IV.—APPLIED AXIOMS.

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As an excuse for writing on so ancient a subject as the respect which is properly due to 'undeniable truths,' a modern instance will be useful as showing that in spite of the antiquity of the problem its solution is still a little obscure. Such an instance is provided by Dr. McTaggart's reply¹ to Mr. Schiller's attack² on a line of argument which at present enjoys considerable vogue among philosophers.

The 'undeniable truth' there criticised is the Law of Contradiction, and the point of the criticism consists in questioning not the undeniability of the law but the assumption that when used as a ground of argument its undeniability remains intact. Mr. Schiller's position—which in general is that of Pragmatism—involves a thorough-going distrust of the well-known argument that because the best account that can be given in words of processes like Change and Causation contradicts itself, therefore the processes in question are not real. Against this it is maintained that a contradiction is something that may always be due to the unavoidable defects of language, and of thought in so far as its structure is the same as language,—that is to say, in so far as thought depends upon antithesis; and that therefore the actuality of anything must be judged on other grounds than that of our power of explaining its nature in words; and that where we are compelled for all practical purposes to recognise that a thing or a process is actual, no verbal argument to show that it is impossible has any standing-ground.³

¹ *Studies in Hegelian Dialectic*, pp. 110-113.

² Republished in *Humanism*, essay.vi. See also *ibid.*, pp. 185-187.

³ To this it may be added that, when once it is admitted that the Law of Contradiction (used as major premiss) cannot be true unless Change is an illusion, it becomes mere question-begging to attempt to prove the illusoriness of Change by assuming the *necessary* truth of the Law of Contradiction in such use. A major premiss can only be called 'unquestionable' by courtesy or exaggeration. To avoid question-begging it must be put forward as open to question.

Dr. McTaggart's contention, on the other hand, is that the reasoning just quoted only holds good if we assume that sensation apart from thought can assure us of reality; and that since nothing can make us aware of reality without thought, any law that can be laid down for thought must be a law that imposes itself on all reality which we can either know or imagine. And, finally, that to predicate reality of a thing whose notion is self-contradictory is to put an end to all possibility of coherent thought anywhere; for if a contradiction is not taken to be a sign of error it will be impossible to make any inferences at all.

What is chiefly interesting in this answer is that it belongs to the time before the distinction between an undeniable law and an undeniable major premiss (or *use* of an undeniable law) had come into notice,—which distinction is involved in Pragmatism. From that former point of view it is natural enough to assume that the only way of objecting to an argument based on the Law of Contradiction is by totally denying the validity of thought and putting mere thoughtless sensation in its place. It is through that assumption that the matter presents itself to Dr. McTaggart as a perfectly simple dilemma. You must either, he thinks, accept or deny the Law of Contradiction; if you accept it, then you are free to use it as a major premiss wherever you please; and if you deny it, then you are talking nonsense. But as soon as we see that it is quite possible to refuse to accept a general rule when used as a major premiss, even though apart from such use we may find no fault with it, this dilemma no longer holds good. There are four alternatives theoretically possible, instead of only two. We may accept or deny the rule regarded as a major premiss, and we may accept or deny it in its other and more ethereal sense. Practically the last of these four alternatives—that of denying the abstract axiom—happens not to commend itself to anybody; we are all content, I believe, to agree that the abstract axiom is undeniable. But what the Pragmatist would maintain is that this admission says nothing at all about the axiom regarded as a major premiss. He contends that when it is so regarded—when it is taken as an 'applicable' law instead of an abstract one—its truth may be denied without talking nonsense; and that the undeniability of what Dr. McTaggart loosely calls "the Law of Contradiction" can only be supported by confusing the applicable law with the abstract one.

Really it would be a very simple matter to upset this position of ours,—if the facts would only allow of it. All that is necessary would be either to explain in what way the

applicable Law of Contradiction can be regarded as universally true, or else to give some reason for condemning as illusory our distinction between the abstract law and the applicable one. Meanwhile I will try to state some of the reasons supporting our side of the question. The fact that here we have a definite instance of a philosopher who rejects our point of view, without appearing to understand what it is, will serve as my excuse for saying some things that might otherwise seem rather too obvious.

