ON THE MOLECULAR PHYSICS OF ORE FLOTATION¹

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Introductory

It is common practice to inquire into the efficiency of a process as soon as it has been made commercially successful. So it has been with the flotation process. Hoover⁴ makes an erroneous statement about the force required to penetrate a liquid surface, and with it as a basis calculates and tabulates the actual and theoretical capacities of several types of flotation machines. Other writers have made similar efforts by using principles incorrectly. The most striking case is that of Van Arsdales.⁵ He begins with the premise that "the maximum weight that can be supported on the surface of water is equal to the surface tension of the area supporting the weight." Beginning thus with a false premise, naturally he ends with false conclusions. It is scarcely hypercritical to say that for these fallacies to go unchallenged after receiving favorable editorial comments and being incorporated in a book, portends a tardy science of flotation.

In this discussion the underlying principles involved in calculating the floatability of solids and constructing profiles arising from surface tension will be pointed out and the calculation of "theoretical capacities" left to the more ambitious.

Maximum Depression against a Vertical Plate

Imagine a plate of unit horizontal length placed vertically with its lower edge below the surface of water with its faces so prepared that on one side $\theta = 90^\circ$ and on the other, $\theta = 180^\circ$, (θ is the angle of contact). It will appear in cross sec-

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⁴ T. J. Hoover: "Concentration of Ores by Flotation," p. 77 (1914).

⁵ G. D. Van Arsdales: "How Flotation Works," Eng. & Min. Jour., 101, 851 (1916).

tion at right angles to its length as shown in Fig. 1. Let h

Fig. 1

Vertical plate with faces prepared so that θ is 90° and 180°

be the depth of the dimple in centimeters. It is obvious that the sum of the horizontal forces must be zero, otherwise there would be motion, and work would be done without the expenditure of energy, which is contrary to the laws of nature.¹

Since, on one side, the surface of the dimple meets the plate at 180° , the horizontal component of the pull due to surface tension is zero. On the other side, it is 0.0715 gram (the assumed tensile strength of the film). Opposite and equal to this force is that of the head of the water from a to b , which is

$$h \times \frac{h}{2} = \frac{h^2}{2}; \frac{h^2}{2} = 0.0715.$$

$$h^2 = 0.143.$$

$h = 0.378$ centimeter, the maximum depth possible in pure water where the face is vertical.

¹ The equilibrium of this system and the one in Fig. 2 was discussed by Le Conte and Riley in the *Philosophical Magazine* in 1883. Le Conte fell into a dispute with Young, who "insisted that according to Laplace's theory the horizontal molecular pressures on the opposite faces being unequal, the composite plate should be drawn in the direction of the center of concavity of the meniscus on the glass face (Fig. 2), and thus cause the entire system to take on a motion of translation.

He, therefore, "made the following arrangements with the view of experimentally testing the question: A plate of well cleaned glass and a plate of polished steel were cemented together so as to constitute a composite plate of these substances. This was floated in a vertical position by securing masses of cork to the two faces, and attaching a leaden sinker of the proper weight to the lower extremity of the plates. If such a composite plate is plunged vertically into alcohol contained in a large vessel, a concave meniscus will be formed by the ascending liquid on the glass face, while on the steel face no meniscus will be formed, and the adjacent surface of the alcohol will be horizontal." There was no evidence of unbalanced forces on the floating faces.

Riley investigated this principle and included conditions represented by Fig. 1 and found that in "every case the identity shows that the magnitude of these tensions is just the same as if the surfaces came up perpendicular to the plates without elevation or depression."

Maximum Rise against a Vertical Plate

The profile of the liquid surface wetting the glass, Fig. 2, is the inverted profile of the one where $\theta = 180^\circ$ in Fig. 1, because hydrostatic pressure is the only variable, and it varies directly with depth—from the top of the meniscus in Fig. 2 to the bottom of the dimple in Fig. 1. The maximum rise is therefore 0.378 centimeter. The radius of curvature at any point is determined by the formula

Fig. 2

Capillary rise against a vertical plate

$$P = \frac{T}{R}.$$

Note that in Fig. 1, where h was found to be 0.378 centimeter, the solid face was vertical. The maximum depth is obtained when the non-wetted face is horizontal. Let this depth be H .

