stars and nebulae that certainly do exist escaped his notice. The fallibility of the observer must not be lost sight of.

Lick Observatory, University of California:
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On a Simple Method of Comparing the Bonn Durchmusterung with Photographic Plates. By H. H. Turner, M.A., F.R.S., Savilian Professor.

1. The *Bonn Durchmusterung* gives R.A.s and Declinations for 1855'0. Measures of stars on a photographic plate give rectilinear coordinates (x, y) which are connected with standard coordinates (ξ, η) by linear relations

$$\xi = x + ax + by + c$$

$$\eta = y + dx + ey + f,$$

and (ξ, η) are connected with R.A. and Decl. for some other epoch (say 1900'0) by the relations

$$\xi = \tan(\alpha - A) \cos q \sec(q - D) \quad \eta = \tan(q - D),$$

where

$$\tan q = \tan \delta \sec(\alpha - A),$$

and (α, δ) are the R.A. and Decl. of the star,

(A, D) " " " " plate centre.

A circuitous method of comparing the B.D. with the plate is thus to perform the following operations:—

(1) Bring up the B.D. places to 1900'0.

(2) Convert into standard coordinates.

(3) Multiply by the plate constants a, b, c , &c., to get coordinates comparable with measures on the plate.

The present paper indicates a method of dispensing with operations (1) and (3), and generally performing the comparison very quickly.

2. The places of the *Durchmusterung* are given to 0^s.1 in R.A. and 0'.1 in Declination. In terms of a *réseau* interval of 5' (a unit now in common use for photographic measures) 0^s.1 on the equator represents 0'005, while 0'.1 represents 0'02. It will probably be sufficient accuracy in both coordinates if we calculate to 0'01 of a *réseau* interval, or in circular measure '000015.

For the plates of the Astrographic Catalogue

x and y range from $-.02$ to $+.02$ in circular measure,

x^2 and y^2 " " 0 to '0004,

x^3 and y^3 " " $-.000008$ to $+.000008$.

Hence we may neglect powers of the coordinates higher than the *second* for the present purpose ; though it is quite easy to take account of these terms afterwards if so desired.

Now to the second order it will be found that

$$\xi = (a - A) \cos \delta, \quad \eta = (\delta - D) + \frac{1}{4}(a - A)^2 \sin 2D.$$

3. The point of the present method lies in choosing the right value of A . If we increase A by A_0 , we add to ξ and η the terms

$$-A_0 \cos \delta, \quad -A_0(a - A) \sin D \cos D + \frac{1}{4}A_0^2 \sin 2D,$$

or if A_0 be of the same order as $(\xi$ and $\eta)$, since we can put $\cos \delta = \cos D - (\delta - D) \sin D$ to the first order, the terms to be added may be written

$$A_0 \eta \sin D + \text{const.} \quad -A_0 \xi \sin D + \text{const.}$$

which is equivalent to a rotation of the axes through an angle $A_0 \sin D$.

4. It is tolerably obvious, geometrically, that we can get the equivalence of a rotation of axes, or rather of the whole plate, by shifting the centre in R.A. ; especially if we visualise the sphere as projected on a plane, say the tangent plane at the Pole. And the significance of the factor $\sin D$ is also clear from the geometrical standpoint : the rotation $A_0 \sin D$ vanishes at the equator whatever A_0 may be ; in other words, the axes of η are all parallel for plates with centres on the equator—we cannot get this rotation there.

5. We proceed to use this principle to get an equivalent for precession. Bringing up the star places from 1855.0 to 1900.0 in the ordinary way is a rather complex numerical process. Viewed as the comparison of two plates of the same stars at different epochs, it merely means that one plate is rotated in its own plane with respect to the other, through a definite angle. To find this angle let us suppose two stars on the axis of η at 1900.0, so that $\xi = 0$ for both ; and let one be at the centre of the plate, the other Δ south of it, so that $\eta = 0$ for the first and $\eta = -\Delta$ for the second. The problem is to choose A , the adopted R.A. of plate centre for 1855.0, so that the value of ξ for the two stars may be, not necessarily zero, but *the same* for both.

We can then reduce both values to zero by subtracting an appropriate constant for the plate, which is a simple matter.

6. If α be the common R.A. of the stars for 1900.0, the precessions for the two stars are

$$m + n \sin \alpha \tan D$$

$$m + n \sin \alpha \tan (D - \Delta).$$

[We are considering differential effects, and the secular variation, even for forty-five years, does not matter.]

Thus the R.A.s for 1855.0 will differ by

$$45n \sin \alpha [\tan D - \tan (D - \Delta)].$$

Write this $\beta \sin \Delta \tan D$,

where $\beta = 90n \sin \alpha \operatorname{cosec} 2D$.

