ART. XLII.—*On the Strain-effect of Sudden Cooling exhibited by Glass and by Steel*,* by C. BARUS and V. STROUHAL.

Introductory.—In the foregoing number of the Journal we communicated a series of results on the structure of steel of a given kind, tempered hard. They showed, in general, that from the circumference to the axis of a quenched non-fileable steel cylinder, hardness continually diminishes; that the first fileable strata are encountered at a distance of about 1^{cm} below the surface. Moreover, as hardness decreases, the density of the elementary conaxial cylindrical shells increases and in proportion as the layers become more and nearly or easily fileable the density is found to approach the density of soft steel. From an examination of rods of different diameters (2^{cm} to 3^{cm}) it appears, steel of a given kind presupposed, that the hardness at a point is essentially dependent on the distance of the point below the surface. The rate at which sudden cooling takes place must be similarly conditioned. Hence it is permissible to associate the one phenomenon with the other and to state that the hardness in a given point below the surface is dependent on the rate at which cooling there takes place. This point of view is suggestive: structure investigations may be made to furnish us the best means we know for the comparison of hardness and rate of cooling.

The results cited apparently make sad havoc with physical theories of temper. They seem indeed to be fatally at variance with the usually accepted views, viz: that in hard tempered steel an abnormally dense shell is arched around an abnormally rare core, the two states of strain mutually conditioning each other. The data may even be looked upon as furnishing evidence sufficient to prove the total absence of strain. It is the object of the present paper to investigate in what measure this evidence is critically sufficient; if it be insufficient, to state the nature and relations of the strain-effect of quenching somewhat more clearly than we were able to do in our earlier work.

In our little book on the iron-carburets† we endeavored to contrast the respective merits of chemical and physical theories of temper, using for this purpose all the data known to us, as well as special results of our own. This deduction seemed warranted, that hardness and strain are distinct and separate properties of a tempered iron-carburet;‡ that they need not necessarily coexist. Leaving therefore hardness pure to be ex-

* Communicated with the permission of the Director of the Geological Survey.

† Bull. U. S. Geolog. Survey, No. 14, 1885, p. 76.

‡ Ibid., p. 103, 197.

plained chemically, we inferred that the category of electrical and magnetic phenomena exhibited by steel on passing from hard to soft are mainly referable to changes in the character and intensity of the strain which the tempered rod carries. We accepted the theory of a dense shell and rare core as being the most satisfactory and elegant conception of the strain in question,—with the proviso* that “it must be regarded as a mere *diagram* of the essential features of the vastly more complicated structure of the glass-hard rod.” These conditions premised we finally interpreted all variation of strain produced by annealing as the combined effect changes of the viscosity of steel due to temperature and of interference of thermal expansion with the said strain.

The experimental question which we are endeavoring to elucidate may therefore be succinctly put as follows: 1. With what degree of sensitiveness do the variations of the density of the rod as a whole, indicate the corresponding variations of strain. 2. Is it possible successively to remove layer after layer of a rod without materially changing the character and intensity of the strain which the rod is supposed to carry. 3. In how far does the actual structure of tempered steel differ from the diagrammatic distribution of density above assumed. 4. Does the process of sudden cooling impart strain alike in kind† but differing enormously in degree to all substances. In our paper on structure we had the second and third of these points principally in mind. We were unable to obtain direct and decisive evidence of the occurrence of shrinkage during the removal of shells from a hard steel rod. But since the structure of steel differs so largely with the kind of steel operated upon we do not regard our experiments as made in sufficient number to be conclusive. An examination of the density of the elementary shells shows that the character of the density at any point regarded as a function of the distance of the point below the surface, is harmonic.‡ To investigate this relation it is unfortunately necessary to make the measurements for thin ($\frac{1}{2}$ mm) shells. Hence the mean amplitude of vibration and the limits of the unavoidable errors of observation are of the same order. To discriminate between the periodic effect of temper, which may show regular or irregular periodicity, and the apparently periodic distribution of mere errors of observation is difficult and requires very nice and careful observation. In the Bulletin (35) cited we attempt to arrive at this discrimination by making minute comparisons of the structures of rods of the same kind of steel, and of the same or of different diameters. This pro-

* Bull. U. S. Geol. Survey, No. 14, p. 96; *ibid.*, pp. 95 to 98.