First as to the exact point in dispute. What we object to is *not* the admission that a contradiction is something that should if possible be removed, or even something that until removed destroys the meaning of an assertion; but to the assumption that it is an infallible mark of error. A contradiction we regard as in itself a mere fault in expression,—a purely verbal affair; we admit that, like other such faults, it may be due sometimes to carelessness, or again to a piecemeal and therefore erroneous way of regarding a complicated set of facts; but we recognise that it may also be sometimes due to the unavoidable clumsiness of language and of antithetical thought. And, if it makes our position any clearer, we should be quite content to deny the practical importance, rather than the truth, of the rule that a contradiction is a sign of error. To deny the practical importance of a rule is the same as to deny the truth of that rule regarded as applicable—regarded as a major premiss and used in supporting conclusions—though it naturally says nothing about the truth of the rule outside such use. Whether indeed there can be anything worth calling truth in a general rule that admittedly cannot be used as a major premiss is one of the chief points at issue between the pragmatist and other people. And so our opponents do not meet our objection—they only ignore it and seek to beg the question at issue—when they talk about the undeniability of the abstract rule. We may freely admit its undeniability and yet draw a distinction between undeniability and truth, since a meaningless statement is just as undeniable¹ as a true one. We contend that the use of a general rule, and the power of testing its truth, both

¹ When this paper was read to the Oxford Philosophical Society the suggestion was made by one member that only true statements deserve to be called 'undeniable,' though meaningless ones may be admitted to be 'not deniable'. The difference escapes me altogether, and I do not think this use of the words has even an established convention behind it. If, however, this were so, the only effect of accepting the convention would be a verbal one. The pragmatist would still be able to agree with other people in *denying the deniability* of the abstract axiom.

depend on the same condition—that it shall be applicable in particular cases and so run a risk of being proved false.

As to our reason for denying that you can safely proceed to infer real error from verbal contradiction, it consists in the fact that any pair of statements which appear contradictory may represent assertions which are really compatible,—the reason for this being that all statements are elliptical and thus run a risk of being ambiguous. If we are right, therefore, the applicable Law of Contradiction can only be taken as universally true when we forget that any statement, however elaborate, may stand in need of further limiting. We should have to assume—exactly in the manner of the most thoughtless Formal Logic—that the risk of ambiguity in a predicate term is a risk that may be ignored. For in so far as the existence of an ambiguity in the statement 'A is B' can come as a surprise to a given person, that person at that stage of his acquaintance with the term B cannot safely apply the Law of Contradiction to the statement. Before the ambiguity is seen by him he is bound by the Law of Contradiction to suppose that because A is B the statement 'A is not B' must be false; and therefore the more confidently he applies the law, the more he shuts himself out from learning in what respects (or in what contexts) B is an ambiguous term.

It is possible to predict some of the rejoinders that are likely to be made. And what I am anxious to find, but have never yet found, is one that possesses any relevance. Since we all agree that the Law of Contradiction—that is to say the abstract law—is not deniable, and since our objection extends only to the supposed universal truth of the applicable law, many of the usual defences are quite beside the point. At the root of them, generally, I suppose, is the fear that any tampering with the law must result in anarchy or deadlock; and I think we are bound, on our part, to do what we can to allay that fear before we consider the more special points that are sometimes raised.

In an earlier passage of McTaggart's book (p. 9) where he defends Hegel against the charge of denying the Law of Contradiction, he has in view two kinds of such denial, one rather less extreme than the other. The extreme form he refers to is that of asserting that a contradiction, instead of being a mark of error, is a mark of truth. This we really need not here consider, since (whatever may have been at times supposed about Hegel) no one is likely to suspect the Pragmatist of wanting to defend the contrary of the Law of Contradiction; our ambition plainly does not extend to the

finding of any permanently unambiguous axiom. It is only the supposed universality of the law that is attacked ; and therefore we need consider only the milder of the two kinds of denial contemplated by McTaggart—that of proposing (as he puts it) ‘to do without the help of the law’. As to this he quite rightly says—what others have often said before him—that then we should be unable to refute any proposition. And we can admit this freely, for the simple reason that we do not propose to do without the help of the law. What we propose is to use it, as so many other general rules are constantly used, with a mental reservation as to its universal truth when applied. It is a mere assumption that this kind of double dealing is impossible,—an assumption which, no doubt, comes very naturally to those who desire to obtain a complete metaphysical system secured against all risk of refutation by an unforeseen course of events. The assumption, however, runs counter not only to the chief part of our daily experience of thinking, and of correcting our thoughts in the act of their growth, but also to the whole method of science. In science, just as in ordinary life, we are in the constant habit of using laws which we know to be not quite beyond improvement. In fact, the better the science is the more it admits this eternal need of correction. Science draws particular conclusions by means of its general rules, but it also recognises that the conclusions—and the rules they depend on—remain open to criticism. And if any defender of the Law of Contradiction will allow us to use that law in the same way we should have no further quarrel with him. But then he must extend this admission of possible error to his own metaphysical conclusions, however clearly he can demonstrate their truth in words.

