,\tau.$$

* We might also investigate another relation, to express the fact that there is no accession or removal of heat during either the second or the fourth operation; but it will be seen that this will not affect the result in the text; although it would enable us to determine both φ and ω in terms of τ.

Hence, the preceding expression for mechanical effect, gained in the cycle of operations, becomes

$$p_0 \, v_0 \, . \, \mathrm{E} \tau \, . \, \frac{d\,v}{v},$$

Or, as we may otherwise express it,

$$\frac{\mathrm{E} p_0 \, v_0}{v \dfrac{d\,q}{d\,v}} \, . \, d\,q \, . \, \tau.$$

Hence, if we denote by M the mechanical effect due to H units of heat descending through the same interval τ, which might be obtained by repeating the cycle of operations described above, $\frac{\mathrm{H}}{d\,q}$ times, we have

$$\mathrm{M} = \frac{\mathrm{E} p_0 \, v_0}{v \dfrac{d\,q}{d\,v}} \, . \, \mathrm{H} \tau \quad . \quad . \quad . \quad . \quad (3)$$

27. If the *amplitudes* of the operations had been finite, so as to give rise to an absorption of H units of heat during the first operation, and a lowering of temperature from S to T during the second, the amount of work obtained would have been found to be expressed by means of a double definite integral, thus;*—

$$\left.\begin{aligned} \mathrm{M} &= \int_0^{\mathrm{H}} d\,q \int_{\mathrm{T}}^{\mathrm{S}} d\,t \, . \, \frac{\mathrm{E} p_0 \, v_0}{v \dfrac{d\,q}{d\,v}} \\ \text{or} \quad \mathrm{M} &= \mathrm{E} \, p_0 \, v_0 \int_0^{\mathrm{H}} \int_{\mathrm{T}}^{\mathrm{S}} \frac{1}{v} \frac{d\,v}{d\,q} \, . \, d\,t\,d\,q; \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4),$$

this second form being sometimes more convenient.

28. The preceding investigations, being founded on the approximate laws of compressibility and expansion (known as the law of MARIOTTE and BOYLE, and the law of DALTON and GAY-LUSSAC), would require some slight modifications, to adapt them to cases in which the gaseous medium employed is such as to present sensible deviations from those laws. REGNAULT's very accurate experiments shew that the deviations are insensible, or very nearly so, for the ordinary gases at ordinary pressures; although they may be considerable for a medium, such as

* This result might have been obtained by applying the usual notation of the integral calculus to express the area of the curvilinear quadrilateral, which, according to CLAPEYRON's graphical construction, would be found to represent the entire mechanical effect gained in the cycle of operations of the air-engine. It is not necessary, however, to enter into the details of this investigation, as the formula (3), and the consequences derived from it, include the whole theory of the air-engine, in the best practical form; and the investigation of it which I have given in the text, will probably give as clear a view of the reasoning on which it is founded, as could be obtained by the graphical method, which, in this case, is not so valuable as it is from its simplicity in the case of the steam-engine.

sulphurous acid, or carbonic acid under high pressure, which approaches the physical condition of a vapour at saturation; and therefore, in general, and especially in practical applications to real air-engines, it will be unnecessary to make any modification in the expressions. In cases where it may be necessary, there is no difficulty in making the modifications, when the requisite data are supplied by experiment.

29.* Either the steam-engine or the air-engine, according to the arrangements described above, gives all the mechanical effect that can possibly be obtained from the thermal agency employed. For it is clear, that, in either case, the operations may be performed in the reverse order, with every thermal and mechanical effect reversed. Thus, in the steam-engine, we may commence by placing the cylinder on the impermeable stand, allow the piston to rise, performing work, to the position $E_3 F_3$; we may then place it on the body B, and allow it to rise, performing work, till it reaches $E_2 F_2$; after that the cylinder may be placed again on the impermeable stand, and the piston may be pushed down to $E_1 F_1$; and, lastly, the cylinder being removed to the body A, the piston may be pushed down to its primitive position. In this inverse cycle of operations, a certain amount of work has been spent, precisely equal, as we readily see, to the amount of mechanical effect gained in the direct cycle described above; and heat has been abstracted from B, and deposited in the body A, at a higher temperature, to an amount precisely equal to that which, in the direct cycle, was *let down* from A to B. Hence it is impossible to have an engine which will derive more mechanical effect from the same thermal agency, than is obtained by the arrangement described above; since, if there could be such an engine, it might be employed to perform, as a part of its whole work, the inverse cycle of operations, upon an engine of the kind we have considered, and thus to continually restore the heat from B to A, which has descended from A to B for working itself; so that we should have a complex engine, giving a residual amount of mechanical effect without any thermal agency, or alteration of materials, which is an impossibility in nature. The same reasoning is applicable to the air-engine; and we conclude, generally, that any two engines, constructed on the principles laid down above, whether steam-engines with different liquids, an air-engine and a steam-engine, or two air-engines with different gases, must derive the same amount of mechanical effect from the same thermal agency.

30. Hence, by comparing the amounts of mechanical effect obtained by the steam-engine and the air-engine from the letting down of the H units of heat from A at the temperature $(t+\tau)$ to B at t, according to the expressions (2) and (3), we have

* This paragraph is the demonstration referred to above, of the proposition stated in § 13; as it is readily seen that it is applicable to any conceivable kind of thermo-dynamic engine.

$$M=(1-\sigma)\frac{\frac{dp}{dt}}{k}.H\tau=\frac{Ep_0 v_0}{v\frac{dq}{dv}}.H\tau \quad . \quad . \quad . \quad . \quad (5).$$

If we denote the coefficient of H τ in these equal expressions by μ, which may be called "CARNOT'S coefficient," we have

$$\mu=(1-\sigma)\frac{\frac{dp}{dt}}{k}=\frac{Ep_0 v_0}{v\frac{dq}{dv}} \quad . \quad . \quad . \quad . \quad (6),$$

and we deduce the following very remarkable conclusions:—

(1.) For the saturated vapours of all different liquids, at the same temperature, the value of

$$(1-\sigma)\frac{\frac{dp}{dt}}{k}$$

must be the same.

(2.) For any different gaseous masses, at the same temperature, the value of

$$\frac{Ep_0 v_0}{v\frac{dq}{dv}}$$

must be the same.

(3.) The values of these expressions for saturated vapours and for gases, at the same temperature, must be the same.