Profile of a Drop of Water

If the liquid is resting on a solid which it meets at an angle of 180° , the drop will appear as shown in Fig. 3. The total

Fig. 3

The arc method of constructing the profile of drops of water

hydrostatic pressure against an imaginary vertical plane of unit horizontal length against the right end is $\frac{H^2}{2}$. The op-

posite and equal force is $2T$, where T is tensile strength in gram-weight per unit length of film.

$$\therefore \frac{H^2}{2} = 2T, H^2 = 4T = 0.286$$

$$H = 0.535 \text{ centimeter.}$$

To draw this, take a horizontal line to represent the upper plane surface. Having the value of H , take another line to represent the surface of the solid on which it rests. Let the surface of the water meet the solid surface at A . The radius of curvature R of the surface of the water where it leaves this solid is found by substituting in,

$$P = \frac{T}{R} = H;$$

$$0.535 = \frac{0.0715}{R}$$

$$R = 0.134 \text{ centimeter.}$$

From "A" measure a distance R vertically upwards, and with the point so chosen, describe a small circular arc of radius R , to represent the lowest arc of the profile. The remaining arcs of the profile are drawn as indicated by the figure.

It is not absolutely correct mathematically to apply the formula

$$P = \frac{T}{R}$$

in this case. It is correct only when a surface element is obtained by bending a flat surface about a straight line as an axis, *i. e.*, only one plane of bending, giving a cylinder. With the flattened drop there are really two planes of bending at right angles to each other, and when the radii are R_1 and R_2 , the correct formula is

$$P = T \left(\frac{1}{R_1} + \frac{1}{R_2} \right).$$

Generally, however, R_2 is so large that $\frac{1}{R_2}$ may be neglected. When $R_1 = R_2$, we have, of course, the spherical drop.

The Proverbial Steel Needle

The following demonstration of equilibrium of the floating needle is taken from Edser:¹

Let Fig. 4 represent the transverse section of a needle of radius r , floating on water.

Let the surface of the water leave the needle tangentially at B, where the radius CB of the needle makes an angle A with the vertical. Let the point B be at a distance, h , below the flat surface of the water.

Fig. 4
Transverse section of wire floating on water when θ is 180°

The following are the upwardly directed forces acting on unit length:

The force due to the surface tension of the water is equal to

$$2 T \sin A. \quad (a)$$

If the portion of the needle below the horizontal plane EB were removed, the hydrostatic pressure of the water would exert an upward force equal to

$$2 r \sin A h \quad (b)$$

The additional upward force due to the segmental portion of the needle below EB is equal to the weight of the water displaced by this portion of the needle; that is,

$$r^2 \left(\pi \cdot \frac{A}{180} - \sin A \cdot \cos A \right) \quad (c)$$

It is assumed that the length of the wire in comparison with the diameter is so great that the surface-tension pull upon the ends may be neglected.

The downward force is

$$7.7 \cdot \pi r^2. \quad (d)$$

We now wish to eliminate h in Equation *b*. Inspection shows that for horizontal forces in FB to be in equilibrium,

¹ Edwin Edser: "General Physics for Students," p. 340 (1911).

$$\begin{aligned}
 \frac{h^2}{2} &= T - T \cos A. \\
 h &= 2\sqrt{T} \sqrt{\frac{1 - \cos A}{2}}. \\
 &= 2\sqrt{T} \sin \frac{A}{2}.
 \end{aligned} \tag{e}$$

Substituting this value for h in Equation b and equating the upward and downward forces, we obtain the equation

$$\begin{aligned}
 2T \sin A + 4r \sin A \sin \frac{A}{2} \cdot \sqrt{T} + \\
 r^2 \left(\pi \cdot \frac{A}{180} - \sin A \cos A \right) &= \pi r^2 7.7.
 \end{aligned} \tag{f}$$