† We do not necessarily refer to mere volume expansion here. See Wrightson, *Journ. Iron and Steel Inst.*, ii, 1879, p. 418.

‡ This volume. p. 386.

cedure is excessively laborious, for it calls for a detailed construction of the relation of the density and the position of the points along any given radius of the rod and subsequent examination of the properties which the divers diagrams have in common. We have gathered many data, but the work is as yet incomplete, and the reader desiring further information will have to be referred to the Bulletin, where the whole subject is discussed. Experiment on the fourth of the above topics we have now in progress and shall publish later. This narrows the purposes of this paper to the consideration of the first and most important of the above points, viz: in what degree the density of a hard steel rod at different points, even when measured with all attainable accuracy, is sufficiently sensitive to represent the character, relations and intensity of the temper-strain.

DENSITY AND RESISTANCE (STRAIN) OF TEMPERED STEEL.

Experimental results.—The discussion may be appropriately commenced by an examination of the variation of density of steel successively annealed from hard to soft. Our experiments on this subject, made in some number, are detailed in Bulletin No. 27, now in press, and for this we select the following digest. The table (I) contains data for length, diameter and temper; temper being expressed by the temperature and time at which the divers glass-hard rods were annealed. The table furthermore contains the specific resistance, (s_0), at 0°C. , in microhms, referred to the centimeter cube; the density (Δ_0) and the volume ($\mathcal{V}_0$) per unit of mass (g) for the same temperature, 0° . In the cases of rods Nos. I and II, density was measured in the ordinary way. The other values of Δ_0 are derived from very painstaking pycnometric measurements, all of which were made at least in duplicate and are warranted to two or three units of the third decimal. Values for resistance which could not be obtained by direct measurement were supplied by interpolation from our earlier researches. All such data are put in parentheses. Parentheses also enclose certain values for the temperature of the annealing furnace to show that they are derived by interpolation from the known power of the furnace under given circumstances. The other temperatures are measured in a way to be indicated elsewhere. The greatest care was taken to so anneal the rods as to *exclude* all errors due to superficial *oxidation or carburization*.

TABLE I.—*Electrical resistance, density and specific volume of steel, when varying with temper.*