Another form of the same defence is that to doubt the universality of the Law of Contradiction leaves us without any reason for condemning inconsistency. In answer to this we should distinguish between merely careless (or worse) inconsistency and the kind that is justified by the known defects of language and of antithetical thought. Against careless inconsistency our provisional use of the Law of Contradiction would be just as effective as any less cautious use of the law could be, while the other kind of inconsistency (we should maintain) is something that needs to be encouraged and not forbidden. There is no merit in misunderstanding the course of Nature through an over-regulated effort to understand it. In the same way there is no merit in falling short of the best kind of conduct through a rigid adherence to restrictive principles.

Another defence sometimes met with is that we have no business to blame a rule merely because it is often misapplied. This is, I think, an interesting kind of defence, involving more subtlety than those already noticed. It raises, in a pointed way, a difficulty which is of common enough occurrence in a dispute,—that of deciding whether a rule is itself defective, or only seems so through a wrong interpretation having been put upon it. Every one would admit that, for example, a rule intended to state a *tendency* is not to be blamed if some one applies it without taking the saving clause into account. Nor, again, need an Act of Parliament be condemned because this or that person interprets it wrongly. But the line is exceedingly hard to draw between a rule that is misapplied merely through the fault of the person applying it, and one that even by its own most careful expression contributes to the mistake. That is the charge we bring against the Law of Contradiction. State the law as you will, it remains impossible to apply securely to statements in general. There is no way of making sure *a priori* that in a given context a given predicate is not ambiguous, since we can only see that it is ambiguous after we have attempted to verify it and so have discovered the ambiguity. We can only be wise, that is, after the event that shows our previous lack of wisdom.

This point was briefly noticed above,¹ but perhaps needs a little further expansion. The difficulty of it lies in the fact that whenever an ambiguous predicate interferes with the Law of Contradiction, the state of things before and after the ambiguity comes clearly into view is different. Before the discovery of the two meanings in the predicate B, the admission that A is both B and not B necessarily seems to be a breach of the law; while directly we can admit that A is literally B but virtually not B, or broadly and generally B but for a special purpose not B, all talk of a breach of the Law of Contradiction is at an end. Thus a breach of the Law of Contradiction insensibly passes into a case where there is no such breach (and by the way this change, like any other, is itself an example of the defect of the law). If we prevent ourselves from admitting the possibility that the Law of Contradiction can ever deceive us in application we are thereby at least hindered in discovering a hidden conflict between a literal and a virtual truth. And the virtual truth of a literal untruth is a thing that often reaches our understanding slowly at the best,—just as when a nearer approach corrects our first impression about a distant

¹ P. 45.

object. So that it is not always a simple matter to make sure that an apparent contradiction is not a real one. We often have to wait for that piece of wisdom until time and trouble, and the habit of expecting such mistakes, have brought us nearer to the facts in question. Ignorance of the defects of the applicable Law of Contradiction stands in the way of our forming that habit.

That is how I should answer the objection that since the Law of Contradiction is supposed to apply only to predicates taken in the same sense, a conflict between literal and virtual truth cannot illustrate a breach of the law. In the abstract it cannot, but in any actual case it can. The Law of Contradiction has, no doubt, the most excellent intentions. It *means* to refer only to unambiguous predicates. But then it cannot guarantee its own correct application. If it pretends that no predicate can ever be ambiguous, it lives in a fools' paradise. As things are, an unrecognised conflict between a literal and a virtual truth is only too frequent. The risk of ambiguity pervades all statement of fact, and wherever the ambiguity becomes actual without being seen, the good intentions of the Law of Contradiction are defeated. And our contention is not only that such cases are far more frequent than is generally supposed, but also that they have in two ways a peculiar importance: they arise chiefly at the point at which old knowledge is modified by new, or broad and rather abstract views by special and concrete facts; and the region of their greatest effectiveness is in disputes which are least easily settled, where the balance between the opposed views is most difficult to overcome. Just in that region where landmarks are changing, and where the changing landmarks seriously confuse our view of the truth, obstinacy in appealing to the Law of Contradiction as a ground of argument tends to bar out the chance of progress and of harmonising the dispute.