Setting $\sin A \cos A$ equal to $0.5 \cdot \sin 2A$

and transforming,

$$\begin{aligned}
 r^2 \left(24.2 + 0.5 \sin 2A - \pi \cdot \frac{A}{180} \right) - 1.07 r \sin A \sin \frac{A}{2} \\
 = 0.143 \sin A.
 \end{aligned} \tag{g}$$

Fig. 5 shows the values of r for values of A lying between 0° and 120° . It will be noted that the maximum value of r corresponds to $A = 100^\circ$, and is equal to 0.0995 centimeter, or roughly 1 mm. Thus the diameter of the largest needle

Fig 5

Graph exhibiting relation of r and A for steel wire on water where θ is 180°

Fig. 6

Steel wire floating on water where θ is 180° and r is a maximum

that can float on water is equal to two millimeters approximately. The position of the needle relative to the surface is shown in Fig. 6. The curve in Fig. 5 shows that the needle is in unstable equilibrium when A is greater than 100° .

Liquid Profiles

A method of constructing liquid profiles is shown in Fig. 3. Its inaccuracy becomes apparent when one attempts to apply it; there is a cumulative error which necessitates "fudging" the curve to make it become tangent at "C." An exact method of construction based on a coördinate system is therefore desirable.

A search of the scientific literature reveals the fact that some of the scientists have been unable to deal with this curve in a scientific manner, using instead "cut and try" methods. A striking example of this is found in the paper of Richards and Coombs,¹ where they made elaborate experiments to "discover" the "diameter necessary in order to attain the flat surface with which the capillary rise is to be compared" while measuring surface tension by the capillary rise method. After investigating the literature and making many tests, they decided that "in the 38 mm. tube, a portion in the middle, apparently about 5 mm. in length, appeared to be flat, coinciding exactly with the horizontal spider-line of the telescope." From this it appears that in a large glass receptacle where θ is zero, or nearly so, the meniscus would coincide with the plane surface of the liquid at a distance 16.5 millimeters $\left(\frac{38-5}{2}\right)$ from the wall. Since the upper portion of the profile (that above B) in Fig. 3 is like the inverted profile of the meniscus referred to, it would be expected therefore that the horizontal distance from B to C would be 1.65 centimeters. The error in the conclusions of Richards and Coombs is indicated below.

A comparison of the theoretical and observed positions of floating cylinders where the specific gravity, angle of contact and surface tension are known would add to the significance of these factors when applied to the study of floating minerals. To do this a correct liquid profile must be constructed by means of coördinates. Gray's² work aids in

¹ T. W. Richards and L. B. Coombs: "The Surface Tension of Water and the Alcohols," *Jour. Am. Chem. Soc.*, **37**, 1656 (1915).

² A. Gray: "A Treatise on Physics," **1**, 651.

this regard. Under the subject of capillarity he gives the equation of this curve but treats it so academically that one finds it difficult to make the technical application. His equation is

$$x = -(4a^2 - y^2)^{1/2} + a \log \frac{2a + (4a^2 - y^2)^{1/2}}{y} + C.$$

It is interesting to note at this point that Lord Kelvin and Tait¹ have likewise applied this equation to the curve of a bent wire. Hall² has also used it. If we set $2a = 0.535$ (the value of H), $C = 0.1427$ and change the Napierian logarithm to the common system by multiplying by 2.3, it takes the form

$$x = -(0.2862 - y^2)^{1/2} + 0.6152 \log \frac{0.535 + (0.2862 - y^2)^{1/2}}{y} + 0.1427.$$

Table I gives the x , y ordinates.

The remarkable coincidence of this curve (Fig. 7) with the portion of Fig. 3 in which the arc method of construction

Fig. 7

Profile of standing water with one plane of bending.

$T = 0.0715$ gram

has the minimum error, and the deviation from it where the error is obviously maximum, dispels all doubt toward the application of the formula to the profile of a standing liquid with one plane of bending.³

¹ Lord Kelvin and P. Tait: "Treatise on Natural Philosophy," Part 2, p. 149.

² T. P. Hall: Phil. Mag., 36, 385 (1893).

³ This equation will be treated in more detail by one of us (A) in another paper. There it will be shown how the two parameters used in the graphical construction of the curve are incorporated into the equation.