No.	Length.	Diameter.	Temper.	s_0 .	Δ_0 .	∇_0 .
	cm.	cm		Microhm		
I	5.3	1.9	Commercial, soft	(15)	7.8372	0.12760
			Glasshard	(44)	7.8092	0.12805
			Annealed, 100°, 1 ^h	(39)	7.8088	0.12806
			" 100°, 4 ^h	(37)	7.8094	0.12805
			" 100°, 8 ^h	(36)	7.8108	0.12803
			" 100°, 12 ^h	(36)	7.8114	0.12802
			" 200°, 30 ^m	(28)	7.8096	0.12805
			" 330°, 5 ^m	(21)	7.8236	0.12782
			" 620°, 5 ^m	(18)	7.8342	0.12765
			" 805°, 30 ^m	(17)	7.8361	0.12762
			" (1300°)	(19)	7.8465	0.12744
II	10.11	0.576	Commercial, soft	(15)	7.7033	0.12981
			Glasshard	(44)	7.6200	0.13123
			Annealed, 190°, 4 ^m	(32)	7.6542	0.13065
			" 330°, 2 ^m	(21)	7.6661	0.13044
			" 645°	(18)	7.7008	0.12986
			" 830°	(18)	7.7170	0.12958
55 to 60 61 to 63	21 pieces, each about 2½ cm long.	0.127	Commercial, soft	15.25	7.7337	0.12931
			Glasshard	43.29	7.6688	0.13040
		0.127	Annealed, 100°, 9 ^h	35.61	7.6976	0.12991
			" 200°, 2 ^m	33.11	7.6981	0.12990
			" 200°, 30 ^m	28.76	7.7030	0.12982
			" 330°, 30 ^m	20.61	7.7215	0.12951
			" 810°, 30 ^m	18.10	7.7872	0.12842
			" (1000°)	19.74	7.7865	0.12843
			" (1300°)	19.29	7.8007	0.12819
O	18* pieces, each about 2½ cm long.	0.226	Commercial, soft	(15)	7.81	0.1280
			Glasshard	(44)	7.6817	0.13018
			Annealed, 100°, 10 ^m	(42)	7.6943	0.12997
			" 100°, 7 ^h	(37)	7.7102	0.12970
			" 190°, 15 ^m	(31)	7.7251	0.12945
			" 330°, 8 ^m	(21)	7.7441	0.12913
			" 460°, 1 ^h	18.27	7.7641	0.12880
			" 720°	17.13	7.7793	0.12854
			" (900°)	18.10	7.7840	0.12846
			" (1100°)	18.60	7.7865	0.12843
			" (1300°)	18.30	7.7856	0.12801
30 to 36	72 pieces about 2½ cm each.	0.081	Commercial, soft	16.27	7.7268	0.12942
			Glasshard	43.86	7.6547	0.13064
		0.081	Annealed, 100°, 1 ^h	39.26	7.6666	0.13044
			" 100°, 13 ^h	35.51	7.6745	0.13030
			" 190°, 4 ^h	30.82	7.6841	0.13014
			" 190°, 4 ^h 30 ^m	27.10	7.6808	0.13020
			" 190°, 4 ^h 30 ^m	27.10	7.6848	0.13030
			" 330°, 10 ^m	20.68	7.6806	0.13020
	71 pieces*		" 470°	18.36	7.7190	0.12955
			" 530°	18.17	7.7227	0.12949
			" 690°	17.24	7.7272	0.12941
			" 810°	17.56	7.7586	0.12889
			" 1000°	18.59	7.7705	0.12869
			" †1100°	----	----	----

* One piece lost.

† Fusion of copper envelope (capsule) destroys the wire.

Discussion.—It is not our object to enter into a full discussion of these results here. We merely advert to the fact that whereas density increases continually on passing from hard to soft, resistance passes through a pronounced minimum.

Whether we regard Δ , as a function of s , or of the temperature t° at which the rod was annealed, the relations contained in the table lead to an important inference, with an immediate bearing on our present purposes. It appears plainly that *the annealing of a rod successively from extreme hard to extreme soft does not exhibit the characteristics of a single and homogeneous phenomenon.* This annealing presents two distinct and independent phases which merge into each other when t° corresponds to incipient redness. A graphic representation of Δ , varying with t° shows this transition with singular clearness. Between $t=200^\circ$ and $t=500^\circ$, all the loci pass through two consecutive circumflexures in such a way as to change the general and pretty uniform contours of concavity downwards, into convexity downwards for the interval in question. The division of the complete phenomenon into two parts being thus suggested, we find on further inspection that the variations of the electrical constants of temper subside almost completely within the first of these parts (annealing between 0° and 350°); whereas in the second part (annealing between 350° and 1000°) they became crowded and complex. It follows plausibly and at once that during the first of the phases under consideration we encounter the subsidence of a mechanical strain. This inference gains much in weight when we find that this strain vanishes in the way peculiar to phenomena of viscosity, at a slowly decreasing rate, through infinite time.* In the second phase, mechanical and chemical occurrences are superimposed and the above results do not enable us to disentangle them.

The final important deduction from the data is this, that during the first phase in which the larger values of resistance are from two to three times the smaller values, the variation of density is slight. In the second phase we encounter large variations of density associated with small variations of resistance.

These curious relations characterize the phenomenon as a whole. We shall interpret them partially at least below.

Density and Strains in Tempered Glass.

Experimental results.—This result substantiates the evidence in favor of the existence of strain in hard steel, adduced in our earlier papers.† Beyond this it shows almost conclusively

* In other words the temper strain vanishes just like the simple "drawn" strain imparted by the wire plate, while the wires carrying them are each exposed to the prolonged action of temperature.—Bull. 14, p. 93.

