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ON A NEWLY DISCOVERED ABSOLUTE LIMIT TO
ECONOMICAL EXPANSION IN THE STEAM ENGINE
AND IN OTHER HEAT-MOTORS.

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It has been universally assumed by engineers, and probably by all physicists familiar with the theory of the steam engine hitherto accepted, that the best ratio of expansion, whether for maximum efficiency of fluid, of engine, of capital, or of plant,* increases with increase of steam pressure without limit, and that such ratio may be indefinitely increased with decrease of the ratio of back-pressure to initial pressure for any one kind of engine, notwithstanding the fact that the value of the ratio of expansion is modified by variation of the conditions of working, even where the ratio $\frac{P_b}{P_1}$ is the same. The writer now proposes to show that, in every actual engine, in every engine operated

* Vide a paper "On the Several Efficiencies of the Steam Engine," etc., JOURNAL OF THE FRANKLIN INSTITUTE, May and June, 1882; *Transactions of the American Society of Mechanical Engineers*, 1882.

under the conditions of real work and of usual practice, there exists a limiting value of any of these "ratios of maximum efficiency" beyond which it cannot be economically raised even with an infinitely elevated boiler-pressure. It will be further shown that this "absolute limit" may be readily, and probably often is, passed in every-day practice, that in the usual forms of steam engine an absolute limit exists within or not far beyond the customary working range of expansion, beyond which expansion cannot be carried with economy however high steam pressure may be adopted; in other words, with infinite pressure, the economical value of the ratio of expansion will be found often not merely finite, but sometimes probably within the limits of familiar practice.

In earlier papers* the writer has shown that the conditions under which engines are worked in actual practice differ in a very important degree from those assumed in the simple thermo-dynamic theory of heat engines, and, especially, that the unjacketed steam cylinder, which is usually assumed to be of non-conducting material, is to such a serious extent a reservoir and a transferrer of waste heat that the efficiency of the engine is greatly and sometimes enormously reduced.

* JOURNAL OF THE FRANKLIN INSTITUTE, May, 1881, February, May, and June 1882; *Transactions of the American Society of Mechanical Engineers*, 1881-2.

The equations which would give values of the ratios of maximum efficiency for the ideal "perfect engine," therefore, do not apply to real engines, and the author has deduced* modified equations, which are at least approximately correct for real practice. He has also produced true "curves of efficiency" from real engines, which were found to be practically the same in form and location, to be practically identical in character with those inductively obtained, so far as the two could be compared.

These curves have the equation

$$\frac{y}{x} = b^2 r^{2q} \frac{n - r^{1-n}}{n - 1}$$

in which b is a constant factor; r is the ratio of expansion and a function of x ; q is an exponent dependent upon the variation of cylinder-condensation with the rate of expansion; and n is the exponent of v in the approximate equations of pressures and volumes $p v^n = \text{constant} = p_1 v_1^n$.

When $b = 1$ and $q = 0$, the equation presented above reduces to

$$y = x \frac{n - r^{1-n}}{n - 1};$$

which is the equation of the curve of efficiency for adiabatic expansion. The value of b usually falls between 0.8 and 0.9 and that of q between -0.3 and -0.1, sometimes becoming nearly 0.

In this new theory of the steam engine, it is seen, the quantity of steam used is to be calculated as for adiabatic expansion, and the amount thus given multiplied by $\frac{1}{b r^q}$ to obtain the quantity actually demanded in non-adiabatic expansion.

The work done per stroke or per unit of weight of steam being calculated for the adiabatic case

$$W_a = p_1 v_1 \frac{n - r^{1-n}}{n - 1},$$

it is obtained for real engines by multiplying by the factor $b r^q$, and we have for net work done,

$$W_n = p_b^I v_1 \frac{n r^q - r^{1-n+q}}{n - 1} - p_b r v_1 = \frac{b p_1 v_2 n r^{q-1} - r^{q-n}}{n - 1} - p_b v_2.$$

* "On the Several Efficiencies of the Steam Engine," etc.

The actual efficiency of any engine thus is less than that calculated for the ideal case in the proportion of $b r^q$ to 1.

THE "GENERAL EQUATION OF STEAM ENGINE EFFICIENCY," as the writer has called it, was shown to be obtained by determining the ratio of the total of annual variable costs to the power obtained from the engine

$$\frac{1}{E} = \frac{A r v_1 + B v}{2 R W_n} = \frac{A v_2 + B v_2 r^{-1}}{2 R W_n}$$

which is a minimum when $\frac{M + r^{-1}}{W_n}$ is a minimum, A being the measure of annual costs, variable with volume of steam supplied, and $M = \frac{A}{B}$. W_n is the work done per stroke of the engine and R the number of revolutions per annum.

