accuracy of the observations, as disturbances in general seem to tend to move the needle much more in the one direction than in the other, which fact removes the errors from the class which can be treated by that method.

Though the exact degree of accuracy of the observations cannot be ascertained, it can be stated conclusively that the error in our final determinations can, in the case of none of the substances experimented upon, with the exception of iron, much exceed 0.2%. We can thus sum up the results of our work thus far as follows: With screens of lead, zinc, mercury, water, alcohol or glycerine occupying one-third of the distance between the attracting masses and the needle, the change produced in the attractive force is certainly less than 0.2% of the total attraction. In the case of the iron screens no certain conclusions can be drawn, as the presence of the iron modified the magnetic field to such an extent that the sensitiveness of the somewhat magnetic needle was almost certainly changed. The change in the gravitational force, if it exists at all, is, we feel sure, less than 1% of the total force.

In the future work which is to be carried on, it is intended to make use of an instrument with a larger tube, so as to reduce the damping and allow the time of swing and relative sensitiveness to be determined with accuracy. With this it is hoped that the questions of the effect of temperature and of the iron screens can be attacked successfully. The work already done will also be repeated, with the hope of reducing the estimated error to less than 0.1%.

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ON THE BEST RESISTANCE FOR A SENSITIVE GALVANOMETER.

BY FRANK A. LAWS.

IT is well known that in the practice of most methods of electrical testing, the precision attainable depends on the proper adjustment of the galvanometer resistance to the work in hand, and that in planning new work a solution for the best galvanometer resistance is always made.

It is our purpose to derive a solution under conditions other than those usually imposed.

To render the discussion more complete, a statement of the usual solution of the problem will be given. It is as follows: If a simple circuit, consisting of a battery, a resistance and a galvanometer, be arranged, the deflection of the instrument will be a maximum when the galvanometer resistance is equal to that of the remainder of the circuit. In obtaining this solution two assumptions are made:

First. That the coils have fixed dimensions.

Second. That the ratio of the diameter of the covered to that of the bare wire is the same for all sizes.

That this second assumption is incorrect is shown by the fact that the ratio for a No. 20 B. & S. gauge double silk-covered wire of the American Electrical Works is 1.12, while for a No. 40 wire it is 2.77.

In the course of the discussion we shall use the following symbols:

G , the galvanometer constant, field at centre of coil per unit current.

I_g , galvanometer current.

D , deflection of instrument.

V , volume of the coil.

b , linear constant of the coil, $b = k(V)^{1/2}$.

R_g , the galvanometer resistance.

R , the resistance of the circuit external to the galvanometer.

C , the diameter of the covered wire.

B , the diameter of the bare wire.

$y = C/B$.

w , the resistance per unit volume of the wire.

n , the number of turns per unit area.

E , E.M.F. of battery.

k , constants.

We shall assume an indefinitely short needle, and for the present retain the first assumption.

The usual demonstration is based on the equation,

$$G = k\sqrt{R_g}.$$

This equation requires correction. It is easily shown that

$$G = 4\pi nb(\varphi \cdot s),$$

where $(\varphi \cdot s)$ is a function the shape of the coil. As the same bobbin is to be used in all cases, we have

$$G = k_1 n = \frac{k_1}{C^2} = k_2 \frac{\sqrt{w}}{y} = k_3 \sqrt{\frac{R_g}{y^2}}.$$

To apply this to the typical circuit we have

$$I_g = \frac{E}{R + R_g}, \quad D = k_4 I_g G = k_5 \frac{E \sqrt{\frac{R_g}{y^2}}}{R + R_g}.$$

Differentiating and solving for a maximum, we have

$$R_g = \frac{R}{\frac{2}{y^2 \frac{d\left(\frac{R_g}{y^2}\right)}{dR_g}} - 1}$$

If γ be constant, this reduces to

$$R_g = R,$$

the relation usually given.

This general formula was first given by Professor Silas W. Holman. In order to employ it the properties of the wire must be found by winding experimental coils.

The assumption of fixed dimensions will now be discarded and attention given to the galvanometer constants of a family of similar instruments. For coils of rectangular cross-section we have, referring to the diagram :

Fig. 1.

$$G = 4 \pi n b \log_e \frac{\cot \frac{\beta}{2}}{\cot \frac{\gamma}{2}}.$$

For coils of this shape,

$$\varphi \cdot s = \log_e \frac{\cot \frac{\beta}{2}}{\cot \frac{\gamma}{2}}.$$

For instruments of the same family, β and γ constant, we have :

$$G = k_6 n b = k_6 \frac{b}{y y B^2}$$

$$b = k_7 \left(\frac{R_g}{w} \right)^{\frac{1}{2}},$$

$$y = \frac{k_8}{B^2 w^{\frac{1}{2}}},$$

$$\therefore G = k_{10} \left(\frac{w^{\frac{1}{2}}}{y} \right) R_g^{\frac{1}{2}}.$$

This shows that the sensitiveness of similar galvanometers wound to various resistances with the same size of wire, is proportional to the cube root of the galvanometer resistance. Also for a family of instruments of the same resistance the best size of wire will be that for which $\frac{w^{\frac{1}{2}}}{y}$ is a maximum. By winding experimental coils we obtain data for determining this function, a table of values of which is given below.

WIRE OF AMERICAN ELECTRICAL WORKS.