TABLE I
Coördinates for Liquid Profile where there is only One Plane of
Bending
 $T = 0.0715$ g, sp. gr. = 1

x	y	x	y
0.143	0.535	0.1828	0.13
0.0809	0.52	0.2032	0.12
0.0645	0.51	0.2240	0.11
0.0517	0.50	0.2479	0.10
0.0332	0.48	0.2749	0.09
0.0149	0.45	0.3052	0.08
0.0050	0.42	0.3409	0.07
0.0011	0.40	0.3799	0.06
0.0000	0.378	0.4278	0.05
0.0018	0.35	0.4768	0.04
0.0085	0.32	0.6714	0.02
0.0154	0.30	0.8663	0.01
0.0244	0.28	0.9331	0.0075
0.0421	0.25	1.0015	0.0050
0.0580	0.23	1.2266	0.0025
0.0746	0.21	1.3630	0.0015
0.0843	0.20	1.4710	0.0010
0.1199	0.17	1.5480	0.00075
0.1496	0.15	2.0866	0.00010
0.1671	0.14		

Reference to the last two values of y and x in Table I shows that the 1.65 centimeters, taken by Richards and Coombs as the distance from the solid at which the meniscus coincides with a plane surface, allows an error of less than 0.00075 centimeter and more than 0.00010 centimeter. This appears in Fig. 7 and Table I.

Having this curve, we can now consider the angle of contact.

The Practical Formula for Steel Wires Floating on Water

In the formula developed for the floating "needle," θ was taken as 180° . Since this value is likely never found in practice, it must be regarded as ideal. By a simple substitution a variable which will make it applicable for all values of θ

TABLE II
Values of A and r in Equation g' . Steel Wires

$\theta = 180^\circ$ D = 0°		$\theta = 168^\circ$ D = 12°		$\theta = 150^\circ$ D = 30°		$\theta = 135^\circ$ D = 45°		$\theta = 105^\circ$ D = 75°		$\theta = 90^\circ$ D = 90°		$\theta = 75^\circ$ D = 105°		$\theta = 45^\circ$ D = 135°	
A	r cm	A	r	A	r	A	r	A	r	A	r	A	r	A	r
0	0.0000	—	—	—	—	—	—	—	—	—	—	—	—	—	—
10	0.0320	—	—	—	—	—	—	—	—	—	—	—	—	—	—
20	—	12	0.0000	—	—	—	—	—	—	—	—	—	—	—	—
30	0.0461	20	0.0291	—	—	—	—	—	—	—	—	—	—	—	—
40	0.0572	30	0.0444	30	0.0000	—	—	—	—	—	—	—	—	—	—
50	0.0667	40	0.0563	40	0.0333	—	—	—	—	—	—	—	—	—	—
60	—	50	—	45	0.0000	—	—	—	—	—	—	—	—	—	—
70	0.0752	60	0.0660	50	0.0482	50	0.0234	—	—	—	—	—	—	—	—
80	0.0829	70	0.0753	60	0.0602	60	0.0421	—	—	—	—	—	—	—	—
90	0.0895	80	0.0829	70	0.0704	70	0.0553	—	—	—	—	—	—	—	—
100	—	90	—	80	—	—	—	75	0	—	—	—	—	—	—
110	0.0941	100	0.0896	90	0.0792	80	0.0668	80	0.0242	—	—	—	—	—	—
120	0.0976	110	0.0946	100	0.0864	90	0.0765	90	0.0435	90	0.0000	—	—	—	—
130	0.0994	120	0.0979	110	0.0921	100	0.0839	100	0.0573	100	0.0345	—	—	—	—
140	—	130	—	120	—	—	—	—	—	—	—	105	0.0000	—	—
150	0.0987	140	0.0989	130	0.0957	110	0.0897	110	0.0682	110	0.0510	110	0.0247	—	—
160	0.0961	150	0.0981	140	0.0974	120	0.0936	120	0.0767	120	0.0630	120	0.0446	—	—
170	0.0906	160	0.0946	150	0.0967	130	0.0947	130	0.0830	130	0.0721	130	0.0572	—	—
180	0.0824	170	0.0885	160	0.0946	140	0.0943	140	0.0869	140	0.0784	140	0.0669	140	0.0248
190	0.0714	180	0.0798	170	0.0880	150	0.0886	150	0.0886	150	0.0828	150	0.0739	150	0.0433
200	0.0570	190	0.0686	180	0.0801	160	0.0857	160	0.0879	160	0.0848	160	0.0784	160	0.0554
210	0.0388	200	0.0547	190	0.0701	170	0.0783	170	0.0853	170	0.0863	170	0.0803	170	0.0635
220	0.0000	210	0.0380	200	0.0580	180	0.0690	180	0.0807	180	0.0821	180	0.0807	180	0.0690

