



276. On Certain Algebraical Factors

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MATHEMATICAL NOTES.

275. [R. 6.] *Change of Kinetic Energy due to Mutual Action of two Particles, and loss of Kinetic Energy by Collision.*

If an action and reaction of any kind take place between two particles, the changes of momentum may be written :

$$I = m(u_1 - u_2) = m'(v_2 - v_1),$$

where m, m' are the masses, u_1, v_1 velocities before and u_2, v_2 velocities after the action.

Let V_1, V_2 be the relative velocities before and after, so that $V_1 = u_1 - v_1$, $V_2 = u_2 - v_2$.

$$\text{Then } I(m + m') = mm'(v_2 - v_1 + u_1 - u_2) = mm'(V_1 - V_2).$$

$$\begin{aligned} \text{Loss of Kinetic Energy} &= \frac{1}{2}m(u_1^2 - u_2^2) + \frac{1}{2}m'(v_1^2 - v_2^2) \\ &= \frac{1}{2}I\{(u_1 + u_2) - (v_1 + v_2)\} \\ &= \frac{1}{2}I(V_1 + V_2) \\ &= \frac{1}{2} \frac{mm'}{m + m'}(V_1^2 - V_2^2). \end{aligned}$$

Hence, in general, there is loss of K.E. in all cases where the relative velocity is diminished, and a gain in cases (*e.g.* attraction) where the relative velocity is increased.

In cases of impact, where Newton's coefficient of restitution applies,

$$V_2 = -e V_1,$$

$$\text{and loss of K.E. is } \frac{1}{2} \cdot \frac{mm'}{m + m'} \cdot V_1^2(1 - e^2),$$

which is positive since $0 < e < 1$.

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[This method was certainly familiar at Cambridge in 1889, and was given by Mr. R. R. Webb to his pupils. It was, I believe, due to Professor Greenhill, who has drawn attention to it by examination questions at the University of London and elsewhere.

The physical meaning of I is the blow or impulse Pt between the colliding bodies, t being the time the collision lasts, and P the average pressure.]

C S. J.

276. [A. 1. b.] *On certain Algebraical Factors.*

Let there be given a quadratic equation $ax^2 + bx + c = 0$ ($P = 0$), in which a, b, c are known coefficients. Then we can write

$$\begin{aligned} (\alpha) \dots\dots\dots ax^2 &= -(bx + c) \dots\dots\dots (\alpha'), \\ (\beta) \dots\dots\dots bx &= -(ax^2 + c) \dots\dots\dots (\beta'), \\ (\gamma) \dots\dots\dots c &= -x(ax + b) \dots\dots\dots (\gamma'), \end{aligned}$$

and any arbitrarily manipulated equation such as $\phi(\alpha, \beta', \gamma) = \phi(\alpha', \beta, \gamma')$ is satisfied by the values of x given by $P = 0$. Consequently

$$\phi(\alpha, \beta', \gamma) - \phi(\alpha', \beta, \gamma')$$

contains P as an algebraic factor.

For instance,

$$\begin{aligned} ax(ax^2+c)-b(bx+c) &\equiv P.(ax-b), \\ bx^2(ax+b)-c(ax^2+c) &\equiv P.(bx-c), \\ ax^2(ax+b)-c(bx+c) &\equiv P.(ax^2-c), \\ a(ax^2+c)^2+b^2(bx+c) &\equiv P.\{a^2x^2-abx+ac+b^2\}, \end{aligned}$$

and so on.

The same principle applies to equations of higher degrees.

Thus

$$\begin{aligned} (x^3+q)^3-p^3(px+q) &\text{ is divisible by } x^3+px+q, \dots\dots\dots(i) \\ (px^2+q)^2(x+p)+qx^4 &\text{ ,, ,, } x^3+px^2+q, \dots\dots\dots(ii) \\ (x^m+q)^2+p^2x^{2n-m}(px+q) &\text{ ,, ,, } x^m+px^m+q, \dots\dots\dots(iii) \end{aligned}$$

the quotients being

$$\begin{aligned} (i) \quad &x^6-px^4+2qx^3+p^2x^2-pqx+q^2-p^3, \\ (ii) \quad &p^2x^2+qx+pq, \\ (iii) \quad &x^m-px^m+p^2x^{2n-m}+q. \end{aligned} \quad \text{R. F. DAVIS.}$$

277. [M. c. a.]. *On the equation of a certain spiral.*

Some years ago a Portuguese writer published an article on a certain "new" spiral to which he gave the name of "the binomial spiral of the first degree," defined by the equation

$$r=(a-a_0)\frac{\theta}{\pi}+r_0=a\theta+b. \dots\dots\dots(1)$$

With Messrs. Brocard, V. Jamet, and the late Prof. Longchamps, I hold that this equation may be reduced to the form $r=a\theta$, under which it is at once recognised as the spiral of Archimedes.

At first sight the curve (1) appears to be a conchoid of the curve $r=a\theta$, and in that case it is likely to be a different curve, for in most cases the conchoid of a curve is a curve of a higher degree. But in this instance the conchoid does not differ essentially from the original since (1) reduces to the form

$$r=a(\theta+k), \dots\dots\dots(2)$$

where k is a constant. It is, in fact, the spiral of Archimedes turned through a constant angle.

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278. [v. a.]. *Notation of Binomial Coefficients.*

It is desirable that there should be a uniform and consistent notation for the binomial coefficients, and also for other expressions which occur in the binomial theorem, Vandermonde's theorem, Taylor's theorem, the ordinary interpolation formula, etc.

The most important expression is the binomial coefficient $\frac{n(n-1)\dots\{r\}}{1.2\dots\{r\}}$,

where $\{m\}$ denotes the presence of m factors. In English works this is usually represented by a C , with n and r associated; *e.g.*, by C_r^n . A fundamental objection to this notation is that C_r^n properly represents, not the number given above, but the number of combinations of n things r together; it is true that the two are equal, but to use them as identical involves a confusion of thought. A further objection is that the n in C_r^n suggests the index of a power. This last difficulty is avoided by using nC_r or ${}_nC_r$; but these again are open to the objection that they take up more space, and further that the n , in printed matter, is liable to be attributed to something immediately preceding.

(The latter is one of the typographical faults of Chrystal's *Algebra*.)