Denote the R.A. of the central star for 1855.0 by A_1 ; that of the southern star by A_2 ; and that of the appropriate plate centre by A_3 . Then we have to find A_3 , so that

$$(A_2 - A_3) \cos (D - \Delta) = (A_1 - A_3) \cos D,$$

or

$$\begin{aligned} A_3 [\cos D - \cos (D - \Delta)] &= A_1 \cos D - A_2 \cos (D - \Delta) \\ &= A_1 [\cos D - \cos (D - \Delta)] \\ &\quad + (A_1 - A_2) \cos (D - \Delta). \end{aligned}$$

Thus $A_3 = A_1 + \beta$

where from above the value of β is

$$\beta = 90n \sin \alpha \operatorname{cosec} 2D.$$

7. It will check this analytical result to look at the question geometrically.

Let
 E be the pole of the ecliptic,
 P be the pole of the equator,
 S a star, of coords. (α, D) .

Precession carries P to R, in the direction perpendicular to EP, and towards the First Point of Aries.

Thus $\angle SPR = \alpha$.

Further $PR = 50''.2 \sin \omega \times t$
 $= 45n$, as in § 6.

Now PS and RS are the directions of the axis of y for the two plates, which are inclined to one another at

$$\begin{aligned} \angle PSR &= PR \sin \alpha \cdot \operatorname{cosec} SP. \\ &= 45n \cdot \sin \alpha \cdot \sec D. \end{aligned}$$

We wish to correct this by choosing a plate centre differing in R.A. by the quantity β of the last paragraph. And by § 3 or

§ 4 the effect of this change of R.A. is to rotate the plate through an angle $\beta \sin D$.

Hence $\beta \sin D = 45n \sin \alpha \sec D$
or $\beta = 90n \sin \alpha \operatorname{cosec} 2D.$

8. It may be a convenience to others to give here a table of the coefficients of $\sin \alpha$ for different declinations, and the rule of signs.

Rule.—To form directly from the *Durchmusterung*, Epoch 1855.0, standard coordinates for 1900.0: if A_0 be the R.A. for 1855.0 of the actual plate centre, add to A_0 the quantity $K \sin A_0$, where K is given in the following table in seconds of time :—

Dec. ° or 90	K. min. ...	Dec. ° or 75	K. s. 240	Dec. 30° or 60°	K. s. 139
1 „ 89	(57)	16 „ 74	226	31 „ 59	136
2 „ 88	(29)	17 „ 73	214	32 „ 58	133
3 „ 87	(19)	18 „ 72	204	33 „ 57	131
4 „ 86	(14)	19 „ 71	194	34 „ 56	130
5 „ 85	(11)	20 „ 70	186	35 „ 55	128
6 „ 84	(10)	21 „ 69	179	36 „ 54	126
7 „ 83	(8)	22 „ 68	173	37 „ 53	125
8 „ 82	sec. 435	23 „ 67	167	38 „ 52	124
9 „ 81	388	24 „ 66	162	39 „ 51	123
10 „ 80	350	25 „ 65	157	40 „ 50	122
11 „ 79	320	26 „ 64	152	41 „ 49	121
12 „ 78	295	27 „ 63	149	42 „ 48	121
13 „ 77	274	28 „ 62	145	43 „ 47	120
14 „ 76	256	29 „ 61	141	44 „ 46	120
				45°	120

The method cannot be advantageously employed for plates near the equator or the poles.

Example.—To compare the B.D. with a plate taken with centre at $14^h 40^m$, $+37^\circ$ at the Epoch 1900.0.

The R.A. of the plate centre for 1855.0 is $14^h 38^m 5^s$.

Take from above table the quantity for $+37^\circ$, viz. 125^s. Multiply this by $\sin 14^h 40^m$, which is $-\sin 40^\circ$ or $-.643$, getting -80^s or $-1^m 20^s$. Add this algebraically to $14^h 38^m 5^s$, getting $14^h 36^m 45^s = A'$ say.

If now (α, δ) be the coordinates of any star for 1855.0, and we form

$$\xi = (\alpha - A') \cos \delta, \quad \eta = (\delta - 37^\circ) + \frac{1}{4} (\alpha - A')^2 \sin 74^\circ$$

as in § 2 (where the angles are expressed in circular measure), then ξ and η only differ by a constant from standard coordinates for 1900.0.

9. We have hitherto supposed the plate correctly oriented for 1900.0. Any error of orientation may be corrected in the same way, by again altering the assumed R.A. of plate centre. If (x, y) denote measures on the plate, and (ξ, η) standard coordinates, an error of orientation is indicated by the relations

$$\begin{aligned}\xi &= x - by \\ \eta &= y + bx,\end{aligned}$$

b being (in circular measure) the angle of rotation.

If this is compensated by a change A_3 in the plate centre, we must have by § 3 or § 4

$$\pm b \times 13751 = A_3 \sin D,$$

assuming that A_3 is expressed in seconds of time.