† Bull. 14, pp. 88 to 103.

that the density-effect of the annealing of hard steel is out of all proportion and symmetry with the strain effect and the corresponding electrical effect. We were desirous therefore of studying the temper-strain in substances other than steel and free from carbon. Long ago, glass had suggested itself to us for this purpose.* We communicate in the following Table II the density effect produced by successive annealing of ordinary Prince Rupert drops. Twelve of these were in hand. Three were broken to test the strain, the remainder examined. All the drops contained included bubbles, one or more, often 0.2^{cm} to 0.3^{cm} in diameter and distributed irregularly. Usually a few large bubbles predominated. The drops moreover showed a purple coloration, which disappeared after annealing at incipient redness.† To anneal the drops at different temperatures we enclosed them in a test-tube, cushioned upon and surrounded by carded asbestos. The end of the tube was submerged sufficiently to cover the P. R. drops in boiling camphor, mercury, sulphur, etc., as given in the table. To anneal at red heat, the drops were suspended in little baskets of platinum foil, in the center of large thick clay crucible and then heated to the desired temperature in an assay furnace. Under all circumstances but one, care was taken to cool the drops slowly. Table II contains under "temper" the temperature and time of annealing, not including the time of cooling, however. The column *m* gives the weight of each drop in grams. Variations of *m* are due to accidental breakage of the frail stems of the drops. Where no such breakage occurs *m* is constant. Δ_0 , finally, is the density of the drops at 0° C., under the conditions given. Each Δ_0 is the mean of two independent determinations and is warranted correct to within one or two units of the third place.

Table III following is constructed to facilitate a comparison of the important data. The second and third columns contain the densities of the extreme states of temper, the original quenched, "hard," and the final "soft" or thoroughly annealed state. The remaining columns contain the increments of density due to quenching, relative to the density of the soft state, the relative increments in other words which are retained at the divers temperatures given.

* Wied. Ann., vii, p. 406, 1879.

† The color of the drops is amethystine; the powdered glass pink.

TABLE II.—*Temper, weight and density of quenched glass (Prince Rupert's drops), successively annealed.*

No.	TEMPER.	<i>m.</i>	Δ_0	REMARKS.
1	Quenched	$\frac{g}{l}$ 1.3500	2.4345	Bubbles. Color.
	Annealed, 360°, 30 ^m	1.3500	2.4401	Do.
	(Mercury.)			
	Annealed (650°), 10 ^m	1.3497	2.4901	Bubbles and color vanish. Form of drop not changed.
				Cooled slowly.
	Annealed, red heat	1.3470	2.4910	Cooled slowly.
	Annealed, red heat	1.3311	2.4883	Cooled <i>in air.</i> from intense redness.
	Annealed, red heat	1.3312	2.4915	Cooled slowly.
2	Quenched	1.3200	2.4364	Bubbles. Color.
	Annealed, 360°, 30 ^m	1.3200	2.4408	Do.
	(Mercury.)			
	Annealed (650°), 10 ^m	1.3191	2.4903	Bubbles and color vanish. Form of drop not changed.
				Cooled slowly.
	Annealed, red heat	1.3172	2.4912	Cooled slowly.
3	Quenched	1.4590	2.4374	Bubbles. Color.
	Annealed, 200°, 15 ^m	1.4591	2.4364	Do.
	(Paraffine.)			
	Annealed, 360°, 30 ^m	1.4590	2.4411	Do.
4	Quenched	1.3305	2.4356	Bubbles. Color.
	Annealed, 200°, 15 ^m	1.3305	2.4351	Do.
	(Paraffine.)			
	Annealed, 360°, 30 ^m	1.3292	2.4383	Do.
5	Quenched	1.3009	2.4349	Bubbles. Color.
	Annealed, * 200° 1 ^h 30 ^m	1.3009	2.4359	Do.
	(Camphor.)			
	Annealed, 360°, 1 ^h	1.3010	2.4402	Do.
	(Mercury.)			
	Annealed, 450°, 30 ^m	1.3012	2.4446	Do.
	(Sulphur.)			
	Annealed (550°), 10 ^m	1.2983	2.4531	Color vanishes. Cooled slowly.
	Annealed, red heat	1.2982	2.4914	Bubbles gone. Form partially changed.
6	Quenched	1.4409	2.4323	Bubbles. Color.
	Annealed, * 200°, 1 ^h 30 ^m	1.4409	2.4333	Do.
	(Camphor.)			
	Annealed, 360°, 1 ^h	1.4409	2.4382	Do.
	(Mercury.)			
	Annealed, 450°, 30 ^m	1.4412	2.4422	Do.
	(Sulphur.)			
	Annealed (550°), 10 ^m	1.4410	2.4534	Color vanishes. Cooled slowly.
	Annealed, red heat	1.4408	2.4893	Bubbles gone. Form partially changed. Slow cooling.
7	Quenched	1.4795	2.4395	Bubbles. Color.