The mere number and frequency of these cases is considerable, since the change of a possible into an actual ambiguity occurs wherever any important detail in a fact is overlooked or ignored in its description or conception. A given fact is described as B when (if a certain detail had been taken into account) it would have been seen to be virtually a case of non-B. The description B is thus correct in the absence of more detailed knowledge, but incorrect in its presence. This piece of progress in knowledge therefore makes a correct description incorrect; and reversely, the habit of supposing that a correct description cannot be incorrect stands in the way of the particular piece of progress.

Perhaps the clearest examples are those where the detail overlooked is one that is growing in importance, and where there is a sufficient interval during which its growth is not much noticed and its existence is rightly kept out of sight. But the growth must not be too slow, else language accommodates itself to the new state of things and no mistakes are made. If an Alpine lake gets silted up with glacier mud in 100,000 years, and becomes first a swamp and then a fertile valley, no one is likely to misdescribe it much during the process; and even when a village or a suburb—like Kensington or Chelsea—is ceasing to be a village or a suburb, and is becoming virtually a part of the town, the people whom the old description would mislead may not be very numerous. Still, that is not always so. Seeley, in the *Expansion of England*, notices an interesting case where an old conception of fact has actually survived its justification for a considerable time, so as to produce an important confusion of thought. In the seventeenth and eighteenth centuries the distance of our colonies from the mother country was a determining factor in the notions we had about them. The sea *divided* us from them—before cables and steamers came into use—and therefore it was natural that we should not think of colonies as parts of our country (like Kent or Cornwall) but rather as appendages to it—'possessions'. The decay of this idea, Seeley points out (writing in 1883), has not kept pace with the rapid decrease in the difficulties of communication. Hence the virtual truth of the matter was obscured—and perhaps it still is so to some extent—by the old forms of expression and the old habits of thought.

But there is no reason to restrict our examples to this kind of situation, where the not-B element in a case of so-called B is properly negligible at one stage and gradually grows in importance. A conflict between literal and virtual truth may be obscure for other reasons beside the fact of slow change or development. It seems better to connect it with the whole problem of ambiguity—that is to say, with the difficulty of applying general names so as to give due weight also to individual peculiarities which are important. Since there is no way of asserting or conceiving any fact except in general names that describe it, and since every fact nevertheless has its own individual peculiarities, all statement or conception of fact is elliptical and therefore liable to suffer through making important omissions. And in so far as the mistake is a plausible one, the correction of it involves (till it is accepted) an apparent breach of the Law of Contradiction, and therefore is hindered by our rigid adherence to that law.

If we take any examples of this extremely common situation—where an important detail is hidden by a general name—we shall find, I think, that the difficulty of viewing them as cases where the Law of Contradiction has to be disregarded arises partly from the change in our point of view *after* the literal and the virtual truth have been disentangled, and partly from the fact that in ordinary life, and even in science, no express reference is usually made to the Law of Contradiction. Except in philosophical writing of a particular kind the axiom is hardly ever expressly mentioned. The man who discovers the importance of a detail which might have been discovered sooner finds it more natural to mention the detail itself; and as soon as he proves its existence, and its importance, the virtual truth at once conquers all verbal obstacles. There is rather a good instance which I have had occasion to quote in another connexion,¹ where some experimental cases reasonably supposed to be cases of splenic fever infection were afterwards unexpectedly found to have been complicated with another kind of infection so rapid in its effects that the splenic fever germs had not time to get to work, and so appeared to be inoperative. The overlooked difference turned upon the fact that the blood used for the experiments was not quite fresh, having been taken twenty-four hours after the death of the animal. These *were* cases of splenic fever infection, and yet in view of the particular experiments they were not so. Literally they were B, and virtually they were not B. Here the axiom that if a disease is a particular kind of fever it is *not* another (different) kind may have had its effect in delaying the discovery, but was naturally not referred to, and therefore not disputed. As soon as the existence and importance of the detail became clear, it was seen that actual splenic fever infection ceases to be virtual splenic fever infection after the lapse of twenty-four hours.