is introduced, after which theoretical and observed values may be compared.

The corrections are simple, though at first somewhat confusing.

Since, in practice, the angle of contact is always less than 180° , an angle D is introduced to represent this difference. Equation a then becomes,

$$2T \sin (A - D). \quad (a')$$

Equation e becomes

$$2\sqrt{T} \sin \frac{A - D}{2},$$

and when this value is substituted in (b) we have

$$4r\sqrt{T} \sin A \sin \left(\frac{A - D}{2} \right). \quad (b')$$

The expressions (c) and (d) are unchanged.

Equation g , which applies to steel wires, then becomes

$$r^2 \left(24.2 + 0.5 \sin 2A - \pi \frac{A}{180} \right) - 1.07 r \sin A \sin \left(\frac{A - D}{2} \right) = 0.143 \sin (A - D). \quad (g')$$

Table II gives the values of A and r for stated values of θ .

Fig. 8 shows the curves conforming to the coördinates given in Table II.

Fig. 8

Curves showing the theoretical values of A for steel wires for stated values of r and θ

The Relation of Angle of Contact to Floatability

We now have sufficient data to show the relation of the angle of contact to the floatability of cylinders. The subject

begins to take the aspect of a science when we see from this figure that if r is sufficiently small a solid is floatable though θ may be as small, for example, as 45° or less. Previously, the fact that pieces of glass floated on water could be explained only by the "air cushion" theory. Such an hypothesis must now give way to facts. Consider, for example, a steel wire with a radius of 0.06 centimeter. Reference to Fig. 8 shows that when $\theta = 180^\circ$, $A = 33^\circ$; $\theta = 168^\circ$, $A = 44^\circ$; $\theta = 150^\circ$, $A = 60^\circ$; $\theta = 135^\circ$, $A = 74^\circ$; $\theta = 105^\circ$, $A = 102^\circ$; $\theta = 90^\circ$, $A = 117^\circ$; $\theta = 75^\circ$, $A = 133^\circ$; $\theta = 45^\circ$, $A = 165^\circ$. The very slight drop in the consecutive crests of the curves from left to right, while θ decreases from 180° to 45° , is striking. But this must not be taken to indicate relative floatability in practice. For, though it is true that increasing "A" brings the object near the point of unstable equilibrium (the descending portion of the curve), yet another factor of no less importance appears in practice. It is the position of the walls of water with reference to the object, and to each other. Note in Fig. 9, *a*, how the object rides safely on a disturbed surface, while in Fig. 9, *h*, there is continual danger of the

Fig. 9

Showing how a steel wire of given diameter ($r = 0.06$ cm) rides on pure water while θ decreases from left to right

walls overlapping and causing submergence. When they overlap and coalesce, the surface tension film, of course, vanishes. A little lycopodium powder or talc on the surface assists in maintaining the interface by preventing coalescence. Is it not probable that the insoluble oil used in flotation acts in the same manner, preventing the coalescence of the walls of the bubble whenever it is in danger of being separated from the mineral?

These theoretical considerations would have greater

significance if supported by physical measurements and observations. Such tests have been made with the cathetometer, and the agreement of observed and calculated values justifies the theory. Let us first consider the principles involved in constructing Fig. 9, after which we shall consider the physical measurements.