B. & S. Gauge No.	Single, Silk- covered. $\frac{w^{\frac{1}{2}}}{y}$	Double, Silk- covered. $\frac{w^{\frac{1}{2}}}{y}$	Ratio.
20	.94	.87	1.08
26	1.40	1.12	1.25
30	1.81	1.27	1.42
34	2.21	1.34	1.65
36	2.19	1.33	1.65
38	2.10	1.28	1.64
40		1.15	

The results are plotted in Fig. 2.

The best size for both single and double-covered wire is seen to be No. 34. This result of course applies only to the particular make of wire examined.

We see that there may be two instruments of a family having the same resistance which will give equal deflections with the same current.

We are now in position to find the best galvanometer resistance. We have

$$D = k_{10} \left(\frac{w^{\frac{1}{2}}}{y} \right) \frac{R_g^{\frac{1}{2}}}{R + R_g}.$$

Considering for the moment that all the instruments are wound with the same size of wire, we have a maximum value of D when

$$R_g = R/2.$$

Therefore the best galvanometer of the family is one which is wound with a No. 34 wire to a resistance equal to one-half that of the remainder of the circuit.

We will now consider two special examples:—

The coils of two-spool reflecting galvanometers very commonly have a volume of about eight cubic inches. Suppose the resistance external to the instrument to be 10ω and that double-silk-covered wire is to be used;

Fig. 2.

Relative galvanometer constants for similar coils wound to the same resistance with various sizes of silk-covered wire.

then by the ordinary rule the ideal galvanometer resistance is 10ω . In accordance with our first deduction No. 20 wire will give the best winding. R_G will then be 6ω ;

$$D = k I_G G = k_1 I_G nb;$$

$$D_{20} = k_1 \frac{E}{16} 775 \cdot (8.)^{\frac{1}{2}}.$$

For the instrument wound with No. 34 wire we have $R_G = 5\omega$, corresponding to a volume of coil of .026 cubic inches;

$$D_{34} = k_1 \frac{E}{15} 7780 \cdot (.026)^{\frac{1}{2}};$$

$$\frac{D_{34}}{D_{26}} = 1.6 \text{ approx.}$$

The deflection of an instrument similar to the first but designed in accordance with our suggestion, will be 60% greater than that of the instrument wound in accordance with what is called the solution for the best galvanometer resistance.

The coils of eight cubic inches would have a diameter of about 2.6" and an axial breadth of about 1.5". The approximate dimensions of the second coil would be diameter .4", breadth .25". The weight of wire in the first case would be 30. oz., in the second, .032 oz.

The results we have just obtained are in emphasis of the plea of Professor Boys for small instruments. In this connection we may observe that if we have two similar galvanometers of equal resistance the first having a constant G_1 , and wound with wire for which the ratio of the diameter covered to bare wire is y_1 , while these quantities for the second instrument are G_2 and y_2 , and if the linear dimensions of the first are N times those of the second, then,

$$\frac{G_2}{G_1} = \sqrt{N} \left(\frac{y_1}{y_2} \right)$$

The factor $\frac{y_1}{y_2}$ shows that there is a limit below which it is unprofitable to reduce the size of the instrument, this being the size for which a No. 34 wire is appropriate.

For galvanometers originally wound with wire finer than No. 34, the more sensitive instrument would be the larger.

We will now consider the case of a Wheatstone bridge where the resistances of the arms are small and nearly equal. Let them be of about one ohm each. Such an arrangement would be used in standardizing one-ohm coils or in bolometric work.

The ordinary solution gives one ohm as the best galvanometer resistance. If the coil has a volume of 8 cubic inches, the winding should be of No. 16 wire making $R_G = .88\omega$. We will suppose the E. M. F. of the battery to be 1 volt, its resistance 1 ohm, and that the bridge is out of balance by $\frac{1}{15000}$ ohm; then with due regard to carrying capacity we have

$$I_G = \frac{1}{15000} \text{ approx.}$$

$$D_{16} = k_1 325 \cdot (8.)^{\frac{1}{3}} \frac{1}{15000}.$$

If wound as here indicated, we should have

$$R_G = .5\omega \quad I_G = \frac{1}{12000}.$$

$$D_{34} = k_1 7780 \cdot (.0026)^{\frac{1}{3}} \frac{1}{12000}.$$

$$\frac{D_{34}}{D_{16}} = 2.04$$

The deflection may be doubled by properly designing the galvanometer. This result is for double-covered wire. If single-covered be used, we should multiply by the ratio factor 1.65, giving

$$\frac{D_{34}}{D_{16}} = 3.36;$$

showing that by the use of a proper galvanometer wound with single-covered wire the sensitiveness of the bridge may be increased about 200 per cent.

The gain indicated in the examples is much greater with very low than with high resistance instruments, the former as usually wound departing more widely from the conditions here imposed.

Of course the numerical results indicate chiefly the order of magnitude of the gain for it would be impossible to reduce the size of the instruments as much as is here indicated. Also we have not considered the magnetic system, or the fact that the space at the centre of the coil must be large enough for free movement of the needle and that turns in very close proximity to a finite needle produce less than the effect indicated by the strength of field at the centre of the coil.

Our conclusions are, that if an existing arrangement of apparatus be taken, we can increase its sensitiveness by replacing the galvanometer by a similar instrument wound with wire of a particular size to a resistance equal to one-half of that of the remainder of the circuit. This best size of wire is that for which $\frac{2v^{\frac{1}{3}}}{y}$ is a maximum.

For low resistance instruments the reduction in size renders magnetic shielding possible without undue clumsiness.

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