* An explosion of the camphor vapor may have slightly raised these values, but we believe them to be correct.

No.	TEMPER.	<i>m.</i>	Δ_0	REMARKS.
8	Quenched	1·0025	2·4395	Bubbles. Color.
	Annealed, 360°, 30 ^m	1·0025	2·4444	Do.
	(Mercury.)			
	Annealed, 360°, 3 ^h	1·0023	2·4482	Do.
	(Mercury.)			
	Annealed, 450°, 1 ^h	1·0023	2·4526	Do.
	(Sulphur.)			
9	Annealed, 450°, 4 ^h	1·0023	2·4586	Do.
	(Sulphur.)			
	Annealed, red heat	0·9982	2·4901	Bubbles and color gone. Form partially changed. Cooled slowly.
	Quenched	1·2759	2·4365	Bubbles. Color.
	Annealed, 360°, 30 ^m	1·2745	2·4423	Do.
	(Mercury.)			
	Annealed, 360°, 3 ^h	1·2745	2·4436	Do.
	Annealed, 450°, 1 ^h	1·2745	2·4472	Do.
	Annealed, 450°, 4 ^h	1·2727	2·4510	Do.
	Annealed, red heat	1·1592	2·4929	Bubbles and color gone. Form partially changed. Cooled slowly.

TABLE III.—*Density of Prince Rupert's Drops, Quenched and Annealed. Digest.*

No.	Hard. Δ_0 .	"Soft." Δ_0 .	Density-increment relative to "soft," when annealed at						
			20°	200°	360°	450°	(550°)	(650°)	Red heat.
1	2·4345	2·4915	—0·0	—0·0	—0·0	—0·0	—0·0	—0·0	—0·0002
2	2·4364	2·4912	229	----	207	----	----	006	—0·0000
3	2·4374	-----	216	220	201	----	----	----	-----
4	2·4356	-----	223	225	212	----	----	----	-----
5	2·4349	2·4914	227	223	206	188	154	----	—0·0001
6	2·4323	2·4893	229	225	205	189	144	----	+0·0007
7	2·4395	-----	207	----	----	----	----	----	-----
8	2·4395	2·4901	203	----	184	150	----	----	+0·0004
9	2·4365	2·4929	227	----	168	127	----	----	—0·0007
					203	184			
					198	168			

Discussion.—The salient features of these data are obvious from an inspection of Table III. The density-effect of quenching is decidedly *negative*; the increase of specific volume is exceptionally large. Moreover, the nine Prince Rupert drops examined exhibit nearly the same initial density, and nearly the same final density. The approximate equality of the values Δ_0 for the soft state is easily explained. They indicate that in proportion as we bring the strain to vanish, we reach the normal density of the glass. Not so easy is it to account for the observation, that the density of the Prince Rupert drops shows almost the same degree of equality in the hard (quenched) state. Indeed, if we take the fact into consideration that the drops invariably contain *bubbles* distributed irregularly and without apparent relation as regards size and number, the difficulties of explanation are increased. In other words the remarkable result that the volume-increase due to quenching is quite as much the same for all drops as is at all possible for the case of so complicated an operation, is an exceedingly striking and important result and leads to this inference: Inasmuch as bubbles are present in like total volume in each of the drops, their presence can not be a circumstance of mere accident; they must be regarded as a normal effect or, better, a necessary result of the operation of quenching applied to glass; they must in some very intimate way be connected with the strain which the glass globules have experienced in virtue of sudden cooling.