Take, for example, Fig. 9, *b*. The radius of the steel wire is 0.06 centimeter and θ is 168° . The angle D is therefore 12° . Look along the line (Fig. 8), corresponding to $r = 0.06$ cm and find where it intersects the curve marked " $\theta = 168$." Here the corresponding angle A is 44° . Now use the modified form of Equation *e*, which is,

$$h = 2\sqrt{T} \sin \frac{A - D}{2}.$$

This gives the value of h , which is then applied to Fig. 9. At the point on the curve so determined strike off the radius of the object so that "A" has the value desired, and draw the circle.

Of course, θ must be determined by test. The classic methods for this determination were considered, but discarded in favor of an indirect method in which the cathetometer was used. Here the capillary rise or depression against a vertical plane surface was determined, and θ was found by means of the above equation by solving for D after setting A equal to 90° . When there is capillary rise, θ equals D, and for capillary depression θ equals 180° minus D. Pure water gives an angle of contact of 168° against a surface prepared by dipping a solid into molten paraffin and sprinkling lycopodium powder on it before the paraffin congeals. Against solid paraffin the angle is 105° .

Table III shows that the observed and theoretical position of steel wires floating on pure water coincide with a satisfactory degree of accuracy. The cathetometer was used to measure the vertical distance from the plane surface of the water to the bottom of the floating wire. No correction was made for the thickness of the paraffin shell, which was found

to be very thin—only 0.01 centimeter. Fig. 10 shows the theoretical position of these wires on the liquid profile. The liquid meets the heavy and dotted circles at 168° and 105° , respectively.

Fig. 10

Theoretical positions of steel wires floating on water where T is 0.0175 and θ is 105° and 168° , respectively

TABLE III

Showing the Observed and Theoretical Depth of Steel Wires Floating on Pure Water

Radii of wires in cm	$\theta = 168$ Depth in cm		$\theta = 105$ Depth in cm	
	Observed	Theoretical	Observed	Theoretical
0.0440	0.090	0.082	0.120	0.115
0.0520	0.125	0.129	0.160	0.156
0.0684	0.205	0.212	0.260	0.253
0.0855	0.330	0.335		

The Flotation of Aluminum Wires

From the standpoint of the metallurgist, we are still dealing with an ideal case because minerals cannot be obtained in the form of right circular cylinders; again, steel wires have a greater specific gravity than many of the metallic sulphides. In order to transform ideal conditions into the practical, let us make a further study of the effect of specific gravity, reserving the question of shape of the floating object for a later discussion.

Having made a study of the conduct of wires heavier than the average of the metallic sulphides, it seems that if we

would make a similar study of those lighter than the minerals and find an agreement also between theoretical and observed values, we could use mathematical processes for predicting the conduct of any mineral. The following results deal, therefore, with aluminum wires:

When aluminum wires (specific gravity 2.7) are used, the equation corresponding to Equation g' is

$$r^2 \left(8.4823 + 0.5 \sin 2A - \pi \frac{A}{180} \right) - 1.07 r \sin A \sin \left(\frac{A - D}{2} \right) = 0.143 \sin (A - D).$$

The values of A and r for stated values of θ are shown in Table IV.

TABLE IV
Values of r and A . Aluminum Wires

$\theta = 168^\circ$ $D = 12^\circ$		$\theta = 105^\circ$ $D = 75^\circ$		$\theta = 90^\circ$ $D = 90^\circ$	
A	r	A	r	A	r
12	0.0000	—	—	—	—
20	0.0499	—	—	—	—
30	0.0775	—	—	—	—
40	0.1004	—	—	—	—
50	0.1216	—	—	—	—
60	0.1422	—	—	—	—
70	0.1626	—	—	—	—
—	—	75	0.0000	—	—
80	0.1812	80	0.0446	—	—
90	0.1986	90	0.0835	90	0.0000
100	0.2129	100	0.1145	100	0.0686
110	0.2210	110	0.1410	110	0.1033
120	0.2262	120	0.1630	120	0.1312
130	0.2217	130	0.1795	130	0.1531
140	0.2083	140	0.1892	140	0.1686
150	0.1860	150	0.1931	150	0.1771
160	0.1564	160	0.1870	160	0.1769
170	0.1182	170	0.1761	170	0.1733
180	0.0744	180	0.1602	180	0.1631

These points are plotted in Fig. 11. The scale is the same as that of Fig. 8. Note how they have the same general

form as those for steel, but run up much higher on account of the lesser density of the aluminum. The remarkable co-

Fig. 11

Curves showing the theoretical values of A for aluminum wires for stated values of r and θ

incidence of the theoretical depth with the observed depth of the bottom of the wire below the plane surface of the liquid again substantiates the theory. See Table V.