The value of W_n may be obtained by multiplying the value of W for adiabatic expansion, such as would be obtained in a non-conducting cylinder, by a factor variable with the ratio of expansion, as has been shown, which shall measure the ratio of actual work done in the metal cylinder to that done in adiabatic expansion, thus: Let b represent the proportion of steam present when expansion commences, as determined by the amount of cylinder condensation; let r^q represent the rate of variation of losses with increase of the ratio of expansion; and let n be the index for the actual expansion line of the mixture, to be determined, if possible, by experiment. Then we shall have:

$$W_n = 2 R \left(b p_1 v_2 \frac{n r^{-1} - r^{-n}}{n - 1} r^q - p_b v_2 \right)$$

"THE GENERAL EQUATION OF ALL STEAM ENGINE EFFICIENCIES," is therefore

$$\frac{1}{E} = \frac{A v_2 + B v_2 r^{-1}}{2 R \left(b p_1 v_2 \frac{n r^{-1} - r^{-n}}{n - 1} r^q - p_b v_2 \right)} \quad (A)$$

which becomes a minimum, and makes the *commercial efficiency* of an engine doing the required work a maximum when, to obtain r , we have made

$$r^q + \frac{q}{M(q-1)} r^{q-1} - \frac{q-n}{n(q-1)} r^{q-n+1} - \frac{q-n+1}{M n (q-1)} r^{q-n} = \frac{n-1}{M n b (q-1)} \frac{p_b}{p_1} \quad (B)$$

The ratio $\frac{p_b}{p_1}$ is the quotient of the total back pressure by initial pressure, all useless resistance being included in p_b .

When instead of M , N is inserted in the equation, N being the quotient of *all* annual expenses, independent of fuel supply, by all such expenses variable with fuel and steam supply — the expression determines the ratio of expansion at “maximum efficiency of capital invested in a *given plant*.”

When M or N is made zero, the expression reduces to :

$$\left(r^{q-1} - \frac{q+1-n}{q n} r^{q-n} \right) \div \frac{n-1}{b q n} = \frac{p_b}{p_1} \quad (C)$$

and gives the ratio of expansion at which maximum “duty” or “efficiency of engine” is attained.

When $b = 1$ and $q = 0$, the case becomes that of the ideal engine working the fluid in a non-conducting cylinder, and these equations become, for efficiency of capital and of engine,

$$r^{-n} - \left\{ \begin{array}{c} M \\ \text{or} \\ N \end{array} \right\} \frac{n - n r^{1-n}}{n - 1} = \frac{p_b}{p_1} \quad (D)$$

$$r^{-n} = \frac{p_b}{p_1} \quad (E)$$

Studying these equations, it will be seen that, in all except the last (E), it is possible to find finite values of r such that their left-hand members shall reduce to zero; as in them n is nearly always equal to unity; q varies from $q = 0$ to $q = 0.3$ in good practice, and b usually ranges between $b = 0.8$ and $b = 0.9$; M or N is usually between 0.02 and 0.15, and the form of the function is such that the first member may always be made to disappear for some finite value of r . Then we shall have in (B)

$$r^q + \frac{q r^{q-1}}{M(q-1)} - \frac{(q-n)r^{q-n+1}}{n(q-1)} - \frac{q-n+1}{M n(q-1)} = 0; \quad (F)$$

and in (C)

$$r^{-n} - M \frac{n - n r^{1-n}}{n - 1} = 0 \quad (G)$$

$\frac{p_b}{p_1} = 0$; $\frac{p_1}{p_b} = \infty$ and the value of r , at which this condition is obtained, constitutes an “absolute limit,” for the care taken, beyond which expansion cannot be carried economically, even with steam increased to

infinite tension ; beyond this point $\frac{p_b}{p_1}$ becomes negative, indicating the assumption of impossible conditions.

This is most plainly exhibited by equation (D). When the value of r is unity the second term disappears, as r increases ; the magnitude of the whole term increases ($n > 1$), and passing through the value obtained from (F), the sign of the first member changes at some finite value of r , indicating, as above stated, the introduction of impossible conditions, since the sign of the second member must always be positive for any actual engine.

Examining equation (E), we find no such limit ; the sign of the first member remains positive for all values of r , and can never become zero for a finite value of that quantity. We are thus taught that an important difference exists between the *ideal* engine, with its non-conducting cylinder, and the *real* engine working steam in a metallic cylinder, as well as between the case of maximum efficiency of engine and that of maximum efficiency of capital. In the cases of maximum efficiency of fluid and of engine for the ideal perfect engine, equation (E) only, is it true that indefinite increase of steam pressure permits indefinitely increased expansion. In all other cases an absolute limit exists, fixed for each case, beyond which expansion cannot be economically carried.