Retaining this very plausible surmise in mind, we proceed to a more minute inspection of the effect of annealing the hard Prince Rupert drops. We obtain results very similar to those investigated above for steel. The density effect of annealing is decidedly positive, and is greater as temperature and time of exposure are increased. Again, we readily divide the physical effect of annealing into two parts or phases. The first of these corresponds to the annealing temperatures 0° to 500° , the other to higher temperatures. The range of temperatures corresponding to the first phase is therefore larger for glass than for steel; and if we compare Tables I and II it appears obviously, that for like density effects the annealing temperatures must be chosen higher in the former case (glass) than in the latter. But the change of density encountered in both instances is small; and yet in spite of the small density-effect during the inferior stage of annealing, by far the greater intensity of strain vanishes. The drops after annealing at 200° are still explosive; if they be broken after having been annealed in boiling sulphur (450°) they are found to have lost all traces of the explosive properties which the originally quenched drops possessed.

Professor Rood* informed us that the polarization figures could be quite wiped out by annealing such ordinary glass as exhibited them only as far as the temperature of melting zinc. We infer that at the end of the first phase of annealing we have in hand a hollow glass globule practically free from strain.

During the second phase of the annealing phenomenon (500° to 1000°), we observe a very pronounced change of the density of the Prince Rupert drops, corresponding to the above result for steel. But the explanation is here readily at hand: at incipient redness the enclosed bubbles *disappear* or are reduced to mere specks. The large increment of density in question is therefore nothing more than an expression for the collapse of the viscous hollow globule in virtue of atmospheric pressure. This important observation enables us to interpret all the phenomena of annealing satisfactorily. It must therefore be carefully examined.

We will endeavor to prove that the bubbles are vacua; that they are not accidental inclusions of gas or aqueous vapor. The temperature from which glass is quenched is certainly less than 1500° . The temperature at which glass is sufficiently viscous to yield easily to atmospheric pressure is certainly greater than 500° . Suppose now that the changes of volume of the bubble were the result of thermal expansion of an included gas. Let v_{1500} and v_{500} be the volumes of the gaseous inclusion at 1500° and 500° , respectively, under normal pressure. Let v_h and v_s be the volumes of the (gas) bubble for the quenched-hard and annealed-soft states. Then we deduce, *a fortiori*,

$$\frac{v_h}{v_s} < \frac{v_{1500}}{v_{500}} = 2.3 \quad (1)$$

with this as a point of departure the following little digest, Table IV, of mean results has been prepared. Here m is the mean mass of all the drops examined; $\mathcal{A}_h$, $\mathcal{A}_s$, V_h , V_s , their mean density and total volume for the hard and the soft states, respectively; v is the mean volume, r the equivalent mean radius of the bubbles. In the second horizontal row V_h has been diminished one-half per cent to refer all volumes to the beginning and end of the second phase of annealing.

* Results obtained by Professor Rood during his experiments with high vacua.

TABLE IV.—Mean volume relations of the Prince Rupert drops and of the hypothetical gas-bubble.

V_h	V_s	v at		r at		
		1500°	500°	1500°	500°	
cc.	cc.	cc.	cc.	c.	c.	
0.5454	0.5334	0.0120	0.0052	0.142	0.107	$\frac{v_h}{v_s} < 2.3$
0.5427	0.5334	0.0093	0.0041	0.130	0.099	$\frac{r_h}{r_s} < 1.3$

Mean $m = 1.3288$ g"Hard:" Mean $\Delta_h = 2.4363 \pm 0.0008$ "Soft:" Mean $\Delta_s = 2.4911 \pm 0.0016$

Inasmuch as the values v_{500} and r_{500} are inferior limits, it is clear that if the bubbles were gas or vapor, they would be of the same order of visibility before and after annealing. In other words, if the bubbles were gas, casual inspection would not detect a difference of their size in the hard-quenched and in the "soft" drops. In the experiments, however, the bubbles all but vanish. Hence they can not be gas.