The overlooking of important details in describing or conceiving a fact is really too common an experience to need further illustration. But if we want examples that are concise, and sharply pointed, and easy to follow, we can get them best from detective stories. Such stories, with their inevitable conflict between a theory formed by the regular police and a theory formed by a gifted amateur with an eye for peculiar details, always illustrate the value of doubting whether a thing that may be correctly described as B may not also be called an unusual kind of non-B; and the conflict is typical of all careful and subtle inquiry into facts and their best description.

¹ *Process of Argument*, p. 95.

Take, for example, Poe's tale of the murders in the Rue Morgue, where the investigation, at one stage, turns upon the point about the fastening of the back windows. The police examined them and found that they were firmly nailed down from inside. There, full in view, were the nails that were driven through the frames of the windows. With some excuse the police concluded that the murderer did not escape that way. Then the amateur detective comes on the scene, and examines the facts more carefully. He finds that while one window (which the police had tried to open) was securely nailed down, the nail in the other had been broken off at some time, leaving the head in the frame. The police took their observed fact as a 'nailed-up window'. For some purposes¹—or at least, apart from the purpose of opening the window—that was a correct description; the window had actually been nailed up, and the nail had never been withdrawn. What made the description importantly incorrect was an individual peculiarity belonging to this case as contrasted with the class of 'nailed-up windows' generally. The police took the responsibility of assuming that none of the special details of the case were of a kind to make the general description virtually incorrect. For them the loose and general predicate B stood in the way of the virtually truer predicate non-B.

Artificially invented points of this kind may at any rate serve to remind us of many similar incidents in our own less dramatic experience. They help us to remember many less striking and less definitely pointed cases where, through the overlooking of some detail either in the fact itself or in what are loosely called its surroundings, the fact (or state of things) has a good apparent right to the name B, and also a better though less apparent right to the name non-B—cases where the fact is B on the surface, or regarded in a broad and general way, and not B when regarded more closely or with a special purpose in view. In all such cases, unless the Law of Contradiction happens to be expressly mentioned its defects do not come naturally into sight. And when the two kinds of meaning have been clearly distinguished the difficulty exists no longer and there is nothing to show that the law has played us false in its application. Nevertheless the Law of Contradiction, applied in the only way it can be applied *before* the new knowledge comes to hand, is a misleading influence. It fights on the sight of prejudice, and delays the advent of the clearer and fuller view of the facts.

¹E.g., for the purpose of noticing the damage done by a tenant.

Common sense, no doubt, often makes light of verbal contradiction and so rushes in where philosophers fear to tread. Still, it does seem more businesslike (and therefore also more truly philosophical) not to entrust the over-riding of this kind of wordy prejudice to our rather uncertain impulses of common sense, but to admit in our Logic and Philosophy the need of keeping a clear course open for doing so. At present, however, the risk of misapplying the Law of Contradiction escapes the notice of some of our most ambitious philosophers. In this respect they seem hardly to have advanced beyond the simple-minded assumptions of the merest Formal Logic. With perfect confidence they assume that a verbal contradiction is an unmistakable sign of real error, and that no further justification is needed for condemning as illusory all the notions that have most abundantly proved their practical value. Conceivably some such criticism of fundamental notions might be undertaken for the sake of improving their usefulness in managing our lives. Take Causation, for example. Even the verbal puzzle about A changing into B would have importance if the moral to be drawn from it were that our notion of the essential elements in a sequence is liable to be vitiated by the drawing of a hard line between antecedent and consequent, and between the whole event and its surroundings. If what we call A is really a little more B than we think it, we naturally make mistakes in foreseeing the line of its development; and merely to be reminded of this possibility—especially in connexion with actual examples—would have, no doubt, some practical value. But unfortunately that is the last thing that this kind of philosopher desires. Everything practical is, for him, a "mere makeshift," and he won't be happy till he gets outside the domain of practice altogether; in other words, out of the region in which his symbols have to be verified in order to mean anything. His whole speculation, in consequence, takes on the character of an ingenious but trivial game, played under rules laid down by verbal axioms which are only safe from criticism when they are subjected to no tests and interpreted as having no bearing on any mundane facts.