The above equations for real engines are approximate, and are practically exact—the values of the constants being determined with accuracy—within the range met with in practice, and the conclusions here deduced are, therefore, correct, although the exact values of the constants, or even the precise form of the variable function, r^a , here may not be fully ascertained.

Should it seem desirable, it is easy to deduce the same conclusions—and thus to confirm, by independent proof, the above proposition—by the examination of the “Curves of Efficiency” of ideal and of real engines. Thus :

Let the curve ABO be the “Curve of Efficiency” for the ideal perfect engine ; let the curves CH , DG , EF be those obtained from real engines as indicated by the writers in earlier papers. Then if the ratio

$\frac{p_b}{p_1} = 0$ and $\frac{p_1}{p_b} = \infty$ the back pressure line coincides with base-line

* With such curves the ordinates measure the work done at various ratios of expansion in any given engine, by quantities of steam proportional to the abscissas.

passing through OX . Using this diagram to determine the ratio of expansion at maximum efficiency of engine, we draw the tangents OP , OK , OL , OM , to the several curves, and thus determine points of tangency— N , M , to the several curves for real engines, which, under the assumed conditions, correspond to the best ratios of expansion, the ratio of work done to cost of doing it being then a maximum.

The tangent point for the *ideal* engine, of which AB is the curve, falls at O , the origin.

Thus it is seen that, while the ratio has no limit for the ideal case, it has such a limit for the real engine, and that this limit may be found at a low ratio of expansion. The writer has made this comparison for the steamers "Michigan," "Georgiana," and "Bache," for which three cases the real curves have been obtained by him,* and finds that, these curves remaining unchanged,† it is impossible economically to increase the ratio of expansion in such engines beyond three, five, and ten respectively, even with unlimited steam pressure; *i. e.*, even when

$$\frac{p_2}{p_1} = 0.†$$

Such a limit must evidently exist in every steam engine, since the real curve of efficiency must fall within the ideal curve, AB , and can never pass without inflection through the origin O . The proposed proof is therefore complete, both as a matter of theory and as the fruit of direct experiment. The fact proven is due to the invariable increase of cylinder condensation with increasing expansion.

It is easily seen that a similar limit exists at still lower values for the ratios of expansion at maximum commercial efficiency, and that this holds true in all engines, the ideal case, AB , included; for, in the figure take the distance OO' proportional to annual "cost of engine," on the scale on which OX measures the "annual cost of steam" used in the same engine without expansion, and including in these items

* *Vide Transactions of the American Society of Mechanical Engineers*, 1882; *JOURNAL OF THE FRANKLIN INSTITUTE*, June, 1882.

† The form and location of these curves at very high pressures would undoubtedly be somewhat altered, but it is evident that there must still be found this absolute limit so long as condensation increases with extended expansion. The writer is inclined to believe that the form of the curves will be substantially as shown, however, at any attainable pressure.

‡ The "Michigan" has an unjacketed condensing engine, using saturated steam the "Georgiana" a similar class of engine, with superheated steam; the "Bache's"; engine is jacketed and compound.

all quantities variable proportionally with each explained by the writer in earlier papers. Then tangents drawn from O^1 to the several curves as O^1P^1 , O^1K^1 , O^1N^1 , and O^1M^1 , will determine points of tangency P^1K^1 , N^1M^1 , which correspond with such expenditures of steam and such ratios of expansion as will give maximum economy of money and minimum annual expense. It is seen by inspection of the diagram that for every case a finite value of the ratio may be obtained even at the

limit at which the value of $\frac{P_1}{P_b} = \infty$ and that with any engine, however perfect, there exist values of the ratios of expansion at maximum efficiency of capital, beyond which it will not pay to carry the point of cut-off, however great the steam pressure; an infinite pressure permits only a finite expansion.

We may therefore conclude:

(1) That in all engines there exists an "absolute limit to the economical expansion of steam," whether considered with reference to efficiency of fluid, of engine, or of capital, which limit can not be passed, whatever pressure of steam may be carried up to the point of cut-off.

(2) That this limit is found at higher ratios of expansion as the type of engine is more efficient, but that the limit is indefinitely removed only in the ideal engine, and then only as affecting the ratios of expansion at maximum efficiency of fluid and engine.

(3) That this limit is found at a small value of the rates of expansion in ordinary engines, and therefore may be readily passed in everyday practice.

The limit is not far from three in the common unjacketed condensing engine, four or five in the same engine using superheated steam, and ten or twelve in the ordinary compound engine.

(4) It is evident that the general propositions of this paper are true of all heat engines having fluid working substances, whether vapors or gases worked in metallic cylinders.

Hoboken, N. J., March, 1882.