31. No conclusion can be drawn *a priori* regarding the values of this coefficient μ for different temperatures, which can only be determined, or compared, by experiment. The results of a great variety of experiments, in different branches of physical science (Pneumatics and Acoustics), cited by CARNOT and by CLAPEYRON, indicate that the values of μ for low temperatures exceed the values for higher temperatures; a result amply verified by the continuous series of experiments performed by REGNAULT on the saturated vapour of water for all temperatures from 0° to 230°, which, as we shall see below, give values for μ gradually diminishing from the inferior limit to the superior limit of temperature. When, by observation, μ has been determined as a function of the temperature, the amount of mechanical effect, M, deducible from H units of heat descending from a body at the temperature S to a body at the temperature T, may be calculated from the expression,

$$M=H\int_T^S \mu\, dt \quad . \quad . \quad . \quad . \quad . \quad (7)$$

which is, in fact, what either of the equations (1) for the steam-engine, or (4) for the air-engine, becomes, when the notation μ, for CARNOT'S multiplier, is introduced.

The values of this integral may be practically obtained, in the most convenient manner, by first determining, from observation, the mean values of μ for the successive degrees of the thermometric scale, and then adding the values for all the degrees within the limits of the extreme temperatures S and T.*

32. The complete theoretical investigation of the motive power of heat is thus reduced to the experimental determination of the coefficient μ; and may be considered as perfect, when, by any series of experimental researches whatever, we can find a value of μ for every temperature within practical limits. The special character of the experimental researches, whether with reference to gases, or with reference to vapours, necessary and sufficient for this object, is defined and restricted in the most precise manner, by the expressions (6) for μ, given above.

33. The object of REGNAULT's great work, referred to in the title of this paper, is the experimental determination of the various physical elements of the steam-engine; and when it is complete, it will furnish all the *data* necessary for the calculation of μ. The valuable researches already published in a first part of that work, make known the latent heat of a given weight, and the pressure, of saturated steam for all temperatures between 0° and 230° cent. of the air-thermometer. Besides these data, however, the density of saturated vapour must be known, in order that k, the latent heat of a unit of volume, may be calculated from REGNAULT's determination of the latent heat of a given weight.† Between the limits of 0° and 100°, it is probable, from various experiments which have been made, that the density of vapour follows very closely the simple laws which are so accurately verified by the ordinary gases;‡ and thus it may be calculated from REGNAULT's table giving the pressure at any temperature within those limits. Nothing as yet is known with accuracy as to the density of saturated steam between 100° and 230°, and we must be contented at present to estimate it by calculation from REGNAULT's table of pressures; although, when accurate experimental researches on the subject shall have been made, considerable deviations from the laws of BOYLE and DALTON, on which this calculation is founded, may be discovered.

* The results of these investigations are exhibited in Tables I. and II. below.

† It is, comparatively speaking, of little consequence to know accurately the value of σ, for the factor $(1-\sigma)$ of the expression for μ, since it is so small (being less than $\frac{1}{1700}$ for all temperatures between 0° and 100°) that, unless all the data are known with more accuracy than we can count upon at present, we might neglect it altogether, and take $\frac{\frac{dp}{dt}}{k}$ simply, as the expression for μ, without committing any error of important magnitude.

‡ This is well established, within the ordinary atmospheric limits, in REGNAULT's Études Météorologiques, in the Annales de Chimie, vol. xv., 1846.

34. Such are the experimental data on which the mean values of μ for the successive degrees of the air-thermometer, from 0° to 230°, at present laid before the Royal Society, is founded. The unit of length adopted is the English foot; the unit of weight, the pound; the unit of work, a "foot-pound;" and the unit of heat that quantity which, when added to a pound of water at 0°, will produce an elevation of 1° in temperature. The mean value of μ for any degree is found to a sufficient degree of approximation, by taking, in place of σ, $\frac{dp}{dt}$, and k; in the expression

$$(1-\sigma)\ \frac{\frac{dp}{dt}}{k};$$

the mean values of those elements; or, what is equivalent to the corresponding accuracy of aproximation, by taking, in place of σ and k respectively, the mean of the values of those elements for the limits of temperature, and in place of $\frac{dp}{dt}$, the difference of the values of p, at the same limits.

35. In REGNAULT's work (at the end of the eighth Mémoire), a table of the pressures of saturated steam for the successive temperatures 0°, 1°, 2°, . . . 230°, expressed in millimetres of mercury, is given. On account of the units adopted in this paper, these pressures must be estimated in pounds on the square foot, which we may do by multiplying each number of millimetres by 2·7896, the weight in pounds of a sheet of mercury, one millimetre thick, and a square foot in area.

36. The value of k, the latent heat of a cubic foot, for any temperature t, is found from λ, the latent heat of a pound of saturated steam, by the equation

$$k=\frac{\mathrm{p}}{760}\cdot\frac{1+\cdot00366\times100}{1+\cdot00366\times t}\cdot\times\cdot036869\cdot\lambda,$$

where p denotes the pressure in millimetres, and λ the latent heat of a pound of saturated steam; the values of λ being calculated by the empirical formula*

$$\lambda=(606\cdot5+0\cdot305\,t)-(t+\cdot00002\,t^2+0\cdot000000\,t^3),$$

given by REGNAULT as representing, between the extreme limits of his observations, the latent heat of a unit weight of saturated steam.

* The part of this expression in the first vinculum (see REGNAULT, end of ninth Mémoire) is what is known as "the total heat" of a pound of steam, or the amount of heat necessary to convert a pound of water at 0° into a pound of saturated steam at t°; which, according to "WATT's law," thus approximately verified, would be constant. The second part, which would consist of the single term t, if the specific heat of water were constant for all temperatures, is the number of thermic units necessary to raise the temperature of a pound of water from 0° to t°, and expresses empirically the results of REGNAULT's experiments on the specific heat of water (see end of the tenth Mémoire), described in the work already referred to.

Explanation of Table I.

37. The mean values of μ for the first, for the eleventh, for the twenty-first, and so on, up to the 231st* degree of the air-thermometer, have been calculated in the manner explained in the preceding paragraphs. These, and interpolated re-results, which must agree with what would have been obtained, by direct calculation from REGNAULT's data, to three significant places of figures (and even for the temperatures between 0° and 100°, the experimental data do not justify us in relying on any of the results to a greater degree of accuracy), are exhibited in Table I.

To find the amount of mechanical effect due to a unit of heat, descending from a body at a temperature S *to a body at* T, *if these numbers be integers, we have merely to add the values of* μ *in Table I. corresponding to the successive numbers.*

$$T+1, T+2, \quad . \quad . \quad . \quad . \quad . \quad S-2, S-1,$$

Explanation of Table II.