The following table indicates the value of A_3 for different values of b and different centres :—

Table showing how to compensate for the value of b by change of Adopted Plate Centre.

b	Dec. = 10° s	20° s	30° s	40° s	50° s	60° s	70° s	80° s	90° s
.001	79	40	28	21	18	16	15	14	14
.002	158	80	55	43	36	32	29	28	28
.003	238	121	82	64	54	48	44	42	41
.004	317	161	110	86	72	64	58	56	55
.005	396	201	138	107	90	80	73	70	69
.006	475	241	165	128	108	95	88	83	83
.007	554	281	193	150	126	111	102	97	97
.008	634	322	220	171	144	127	117	111	110
.009	713	362	248	193	162	143	131	125	124
.010	792	402	275	214	180	159	146	139	138

Rule of Signs.—If b denotes a rotation in the direction above indicated, so that

$$\xi = x - by, \quad \eta = y + bx,$$

(x, y) denoting measures, and (ξ, η) standard coordinates, then the adopted R.A. of the plate centre is to be *increased* by the above quantity before deriving (ξ, η) ; and we shall then get

$$\xi = x + \text{const.}, \quad \eta = y + \text{const.}$$

10. If the plate has a scale value different from that adopted

for (ξ, η) , this can be allowed for in a variety of ways. Thus, in forming

$$\xi = (a - A) \cos \delta$$

if we wish to form instead

$$(1 + p)\xi = (a - A) (1 + p) \cos \delta,$$

we can do so indirectly by writing $(\delta + \epsilon)$ for δ , where

$$\begin{aligned} \cos (\delta + \epsilon) &= (1 + p) \cos \delta \\ \text{or } \epsilon &= -p \cot \delta. \end{aligned}$$

It must be remembered that great refinements are not necessary for the comparison of B.D. with our measures, and the mean scale value for a number of plates will probably be near enough for all, in spite of refraction and aberration. At Oxford the focal length of the instrument is nearly one per cent. too great, so that measures made with a standard *réseau* of 5^{mm} spaces have all to be diminished in the ratio 1.0078 to 1. Recently, this troublesome correction (which ought to have been avoided by greater care on the part of the maker of the object-glass) has been obviated by measuring with a *réseau* of 5^{mm}.04 spaces, so that the scale value is now approximately correct. Suppose then that we have made tables for the more recent plates, as below.

Table for $(a - A) \cos \delta$ in *Réseau* intervals.

	Dec' = 25° 0'	10'	20'	30'	40'	50'	26° 0'
(a - A)							
m s							
0 10	0.453	0.453	0.452	0.451	0.451	0.450	0.449
20	0.906	0.905	0.904	0.903	0.901	0.900	0.899
30	1.360	1.358	1.356	1.354	1.352	1.350	1.348
40	1.813	1.810	1.853	1.850	1.848	1.845	1.843
50	2.266	2.263	2.259	2.257	2.254	2.250	2.247
1 0	2.719	2.716	2.711	2.708	2.704	2.700	2.696
10	3.172	3.168	3.163	3.159	3.155	3.150	3.146
20	3.626	3.621	3.615	3.610	3.606	3.600	3.595
&c.	&c.	&c.					

the equivalent of the seconds and tenths being found from a supplementary table, and added to the number taken from above before contracting to two figures.

To use this table for comparing the B.D. with measures on

the old plates with defective scale value, we need only substitute for the headings,

25° 0', 25° 10', 25° 20', 25° 30', 25° 40', &c.
the values 25° 56', 26° 6', 26° 15', 26° 25', 26° 34', &c.

and the tables are then adapted to the old plates.

For the formation of $\eta = (\tilde{c} - D) + \frac{1}{4}(a - A)^2 \sin 2D$, the second term will not generally need correction, and the first term can be multiplied by the proper factor by means of a small table.

The Use of a Coloured Screen in Photographing the Corona during an Eclipse. By W. Shackleton, A.R.C.Sc.

It has long been known that the distribution of *coronium* throughout the corona was not concurrent with the streamers themselves, the '1474 K' line often being as strong in rifts as elsewhere. From observations made at the eclipse of 1898 both Mr. Newall and Mr. Fowler pointed out that the image of the corona, viewed in the monochromatic light of the green corona line, had decided variations from that seen or photographed directly. In the main the image of the former corresponds to an *inner corona*; this *inner corona* not being merely a sort of under-exposed picture of the whole, but one having a delineation of its own which does not agree with the corona proper. If, then, it were possible to photograph the corona in '1474 K' light alone, we should determine the distribution of *coronium* throughout the corona.

The most obvious way of photographing the corona in the light of the green corona line would be, of course, to use some spectroscopic method; but I wished rather to employ simple apparatus, and from the following considerations I think it possible this may be done by the use of a camera provided with a suitably coloured screen. It will be seen, on examining the photographs taken with prismatic cameras or other spectroscopes during the eclipses of 1893, 1896, and 1898,* that the light of the corona consists principally of continuous spectrum, the line spectrum, with the exception of '1474 K,' being feeble. This line, however, is strong enough to give good decided impressions, although falling in a region of weakness on the isochromatic plates used; indeed, it is so intense that I have seen it recorded on plates supposed to be insensitive to the green.

Referring again to the above photographs, it appears that the continuous spectrum has been photographed to a greater distance from the Sun's limb than has the line spectrum.

* *Phil. Trans.*, A, vol. clxxxvii. (1896), Plates II, 12; vol. clxxxix. (1897), Plate 9. *Monthly Notices*, R.A.S., vol. lviii., Appendix, Plate 9.