To obtain additional assurance on this point we ground flat faces on the annealed drops, Nos. 1, 5 and 8, and then measured the size of the inclusions. We found these values:

No. 1: $v_s = 0.000037$ c. c.No. 5: $v_s = 0.000062$ c. c.No. 8: $v_s = 0.000040$ c. c.Mean: $v_s = 0.000046$ c. c.

From actual measurement therefore we find,

$$\frac{v_h}{v_s} > 100 \quad (2)$$

a result wholly incompatible with $\frac{v_h}{v_s} < 2.4$ as derived above (Table IV). It follows conclusively that the bubbles are not accidental gas inclusions; that only an insignificant part of the variations of volume produced by quenching can be referred to thermal expansion of gas.

If the gas were aqueous vapor, and if dissociation were complete, then the upper limit of (1) would have to be increased in the ratio of 3:2. This does not remove the discrepancy between (1) and (2). Moreover, since dissociation of water is only incipient at 1500° and is not even complete at the temperature of melting platinum, the volume variation of the bubbles can not be due to the presence of water in the bubbles.

If on the other hand we endeavor to explain the bubbles as vacuities* left by the contracting glass, we arrive at accordant and plausible results. Let t be the temperature to which the

* Very analogous to Torricellian vacua.

homogeneous glass drop must be heated in order that the observed mean volume of the soft state may be equal to the observed mean volume of the hard state. Then

$$t < \frac{1}{3\alpha} \frac{\Delta_0 - \Delta_i}{\Delta_i} \quad (3)$$

where α is the ordinary coefficient of linear expansion of glass. By inserting the values of Table III into (3) we derive

$$t < 900^\circ.$$

In other words the quenched globule has retained the volume which the hot glass possessed at a temperature certainly smaller than 900° . This result taken together with the other is conclusive.

The general result of our investigation on the process of sudden cooling, therefore, points out the fundamental importance of a rigid shell. As the material (glass or steel) cools, contraction as a whole takes place not centripetally but centrifugally, i. e., not towards the center of figure but away from it. So long as the interior remains liquid or viscous, the result is simple separation, commencing at points where minute air bubbles may preëxist,* rather than at points of the continuity of glass. The final result is a vacuum bubble such as we have observed it. In proportion as the interior becomes rigid, the temper strain appears. Should the glass hold gases in solution, they would tend to escape into the vacua in question. There is probably another reason why in most instances the bubbles cannot be brought completely to vanish.

Annealing reverses the whole phenomenon. The small changes of density observed in the first phase may be easily accounted for; it is probable that during these stages of annealing the re-arrangement of molecules and disappearance of strain is accompanied by expansion inward, toward the vacua. Hence the incommensurately small variation of mean density which accompanies the great optical effects (glass), and the great electrical effects (steel).† During the second stage of annealing, again, the density-effect is incommensurately large; for glass it is the expression of a mere collapse due to atmospheric pressure. At all events the density-effect (error) due to bubbles quite swallows up the density-effect due to strain. Hence to represent the true relation of density and resistance or of density and annealing temperature in case of steel, it may be necessary to lower the part of the curves corresponding to the second place of annealing by amounts equivalent to the bubble

* P. R. drop No. 1, heated intensely in a blast lamp and cooled in air shows a decrement of density. This is due to the rapid cooling. For when this drop is heated and cooled in the crucible, density is again incremented. The experiment shows the tendency of glass to retain strains. During the heating to 1000° no expansion of the very small bubbles was observable.

† The electrical and optical criteria may be considered equally sensitive.

or fissure discrepancy, or else for like reasons to raise the parts of the curves corresponding to the first phase. If this be done the circumflexures become less pronounced or disappear, and the points for the commercial soft state are more easily referred to the curves to which they belong. If, therefore, allowance be made for the distortion due to internal sensible pores, the relations become more uniform.