38. The calculation of the mechanical effect, in any case, which might always be effected in the manner described in § 37 (with the proper modification for fractions of degrees, when necessary), is much simplified by the use of Table II., where the first number of Table I., the sum of the first and second, the sum of the first three, the sum of the first four, and so on, are successively exhibited. The sums thus tabulated are the values of the integrals

$$\int_0^1 \mu\, d\, t, \int_0^2 \mu\, d\, t, \int_0^3 \mu\, d\, t, \quad . \quad . \quad . \quad . \int_0^{231} \mu\, d\, t;$$

and, if we denote $\int_0^t \mu\, d\, t$ by the letter M, Table II. may be regarded as a table of the values of M.

To find the amount of mechanical effect due to a unit of heat descending from a body at a temperature S *to a body at* T, *if these numbers be integers, we have merely to subtract the value of* M, *for the number* T + 1, *from the value for the number* S, *given in Table II.*

* In strictness, the 230th is the last degree for which the experimental data are complete; but the data for the 231st may readily be assumed in a sufficiently satisfactory manner.

TABLE I.* *Mean Values of μ for the successive Degrees of the Air-Thermometer from 0° to 230°.*

	μ		μ		μ		μ		μ
1°	4·960	48°	4·366	94°	3·889	140°	3·549	186°	3·309
2	4·946	49	4·355	95	3·880	141	3·543	187	3·304
3	4·932	50	4·343	96	3·871	142	3·537	188	3·300
4	4·918	51	4·331	97	3·863	143	3·531	189	3·295
5	4·905	52	4·319	98	3·854	144	3·525	190	3·291
6	4·892	53	4·308	99	3·845	145	3·519	191	3·287
7	4·878	54	4·296	100	3·837	146	3·513	192	3·282
8	4·865	55	4·285	101	3·829	147	3·507	193	3·278
9	4·852	56	4·273	102	3·820	148	3·501	194	3·274
10	4·839	57	4·262	103	3·812	149	3·495	195	3·269
11	4·826	58	4·250	104	3·804	150	3·490	196	3·265
12	4·812	59	4·239	105	3·796	151	3·484	197	3·261
13	4·799	60	4·227	106	3·788	152	3·479	198	3·257
14	4·786	61	4·216	107	3·780	153	3·473	199	3·253
15	4·773	62	4·205	108	3·772	154	3·468	200	3·249
16	4·760	63	4·194	109	3·764	155	3·462	201	3·245
17	4·747	64	4·183	110	3·757	156	3·457	202	3·241
18	4·735	65	4·172	111	3·749	157	3·451	203	3·237
19	4·722	66	4·161	112	3·741	158	3·446	204	3·233
20	4·709	67	4·150	113	3·734	159	3·440	205	3·229
21	4·697	68	4·140	114	3·726	160	3·435	206	3·225
22	4·684	69	4·129	115	3·719	161	3·430	207	3·221
23	4·672	70	4·119	116	3·712	162	3·424	208	3·217
24	4·659	71	4·109	117	3·704	163	3·419	209	3·213
25	4·646	72	4·098	118	3·697	164	3·414	210	3·210
26	4·634	73	4·088	119	3·689	165	3·409	211	3·206
27	4·621	74	4·078	120	3·682	166	3·404	212	3·202
28	4·609	75	4·067	121	3·675	167	3·399	213	3·198
29	4·596	76	4·057	122	3·668	168	3·394	214	3·195
30	4·584	77	4·047	123	3·661	169	3·389	215	3·191
31	4·572	78	4·037	124	3·654	170	3·384	216	3·188
32	4·559	79	4·028	125	3·647	171	3·380	217	3·184
33	4·547	80	4·018	126	3·640	172	3·375	218	3·180
34	4·535	81	4·009	127	3·633	173	3·370	219	3·177
35	4·522	82	3·999	128	3·627	174	3·365	220	3·173
36	4·510	83	3·990	129	3·620	175	3·361	221	3·169
37	4·498	84	3·980	130	3·614	176	3·356	222	3·165
38	4·486	85	3·971	131	3·607	177	3·351	223	3·162
39	4·474	86	3·961	132	3·601	178	3·346	224	3·158
40	4·462	87	3·952	133	3·594	179	3·342	225	3·155
41	4·450	88	3·943	134	3·586	180	3·337	226	3·151
42	4·438	89	3·934	135	3·579	181	3·332	227	3·148
43	4·426	90	3·925	136	3·573	182	3·328	228	3·144
44	4·414	91	3·916	137	3·567	183	3·323	229	3·141
45	4·402	92	3·907	138	3.561	184	3·318	230	3·137
46	4·390	93	3·898	139	3·555	185	3·314	231	3·134
47	4·378								

* The numbers here tabulated may also be regarded as, *the actual values of μ for $t=\frac{1}{2}$, $t=1\frac{1}{2}$, $t=2\frac{1}{2}$, $t=3\frac{1}{2}$*, &c.

TABLE II. *Mechanical Effect in Foot-Pounds due to a Thermic Unit Centigrade, passing from a body, at any Temperature less than 230° to a body at 0°.*