TABLE V

Showing the Observed and Theoretical Depth of Aluminum Wires Floating on Pure Water

Radii of wire in cm	$\theta = 168^\circ$ Depth in centimeters		$\theta = 105^\circ$ Depth in centimeters	
	Observed	Theoretical	Observed	Theoretical
0.065	0.067	0.065	0.102	0.100
0.095	0.136	0.140	0.190	0.195
0.192	0.494	0.500	—	—
0.215	0.680	0.642	—	—

Fig. 12 shows the theoretical position of these aluminum wires on the liquid profile. The liquid meets the heavy and

dotted circles at 168° and 105° , respectively, as in Fig 10, where the steel wires were shown.

Fig. 12

Theoretical position of aluminum wires of given radii, floating on pure water, where θ is 105° and 168° , respectively

Anderson¹ says: "A needle, especially if slightly oily, will float if carefully placed upon water. We may note in passing that the weight of the water displaced by the sagging of the surface film is equal to the weight of the needle (Archimedes' Principle)." This statement will be discussed in another paper.

The Flotation of Galena

To compare the conduct of the mineral grains fed to a H. E. Wood flotation machine to a group of skaters where some glide over the ice and others break through, is an analogy that is fruitless. Ice breaks when the molecules are removed beyond the limit of influence, but at the instant when galena sinks the molecules in the surface of the liquid are under the same stress as those on an undisturbed surface. To speak of the membrane rupturing or breaking at the instant the galena becomes submerged, is as far from accuracy as is the North from the South. We must have the facts or continue the guessing process in flotation, which, it must be admitted, has, in some cases at least, produced such favorable results that a "small piece of cheese has been left for the cats after the monkey got his portion."

¹ W. B. Anderson: "Physics for Technical Students."

What is the position of floating galena relative to the liquid profile? The angle of contact of pure water against cleavage face is approximately 45° .

Fig. 13

Diagram showing how galena floats on water when θ is 45°

Is a mineral floatable when the water turns up at this angle to meet a vertical face? Figs. 8 and 9 show that wires are, and it seems consistent to assume that the same conclusion applies to minerals. Conjecture offers only one explanation—diagrammed

in Fig. 13. Cathetometer observations show that this is a reality. The liquid profile meets the upper horizontal face (at 45°), the proof of which is shown in Table VI.

TABLE VI
Cathetometer Observations on Floating Galena

Approximate mesh	Vertical dimensions in centimeters	
	Of galena	Bottom of galena to intersection of liquid profile with galena
36-mesh	0.040	0.055
33-mesh	0.045	0.055
13-mesh	0.130	0.150
12-mesh	0.140	0.150

In each observation the readings showed that the mineral was wholly below the bottom of the dimple.

Summary

The maximum rise and depression of a given liquid against vertical plates are determined and shown to be the same for angles of contact of 0° and 180° , respectively. The horizontal pull of the film is independent of the angle of contact.

A graphical method of constructing a liquid profile with one plane of bending is reviewed.

A review is given of the mathematical discussion of the floating needle when the angle of contact is ideal.

A liquid profile with one plane of bending is constructed on a coördinate system, and compared with the arc and "cut and try" methods.

A practical formula for steel wires floating on water is developed from the ideal and theoretical and observed values are compared.

A formula is developed for calculating angle of contact from the cathetometer measurements of rise or depression of a liquid against a vertical plate. The relation of angle of contact to floatability is indicated by curves and checked by observations.

The flotation of aluminum wires is treated scientifically and practically.

The manner in which "floating" galena is suspended at the surface of water is shown.