It is interesting to observe that glass retains the volume-expansion corresponding to a temperature somewhat below 900° ; whereas steel retains the volume-expansion of a temperature below 400° . Similarly the strain in steel is very perceptibly affected by annealing temperatures as low as 50° , whereas for glass the perceptible annealing effects are only incipient even at 200° . The amount of strain retained is, *cæt. par.*, not merely a function of the viscosity of the material subjected to quenching. It must depend also on the heat conductivity of this substance. For instance, if like figures of glass and of steel be quenched alike, then at the same time and depth the thermal gradient would be much steeper in the case of glass than in the case of steel. Hence, *cæt. par.*, during quenching a rigid shell is possible in the case of glass for higher temperatures of the core than in the case of steel. The sudden contraction of the shell pressure has a marked effect on the melting point or degree of fluidity of the core. But it is best to waive this observation here, for the want of data to interpret it.

We have finally to consider the bearing of the results of this paper on the structure of steel. The detailed similarity observed in the annealing of glass and of steel suggest the inference that the interior of hard steel may be sensibly fissured. Under all circumstances the diagrammatic structure of dense shell and rare core would be inaccurate to the extent in which these fissures are irregularly distributed. Hence the difficulty of developing the true character of the dependence of the density (δ) at any point upon the distance of this point below the surface. We are able to account in part for the quasi-harmonic relations obtained both by Dr. Fromme* and by ourselves.† While the consecutive shells are being removed by solution, periodic fluctuation of δ must result whenever fissures are invaded. If, furthermore, we take into consideration that the density effect of even great intensities of temper-strain is small, it appears that the true nature of the strained structure of tempered steel may be beyond the discernment of the density-method of investigation, altogether. The *gross* variation of density along the radius is a carburization phenomenon.

Retrospective.

In our earlier work on steel we adverted to the singular co-

* Wied. Ann., viii, p. 356, 1879.

† Bull. 35. This volume, p. 386.

incidence of the resistance increments corresponding to like volume increments, no matter whether the latter be the result of temper or of temperature. The light which the new data throw upon the internal condition of tempered steel—the tendency to a fissured structure—shows, however, that this material is not as well adapted for the quantitative evaluation of the *true* relation between volume and resistance, as we supposed. So long, moreover, as the meaning of the minimum resistance and of certain carburization phenomena encountered during annealing at red heat remains obscure, it is safest to give the said relations no more than qualitative importance. The best means to the end in question is probably to be sought for in the compression of mercury.

With this unimportant exception, the present results have materially substantiated our earlier views at every essential point. Hence we infer as before that “the annealing of steel, considered physically, is at once referable to the category of viscous phenomena. In the ordinary cases of viscosity measurements, the phenomenon is evoked by sudden application of stress (torsion, flexure, tension, volume-compression, etc.) under conditions of constant viscosity; in the case of annealing by sudden decrease of viscosity under conditions of initially constant stress. Thermal expansion interferes with the purity of these phenomena by destroying the conditions of existence of the strain which accompanies hardness, and this in proportion as the expansion is greater.”* Again, irrespective of the manifestation of mere hardness, “the existence of the characteristic strain in glass-hard steel is the cause of electrical effects so enormous, that such additional effects which any change of carburization may involve may be disregarded and the electrical and magnetic results interpreted as due to variations in the intensity of the said strain.”†

This deduction applies of course to the first phase of the phenomena of annealing, since it is within these limits that the strain in question is brought to vanish. With these results in hand we may proceed justifiably toward a study of the question, whether the conditions for the permanent retention of magnetism in an iron-carburet are not the identical conditions for the permanent retention of any strain. If we select the temper-strain for comparison, we do it not merely because our experience has familiarized us with this strain, but because of the clear-cut beauty of its manifestations, and because of the simplicity and pronounced character of the functions which describe it.‡

Washington—Prague. April. 1886.

* Bulletin, U. S. Geolog. Survey, No. 14, p. 196.

† Ibid., p. 97.

‡ We desire to remark that the principal inferences of this paper have since been substantiated by polariscopic evidence, and by an investigation of the density of the consecutive shells of the P. R. drop. These results will be given in our next paper, nearly ready. May, 1886.