Superior Limit of Temperature.	Mechanical Effect.	Superior Limit of Temperature.	Mechanical Effect.	Superior Limit of Temperature.	Mechanical Effect.	Superior Limit of Temperature.	Mechanical Effect.	Superior Limit of Temperature.	Mechanical Effect.
	Foot-pounds.		Foot-pounds.		Foot-pounds.		Foot-pounds.		Foot-pounds.
1°	4·960	48°	223·487	94°	412·545	140°	582·981	186°	740·310
2	9·906	49	227·842	95	416·425	141	586·524	187	743·614
3	14·838	50	232·185	96	420·296	142	590·061	188	746·914
4	19·756	51	236·516	97	424·159	143	593·592	189	750·209
5	24·661	52	240·835	98	428·013	144	597·117	190	753·500
6	29·553	53	245·143	99	431·858	145	600·636	191	756·787
7	34·431	54	249·439	100	435·695	146	604·099	192	760·069
8	39·296	55	253·724	101	439·524	147	607·656	193	763·347
9	44·148	56	257·997	102	443·344	148	611·157	194	766·621
10	48·987	57	262·259	103	447·156	149	614·652	195	769·890
11	53·813	58	266·509	104	450·960	150	618·142	196	773·155
12	58·625	59	270·748	105	454·756	151	621·626	197	776·416
13	63·424	60	274·975	106	458·544	152	625·105	198	779·673
14	68·210	61	279·191	107	462·324	153	628·578	199	782·926
15	72·983	62	283·396	108	466·096	154	632·046	200	786·175
16	77·743	63	287.590	109	469·860	155	635·508	201	789·420
17	82·490	64	291·773	110	473·617	156	638·965	202	792·661
18	87.225	65	295·945	111	477·366	157	642·416	203	795·898
19	91·947	66	300·106	112	481·107	158	645·862	204	799·131
20	96·656	67	304·256	113	484·841	159	649·302	205	802·360
21	101·353	68	308·396	114	488·567	160	652·737	206	805·585
22	106·037	69	312·525	115	492·286	161	656·167	207	808·806
23	110·709	70	316·644	116	495·998	162	659·591	208	812·023
24	115·368	71	320·752	117	499·702	163	663·010	209	815·236
25	120·014	72	324·851	118	503·399	164	666·424	210	818·446
26	124·648	73	328·939	119	507·088	165	669·833	211	821·652
27	129·269	74	333·017	120	510·770	166	673·237	212	824·854
28	133·878	75	337·084	121	514·445	167	676·636	213	828·052
29	138·474	76	341·141	122	518·113	168	680·030	214	831·247
30	143·058	77	345·188	123	521·174	169	683·419	215	834·438
31	147·630	78	349·225	124	525·428	170	686·803	216	837·626
32	152·189	79	353·253	125	529·075	171	690·183	217	840·810
33	156·736	80	357·271	126	532·715	172	693·558	218	843·990
34	161·271	81	361·280	127	536·348	173	696·928	219	847·167
35	165·793	82	365·279	128	539·975	174	700·293	220	850·340
36	170·303	83	369·269	129	543·595	175	703·654	221	853·509
37	174·801	84	373·249	130	547·209	176	707·010	222	856·674
38	179·287	85	377·220	131	550·816	177	710·361	223	859·836
39	183·761	86	381·181	132	554·417	178	713·707	224	862·994
40	188·223	87	385·133	133	558·051	179	717·049	225	866·149
41	192·673	88	389·076	134	561·597	180	720·386	226	869·300
42	197·111	89	393·010	135	565·176	181	723·718	227	872·448
43	201·537	90	396·935	136	568·749	182	727·046	228	875·592
44	205·951	91	400·851	137	572·316	183	730·369	229	878·733
45	210·353	92	404·758	138	575·877	184	733·687	230	881·870
46	214·743	93	408·656	139	579·432	185	737·001	231	885·004
47	219·121								

Note.—On the curves described in CLAPEYRON'S graphical method of exhibiting CARNOT'S Theory of the Steam-Engine.

39. At any instant when the temperature of the water and vapour is t, during the fourth operation (see above, § 16), the latent heat of the vapour must be precisely equal to the amount of heat that would be necessary to raise the temperature of the whole mass, if in the liquid state, from t to S.* Hence, if v' denote the volume of the vapour, c the mean capacity for heat of a pound of water between the temperatures S and t, and W the weight of the entire mass, in pounds, we have

$$k\,v' = c\,(\mathrm{S}-t)\,\mathrm{W}.$$

Again, the circumstances during the second operation are such that the mass of liquid and vapour possesses H units of heat more than during the fourth; and consequently, at the instant of the second operation, when the temperature is t, the volume v of the vapour will exceed v' by an amount of which the latent heat is H, so that we have

$$v = v' + \frac{\mathrm{H}}{k}$$

40. Now, at any instant, the volume between the piston and its primitive position is less than the actual volume of vapour by the volume of the water evaporated. Hence, if x and x' denote the abscissæ of the curve at the instants of the second and fourth operations respectively, when the temperature is t, we have

$$x = v - \sigma\,v,\quad x' = v' - \sigma\,v',$$

and, therefore, by the preceding equations,

$$x = \frac{1-\sigma}{k}\,\{\mathrm{H} + c\,(\mathrm{S}-t)\,\mathrm{W}\} \quad . \quad . \quad . \quad . \quad (a)$$

$$x' = \frac{1-\sigma}{k}\,c\,(\mathrm{S}-t)\,\mathrm{W} \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

These equations, along with

$$y = y' = p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$

enable us to calculate, from the data supplied by REGNAULT, the abscissa and ordinate for each of the curves described above (§ 17), corresponding to any as-

* For, at the end of the fourth operation, the whole mass is liquid, and at the temperature t. Now, this state might be arrived at by first compressing the vapour into water at the temperature t, and then raising the temperature of the liquid to S; and however this state may be arrived at, there cannot, on the whole, be any heat added to or subtracted from the contents of the cylinder, since, during the fourth operation, there is neither gain nor loss of heat. This reasoning is, of course, founded on CARNOT'S fundamental principle, which is tacitly assumed in the commonly-received ideas connected with "WATT'S law," the "latent heat of steam," and "the total heat of steam."

sumed temperature t. After the explanations of §§ 33, 34, 35, 36, it is only necessary to add that c is a quantity of which the value is very nearly unity, and would be exactly so were the capacity of water for heat the same at every temperature as it is between 0° and 1°; and that the value of $c\,(S-t)$, for any assigned values of S and t, is found, by subtracting the number corresponding to t from the number corresponding to s, in the column headed "*Nombre des unités de chaleur abandonnées par un kilogramme d'eau en descendant de* T° *à* 0°", of the last table (at the end of the Tenth Mémoire) of REGNAULT's work. By giving S the value 230°, and by substituting successively 220, 210, 200, &c., for t, values for x, y, x', y', have been found, which are exhibited in the following Table:—

Temperatures. t	Volumes to be described by the piston, to complete the fourth operation. x'	Volumes from the primitive position of the piston to those occupied at instants of the second operation. x	Pressures of saturated steam, in pounds on the square foot. $y=y'=p$
0°	1269.W	x' + 5·409.H	12·832
10	639·6.W	x' + 2·847.H	25·567
20	337·3.W	x' + 1·571.H	48·514
30	185·5.W	x' + ·9062.H	88·007
40	105·9.W	x' + ·5442.H	153·167
50	62·62.W	x' + ·3392.H	256·595
60	38·19.W	x' + ·2188.H	415·070
70	21·94.W	x' + ·1456.H	650·240
80	15·38.W	x' + ·09962.H	989·318
90	10·09.W	x' + ·06994.H	1465·80
100	6·744.W	x' + ·05026.H	2120·11
110	4·578.W	x' + ·03688.H	2999·87
120	3·141.W	x' + ·02758.H	4160·10
130	2·176.W	x' + ·02098.H	5663·70
140	1·519.W	x' + ·01625.H	7581·15
150	1·058.W	x' + ·01271.H	9990·26
160	0·7369.W	x' + ·01010.H	12976·2
170	0·5085.W	x' + ·008116.H	16630·7
180	0·3454.W	x' + ·006592.H	21051·5
190	0·2267.W	x' + ·005406.H	26341·5
200	0·1409.W	x' + ·004472.H	32607·7
210	0·0784.W	x' + ·003729.H	39960·7
220	0·3310.W	x' + ·003130.H	48512·4
230	0	x' + ·002643.H	58376·6

Appendix.

(Read April 30, 1849.)

41. In p. 30, some conclusions drawn by CARNOT from his general reasoning were noticed; according to which it appears, that if the value of μ for any temperature is known, certain information may be derived with reference to the saturated vapour of any liquid whatever, and, with reference to any gaseous mass, without the necessity of experimenting upon the specific medium considered. Nothing in the whole range of Natural Philosophy is more remarkable than the establishment of general laws by such a process of reasoning. We have seen, however, that doubt may exist with reference to the truth of the axiom on which the entire theory is founded, and it therefore becomes more than a matter of mere curiosity to put the inferences deduced from it to the test of experience. The importance of doing so was clearly appreciated by CARNOT; and, with such data as he had from the researches of various experimenters, he tried his conclusions. Some very remarkable propositions which he derives from his Theory, coincide with DULONG and PETIT's subsequently-discovered experimental laws with reference to the heat developed by the compression of a gas; and the experimental verification is therefore in this case (so far as its accuracy could be depended upon) decisive. In other respects, the data from experiment were insufficient, although, so far as they were available as tests, they were confirmatory of the theory.

42. The recent researches of REGNAULT add immensely to the experimental data available for this object, by giving us the means of determining with considerable accuracy the values of μ within a very wide range of temperature, and so affording a trustworthy standard for the comparison of isolated results at different temperatures, derived from observations in various branches of physical science.

In the first section of this Appendix the Theory is tested, and shewn to be confirmed by the comparison of the values of μ found above, with those obtained by CARNOT and CLAPEYRON from the observations of various experimenters on air, and the vapours of different liquids. In the second and third sections some striking confirmations of the theory arising from observations by DULONG, on the specific heat of gases, and from Mr JOULE's experiments on the heat developed by the compression of air, are pointed out; and in conclusion, the actual methods of obtaining mechanical effect from heat are briefly examined with reference to their economy.

I. *On the values of μ derived by Carnot and Clapeyron from observations on Air, and on the Vapours of various liquids.*

43. In CARNOT'S work, p. 80–82, the mean value of μ between $0°$ and $1°$ is derived from the experiments of DELAROCHE and BERARD on the specific heat of gases, by a process approximately equivalent to the calculation of the value of $\frac{E p_0 v_0}{v \frac{dq}{dv}}$ for the temperature $\frac{1}{2}°$. There are also, in the same work, determinations of the values of μ from observations on the vapours of alcohol and water; but a table given in M. CLAPEYRON'S paper, of the values of μ derived from the data supplied by various experiments with reference to the vapours of ether, alcohol, water, and oil of turpentine, at the respective boiling-points of these liquids, afford us the means of comparison through a more extensive range of temperature. In the cases of alcohol and water, these results ought of course to agree with those of CARNOT. There are, however, slight discrepancies which must be owing to the uncertainty of the experimental data.* In the following table, CARNOT'S results with reference to air, and CLAPEYRON'S results with reference to the four different liquids, are exhibited, and compared with the values of μ which have been given above (Table I.) for the same temperatures, as derived from REGNAULT'S observations on the vapour of water.

Names of the Media.	Temperatures.	Values of μ	Values of μ deduced from Regnault's Observations.	Differences.
Air, . . .	$0°5$	(CARNOT) 4·377	4·960	·383
Sulphuric Ether,	(Boiling point) 35·5	(CLAPEYRON) 4·478	4·510	·032
Alcohol, . .	.... 78·8	.... 3·963	4·030	·071
Water, . .	 100	.... 3·658	3·837	·179
Essence of Turpentine,	.... 156·8	.... 3·530	3·449	−·081

44. It may be observed that the discrepancies between the results founded on the experimental data supplied by the different observers with reference to water at the boiling-point, are greater than those which are presented between the results deduced from any of the other liquids, and water at the other temperatures; and we may therefore feel perfectly confident that the verification is com-

* Thus, from CARNOT'S calculations, we find, in the case of alcohol, 4·035; and in the case of water, 3·648, instead of 3·963, and 3·658, which are CLAPEYRON'S results in the same cases.

plete to the extent of accuracy of the observations.* The considerable discrepancy presented by CARNOT'S result, deduced from experiments on air, is not to be wondered at when we consider the very uncertain nature of his data.

45. The fact of the gradual decrease of μ through a very extensive range of temperature, being indicated both by REGNAULT'S continuous series of experiments, and by the very varied experiments on different media, and in different branches of Physical Science, must be considered as a striking verification of the theory.

II. *On the Heat developed by the compression of Air.*

46. Let a mass of air, occupying initially a given volume V, under a pressure P, at a temperature t, be compressed to a less volume V, and allowed to part with heat until it sinks to its primitive temperature t. The quantity of heat which is evolved may be determined, according to CARNOT'S theory, when the particular value of μ, corresponding to the temperature t, is known. For, by equation § 30, equation (6), we have

$$v \frac{d q}{d v} = \frac{E p_0 v_0}{\mu},$$

where $d q$ is the quantity of heat absorbed, when the volume is allowed to increase from v to $v + d v$; or the quantity evolved by the reverse operation. Hence we deduce

$$d q = \frac{E p_0 v_0}{\mu} \frac{d v}{v} \quad . \quad . \quad . \quad . \quad (8),$$

Now, $\frac{E p_0 v_0}{\mu}$ is constant, since the temperature remains unchanged; and therefore, we may at once integrate the second number. By taking it between the limits V′ and V, we thus find

$$Q = \frac{E p_0 v_0}{\mu} \log \frac{V}{V'}\dagger \quad . \quad . \quad . \quad . \quad (9),$$

where Q denotes the required amount of heat, evolved by the compression from V to V′. This expression may be modified by employing the equations $PV = P'V' = p_0 v_0 (1 + E t)$; and we thus obtain

$$Q = \frac{E P V}{\mu(1 + E t} \log \frac{V}{V'} = \frac{E P' V'}{(\mu(1 + E t)} \log \frac{V}{V'} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

* A still closer agreement must be expected, when more accurate experimental data are afforded with reference to the other media. Mons. REGNAULT informs me that he is engaged in completing some researches, from which we may expect, possibly before the end of the present year, to be furnished with all the data for five or six different liquids which we possess at present for water. It is therefore to be hoped that, before long, a most important test of the validity of CARNOT'S theory will be afforded.

† The *Napierian* logarithm of $\frac{V}{V'}$ is here understood.

From this result we draw the following conclusion :—

47. *Equal volumes of all elastic fluids, when compressed to smaller equal volumes, disengage equal quantities of heat.*

This extremely remarkable theorem of CARNOT'S was independently laid down as a probable experimental law by DULONG, in his "Recherches sur la Chaleur Spécifique des Fluides Élastiques," and it therefore affords a most powerful confirmation of the theory.*

48. In some very remarkable researches made by Mr JOULE upon the heat developed by the compression of air, the quantity of heat produced in different experiments has been ascertained with reference to the amount of work spent in the operation. To compare the results which he has obtained with the indications of theory, let us determine the amount of work necessary actually to produce the compression considered above.

49. In the first place, to compress the gas from the volume $v+dv$ to v, the work required is $p\,dv$, or, since $pv=p_0 v_0 (1+\mathrm{E}t)$,

$$p_0 v_0 (1+\mathrm{E}t)\frac{dv}{v},$$

Hence, if we denote by W the total amount of work necessary to produce the compression from V to V′, we obtain, by integration,

$$\mathrm{W}=p_0 v_0 (1+\mathrm{E}t)\log\frac{\mathrm{V}}{\mathrm{V}'},$$

Comparing this with the expression above, we find

$$\frac{\mathrm{W}}{\mathrm{Q}}=\frac{\mu(1+\mathrm{E}t)}{\mathrm{E}} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

50. Hence we infer that

(1.) The amount of work necessary to produce a unit of heat by the compression of a gas, is the same for all gases at the same temperature.

(2.) And that the quantity of heat evolved in all circumstances, when the temperature of the gas is given, is proportional to the amount of work spent in the compression.

* CARNOT varies the statement of his theorem, and illustrates it in a passage, pp. 52, 53, of which the following is a translation :—

"*When a gas varies in volume without any change of temperature, the quantities of heat absorbed or evolved by this gas are in arithmetical progression, if the augmentation or diminutions of volume are in geometrical progression.*

"When we compress a litre of air maintained at the temperature 10°, and reduce it to half a litre, it disengages a certain quantity of heat. If, again, the volume be reduced from half a litre to a quarter of a litre, from a quarter to an eighth, and so on, the quantities of heat successively evolved will be the same.

"If, in place of compressing the air, we allow it to expand to two litres, four litres, eight litres, &c., it will be necessary to supply equal quantities of heat to maintain the temperature always at the same degree."

51. The expression for the amount of work necessary to produce a unit of heat is

$$\frac{\mu(1+\mathrm{E}\,t)}{\mathrm{E}},$$

and therefore REGNAULT'S experiments on steam are available to enable us to calculate its value for any temperature. By finding the values of μ at 0°, 10°, 20°, &c., from Table I., and by substituting successively the values 0, 10, 20, &c., for t, the following results have been obtained.

Table of the Values of $\frac{\mu(1+\mathrm{E}\,t)}{\mathrm{E}}$,

Work requisite to produce a unit of Heat by the compression of a Gas.	Temperature of the Gas.	Work requisite to produce a unit of Heat by the compression of a Gas.	Temperature of the Gas.
Ft.-lbs.	°	Ft.-lbs.	°
1357·1	0	1446·4	120
1368·7	10	1455·8	130
1379·0	20	1465·3	140
1388·0	30	1475·8	150
1395·7	40	1489·2	160
1401·8	50	1499·0	170
1406·7	60	1511·3	180
1412·0	70	1523·5	190
1417·6	80	1536·5	200
1424·0	90	1550·2	210
1430·6	100	1564·0	220
1438·2	110	1577·8	230

Mr JOULE'S experiments were all conducted at temperatures from 50° to about 60° Fahr., or from 10° to 16° cent.; and, consequently, although some irregular differences in the results, attributable to errors of observation inseparable from experiments of such a very difficult nature are presented, no regular dependance on the temperature is observable. From three separate series of experiments, Mr JOULE deduces the following numbers for the work, in foot-pounds, necessary to produce a thermic unit Fahrenheit by the compression of a gas.

820, 814, 760.

Multiplying these by 1·8, to get the corresponding number for a thermic unit centigrade, we find

1476, 1465, and 1368.

The largest of these numbers is most nearly conformable with Mr JOULE'S views of the relation between such experimental "equivalents," and others which he obtained in his electro-magnetic researches; but the smallest agrees almost perfectly with the indications of CARNOT'S theory; from which, as exhibited in the preceding Table, we should expect, from the temperature in Mr JOULE'S experiments, to find a number between 1369 and 1379 as the result.

III. *On the Specific-Heats of Gases.*

52. The following proposition is proved by CARNOT as a deduction from his general theorem regarding the specific heats of gases.

The excess of the specific heat under a constant pressure above the specific heat at a constant volume, is the same for all gases at the same temperature and pressure.*

53. To prove this proposition, and to determine an expression for the "excess" mentioned in its enunciation, let us suppose a unit of volume of a gas to be elevated in temperature by a small amount, τ. The quantity of heat required to do this will be $A\tau$, if A denote the specific heat at a constant volume. Let us next allow the gas to expand without going down in temperature, until its pressure becomes reduced to its primitive value. The expansion which will take place will be $\frac{E\tau}{1+Et}$, if the temperature be denoted by t; and hence, by (8), the quantity of heat that must be supplied, to prevent any lowering of temperature, will be

$$\frac{Ep_0 v_0}{\mu} \cdot \frac{E\tau}{1+Et}, \quad \text{or} \quad \frac{E^2 p}{\mu(1+Et)^2}\tau.$$

Hence, the total quantity added is equal to

$$A\tau + \frac{E^2 p}{\mu(1+Et)^2}\tau$$

But, since B denotes the specific heat under constant pressure, the quantity of heat requisite to bring the gas into this state, from its primitive condition, is equal to $B\tau$; and hence we have

$$B = A + \frac{E^2 p}{\mu(1+Et)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

IV. *Comparison of the Relative advantages of the Air-Engine and Steam-Engine.*

54. In the use of water-wheels for motive power, the economy of the engine depends not only upon the excellence of its adaptation for actually transmitting any given quantity of water through it, and producing the equivalent of work, but upon turning to account the entire available fall; so, as we are taught by CARNOT, the object of a thermodynamic engine is to economize in the best possible way the transference of all the heat evolved, from bodies at the temperature of the source, to bodies at the lowest temperature at which the heat can be discharged. With reference then to any engine of the kind, there will be two points to be considered.

(1.) The extent of the *fall* utilised.

(2.) The economy of the engine, with the fall which it actually uses.

55. In the first respect, the air-engine, as CARNOT himself points out, has a

* Or the capacity of a unit of volume for heat.

vast advantage over the steam-engine; since the temperature of the hot part of the machine may be made very much higher in the air-engine than would be possible in the steam-engine, on account of the very high pressure produced in the boiler, by elevating the temperature of the water which it contains to any considerable extent above the atmospheric boiling point. On this account, a "perfect air-engine" would be a much more valuable instrument than a "perfect steam-engine."*

Neither steam-engines nor air-engines, however, are nearly perfect; and we do not know in which of the two kinds of machine the nearest approach to perfection may be actually attained. The beautiful engine invented by Mr STIRLING of Galston, may be considered as an excellent beginning for the air-engine;† and it is only necessary to compare this with NEWCOMEN'S steam-engine, and consider what WATT has effected, to give rise to the most sanguine anticipations of improvement.

V. *On the Economy of actual Steam-Engines.*

56. The steam-engine being universally employed at present as the means for deriving motive power from heat, it is extremely interesting to examine, according to CARNOT'S theory, the economy actually attained in its use. In the first place, we remark that, out of the entire "fall" from the temperature of the coals to that of the atmosphere, it is only part—that from the temperature of the boiler to the temperature of the condenser—that is made available; while the very great fall from the temperature of the burning coals to that of the boiler, and the comparatively small fall from the temperature of the condenser to that of the atmosphere, are entirely lost as far as regards the mechanical effect which it is desired to obtain. We infer from this, that the temperature of the boiler ought to be kept as high as, according to the strength, is consistent with safety, while that of the condenser ought to be kept as nearly down at the atmospheric temperature as possible. To take the entire benefit of the actual fall, CARNOT shewed that the "principle of expansion" must be pushed to the utmost.‡

* CARNOT suggests a combination of the two principles, with air as the medium for receiving the heat at a very high temperature from the furnace; and a second medium, alternately in the state of saturated vapour and liquid water, to receive the heat, discharged at an intermediate temperature from the air, and transmit it to the coldest part of the apparatus. It is possible that a complex arrangement of this kind might be invented, which would enable us to take the heat at a higher temperature, and discharge it at a lower temperature than would be practicable in any simple air-engine or simple steam-engine. If so, it would no doubt be equally possible, and perhaps more convenient, to employ steam alone, but to use it at a very high temperature not in contact with water in the hottest part of the apparatus, instead of, as in the steam-engine, always in a saturated state.

† It is probably this invention to which CARNOT alludes in the following passage (p. 112):—"Il a été fait, dit-on, tout recemment en Angleterre des essais heureux sur le développement de la puissance motrice par l'action de la chaleur sur l'air atmosphérique. Nous ignorons entièrement ne quoi ces essais ont consisté, si toutefois ils sont réels."

‡ From this point of view, we see very clearly how imperfect is the steam-engine, even after all WATT'S improvements. For to "push the principle of expansion to the utmost," we must allow the

57. To obtain some notion of the economy which has actually been obtained, we may take the alleged performances of the best Cornish engines, and some other interesting practica cases as examples.*

(1.) The engine of *the Fowey Consols mine* was reported, in 1845, to have given 125,089,000 foot-pounds of effect, for the consumption of one bushel or 94 lbs. of coals. Now, the average amount evaporated from Cornish boilers, by one pound of coal, is $8\frac{1}{2}$ lbs. of steam; and hence, for each pound of steam evaporated 156,556 foot-pounds of work are produced.

The pressure of the saturated steam in the boiler may be taken as $3\frac{1}{2}$ atmospheres;† and, consequently, the temperature of the water will be 140°. Now (REGNAULT, end of Memoire X.), the latent heat of a pound of saturated steam at 140° is 508, and since, to compensate for each pound of steam removed from the boiler in the working of the engine, a pound of water, at the temperature of the condenser, which may be estimated at 30°, is introduced from the hot well; it follows that 618 units of heat are introduced to the boiler for each pound of water evaporated. But the work produced, for each pound of water evaporated, was found above to be 156,556 foot-pounds. Hence, $\frac{156556}{618}$, or 253 foot-pounds is the amount of work produced for each unit of heat transmitted through the Fowey Consols engine. Now, in Table II., we find 583·0 as the theoretical effect due to a unit descending from 140° to 0°, and 143 as the effect due to a unit descending from 30° to 0°. The difference of these numbers, or 440,‡ is the number of foot-pounds of work that a *perfect* engine with its boiler at 140°, and its condenser at 30° would produce for each unit of heat transmitted. Hence, the Fowey Consols engine, during the experiments reported on, performed $\frac{253}{440}$ of its theoretical duty, or $57\frac{1}{2}$ per cent.

(2.) The best duty on record, as performed by an engine at work (not for merely experimental purposes), is that of TAYLOR's engine, at the United mines, which, in 1840, worked regularly, for several months, at the rate of 98,000,000 foot-pounds for each bushel of coals burned. This is $\frac{98}{125}$, or ·784 of the experimental

steam, before leaving the cylinder, to expand until its pressure is the same as that of the vapour in the condenser. According to "WATT's law," its temperature would then be the same as (actually a little above, as REGNAULT has shewn) that of the condenser, and hence the steam-engine worked in this most advantageous way, has in reality the very fault that WATT found in NEWCOMEN's engine. This defect is partially remedied by HORNBLOWER's system of using a separate expansion cylinder, an arrangement, the advantages of which did not escape CARNOT's notice, although they have not been recognised extensively among practical engineers, until within the last few years.

* I am indebted to the kindness of Professor GORDON of Glasgow, for the information regarding the various cases given in the text.

† In different Cornish engines, the pressure in the boiler is from $2\frac{1}{2}$ to 5 atmospheres; and, therefore, as we find from REGNAULT's table of the pressure of saturated steam, the temperature of the water in the boiler must, in all of them, lie between 128° and 152°. For the better class of engines, the average temperature of the water in the boiler may be estimated at 140°, the corresponding pressure of steam being $3\frac{1}{2}$ temperatures.

‡ This number agrees very closely with the number corresponding to the fall from 100° to 0°, given in Table II. Hence, the fall from 140° to 30° of the scale of the air-thermometer is equivalent, with reference to motive power, to the fall from 100° to 0°.

duty reported in the case of the Fowey Consols engine. Hence, the best useful work on record, is at the rate of 198·3 foot-pounds for each unit of heat transmitted, and is $\frac{198\cdot3}{440}$, or 45 per cent. of the theoretical duty, on the supposition that the boiler is at 140°, and the condenser at 30°.

(3.) French engineers contract (in Lille, in 1847, for example) to make engines for mill power which will produce 30,000 metre-lbs., or 98,427 foot-lbs. of work for each pound of steam used. If we divide this by 618, we find 159 foot-pounds for the work produced by each unit of heat. This is 36·1 per cent. of 440, the theoretical duty.*

(4.) English engineers have contracted to make engines and boilers which will require only $3\frac{1}{2}$ lbs. of the best coal per horse-power per hour. Hence, in such engines, each pound of coal ought to produce 565,700 foot-pounds of work, and if 7 lbs. of water be evaporated by each pound of coal, there would result 80,814 foot-pounds of work for each pound of water evaporated. If the pressure in the boiler be $3\frac{1}{2}$ atmospheres (temperature 140°) the amount of work for each unit of heat will be found, by dividing this by 618, to be 130·7 foot-pounds, which is $\frac{130\cdot7}{440}$ or 29·7 per cent. of the theoretical duty.†

(5.) The actual average of work performed by good Cornish engines and boilers is 55,000,000 foot-pounds for each bushel of coal, or less than half the experimental performance of the Fowey Consols engine, more than half the actual duty performed by the United Mines engine in 1840; in fact about 25 per cent. of the theoretical duty.

(6.) The average performances of a number of Lancashire engines and boilers have been recently found to be such as to require 12 lbs. of Lancashire coal per horse-power per hour (*i. e.*, for performing $60 \times 33,000$ foot-pounds) and of a number of Glasgow engines, such as to require 15 lbs. (of a somewhat inferior coal) for the same effect. There are, however, more than twenty large engines in Glasgow at present,‡ which work with a consumption of only $6\frac{1}{2}$ lbs. of dross, equivalent to 5 lbs. of the best Scotch, or 4 lbs. of the best Welsh coal, per horse-power

* It being assumed that the temperatures of the boiler and condenser are the same as those of the Cornish engines. If, however, the pressure be lower, two atmospheres, for instance, the numbers would stand thus : The temperature in the boiler would be only 121. Consequently, for each pound of steam evaporated, only 614 units of heat would be required ; and, therefore, the work performed for each unit of heat transmitted would be 160·3 foot-pounds, which is *more* than according to the estimate in the text. On the other hand, the range of temperatures, or the fall utilised, is only from 131 to 30, instead of from 140 to 30°, and, consequently (Table II.), the theoretical duty for each unit of heat is only 371 foot-pounds. Hence, if the engine, to work according to the specification, requires a pressure of only 15 lbs. on the square inch (*i. e.*, a total steam pressure of two atmospheres), its performance is $\frac{160\cdot3}{371}$, or 43·2 per cent. of its theoretical duty.

† If, in this case again, the pressure required in the boiler to make the engine work according to the contract were only 15 lbs. on the square inch, we should have a different estimate of the economy, for which, see Table B, at the end of this paper.

‡ These engines are provided with separate expansive cylinders, which have been recently added to them by Mr M'NAUGHT of Glasgow.

per hour. The economy may be estimated from these data, as in the other cases, on the assumption which, with reference to these, is the most probable we can make, that the evaporation produced by a pound of best coal is 7 lbs. of steam.

58. The following Tables afford a synoptic view of the performances and theoretical duties in the various cases discussed above.

In Table A the numbers in the second column are found by dividing the numbers in the first by $8\frac{1}{2}$ in cases (1.), (2.), and (5.), and by 7 in cases (4.), (6.), and (7.), the estimated numbers of pounds of steam actually produced in the different boilers by the burning of 1 lb. of coal.

The numbers in the third column are found from those in the second, by dividing by 618, in Table A, and 614 in Table B, which are respectively the quantities of heat required to convert a pound of water taken from the hot well at 30°, into saturated steam, in the boiler, at 140° or at 121°.

With reference to the cases (3.), (4.), (6.), (7.), the hypothesis of Table B is probably in general nearer the truth than that of Table A. In (4.), (6.), and (7.), especially upon hypothesis B, there is much uncertainty as to the amount of evaporation that will be actually produced by 1 lb. of fuel. The assumption on which the numbers in the second column in Table B are calculated, is, that each pound of coal will send the same number of units of heat into the boiler whether hypothesis A or hypothesis B be followed. Hence, except in the case of the French contract, in which the *evaporation*, not the fuel, is specified, the numbers in the third column are the same as those in the third column of Table A.

TABLE A. *Various Engines in which the temperature of the Boiler is* 140°, *and that of the Condenser* 30°.

Theoretical Duty for each Unit of Heat transmitted, 440 *foot-pounds.*

CASES.	Work produced for each pound of coal consumed.	Work produced for each pound of water evaporated.	Work produced for each unit of heat transmitted.	Per centage of theoretical duty.
	Foot-Pounds.	Foot-Pounds.	Foot-Pounds.	
(1.) Fowey Consols Experiment, reported in 1845,	1,330,734	156,556	253	57·5
(2.) Taylor's Engine at the United Mines, working in 1840,	1,042,553	122,653	198·4	45·1
(3.) French Engines, according to contract,	* * * *	98,427	159	36·1
(4.) English Engines, according to contract,	565,700	80,814	130·8	29·7
(5.) Average actual performance of Cornish Engines,	585,106	68,836	111·3	25·3
(6.) Common Engines, consuming 12 lbs. of best coal per hour per horse-power,	165,000	23,571	38·1	8·6
(7.) Improved Engines with Expansion Cylinders, consuming an equivalent to 4 lbs. of best coal per horse-power per hour,	495,000	70,710	114·4	26

TABLE B. *Various Engines in which the Temperature of the Boilers is* 121°,* *and that of the Condenser* 30°.

Theoretical Duty for each Unit of Heat transmitted, 371 *foot-pounds.*

CASES.	Work produced for each pound of coal consumed.	Work produced for each pound of water evaporated.	Work produced for each unit of heat transmitted.	Per centage of theoretical duty.
	Foot-Pounds.	Foot-Pounds.	Foot-Pounds.	
(3.) French Engines, according to contract, .	* * *	98,427	160·3	43·2
(4.) English Engines, according to contract, .	565,700	$\frac{614}{618} \times 80{,}814$	130·8	35
(6.) Common Engines, consuming 12 lbs. of coal per horse-power per hour, . .	165,000	$\frac{614}{618} \times 23{,}571$	38·1	10·3
(7.) Improved Engines with expansion cylinders, consuming an equivalent to 4 lbs. best coal per horse-power per hour, . .	495,000	$\frac{614}{618} \times 70{,}710$	114·4	30·7

* Pressure 15 lbs. on the square inch.