The particular example I started with has perhaps drawn us into a good deal of disputable matter on its own account. But exactly the same disputable matter belongs also to the wider question how far and in what manner the defect here imputed to the Law of Contradiction should be recognised in other general rules as well. I have only taken the Law

of Contradiction as a notable instance of a whole class of general rules—those where it is possible to delude ourselves by the undeniability of the rule regarded otherwise than as a major premiss. Whenever a rule can thus claim axiomatic value the defect in question attaches to it. Our contention is that no clear distinction can be made between the defects that really belong to a rule and those that falsely appear to belong to it because it is misapplied. Some way this distinction will carry us, but not quite far enough. There seems to be no clearly distinctive feature of careless or faulty interpretation as contrasted with interpretation which is unfortunate through no one's fault. For since all general rules—even axioms—are liable to unforeseen ambiguity therefore any particular misinterpretation of a rule may be one of those accidents that will happen even in the best regulated kinds of thought. In such cases we do not seem to gain anything by maintaining that the rule *in itself* is not defective, for nobody can tell (except negatively) what this 'rule in itself' may be—the rule regarded apart from all application. Such a defence is really no defence at all. It is either an empty, resultless courtesy paid to the rule, or else its result must be to make us treat the risk of misinterpretation lightly.

The way in which this kind of defence of a rule misleads us is by restricting our notion of what makes a rule defective, so as to leave out of account the subtlest and therefore most troublesome source of error in the use of rules. Assuming that it is important to know exactly how far a given rule may be trusted—and that assumption is, I suppose, involved in all attempts to make our inferences correct—it seems to follow that we ought to get as far as possible from an optimistic or happy-go-lucky view of the risks; that we ought not to be content with calling a rule defective only in so far as we can already see ways of improving its statement, but rather to recognise that the worst defects are those that no improvement of statement can ever remedy,—those that are inherent in the applicable rule however we try to guard against them. Mistakes of application are then chargeable to the rule itself; or at any rate if the actual mistakes may be charged against the individuals who make them, the general risk is something that affects not this or that particular application but all applications alike. The only general account of misapplications is that they are *facilitated* by a rigid belief in the universal truth of the rule, while this in its turn is facilitated by confusing the applicable with the abstract rule. Be an axiom never so axiomatic it has got to contemplate being applied,

or else gives up its claims to a meaning. And to contemplate being applied involves either the contemplation or else the ignoring of the risks of wrong application.

In this connexion it may be asked what is our attitude towards mathematical axioms. They are not, I should say, an exception to the rule that all rules exist only to be used as major premisses and were originally discovered with that purpose more or less dimly in view. But our opponents may grant this and still insist on asking whether, in our opinion, mathematical axioms are, like other general rules, open to question when applied. Our opponents may contend that it would be folly or perversity to call (*e.g.*) the multiplication table questionable. When we make a mistake in addition—even the kind of mistake that is likely in counting infusoria while they are in process of division—we blame our *minor* premiss as a matter of course. Though the apparently one and the apparently one may make some other number than two, we at once in such cases admit our observation to have been at fault. Here, then, the opponents may say, is a considerable class of abstract truths used as major premisses and yet quite beyond our power to question. If so, what becomes of our argument against using the abstract axioms of Logic with the same entire conviction of their truth?

I confess to having only the vaguest ideas as to the present or future possibility of questioning the truth of the mathematical axioms otherwise than idly. It is for those who are able to find, or to foresee, the faults of them to show what value such researches may have. But even a person thus ignorant of mathematics may, without venturing out of his depth, deal with the argument that if mathematical axioms are admitted to be abstract and also true and useful as major premisses, the ground of our objection against the abstract logical axioms disappears. This argument is irrelevant in two respects. In the first place the defect of the abstract logical axioms, as explained above, does not consist in their abstractness *per se*, but in their abstractness relatively to their only possible applications; the logical axioms (when universally true) refer to *assertions* while their only application is to *sentences*. But when we apply (*e.g.*) the multiplication table, though the laws of number are always admitted to be abstract the numbers counted are also always admitted to be only one aspect of the particular facts in which the numbers are found. Behind an inference resting on an appeal to arithmetical principles there is always implied the saving clause 'so far as these are numbers'—which includes

both the proviso 'if I have counted as I intended' (i.e. correctly) and the proviso 'if the laws of number are applicable to the subject'. And where do we find any similar safeguards in the application of logical axioms? Only where the conclusion is held tentatively or at least with a full consciousness of the risk of identifying sentence with assertion; only, for example, where the confident use of the Law of Contradiction to rule out inquiry has been most carefully avoided.

In the second place, if it be the fact—as we need not mind assuming provisionally—that no present or future application of a particular mathematical axiom will ever force us to reconsider and revise it, all we can say is, So much the better for that remarkably wary and farsighted axiom. Still the prophecy is gratuitous, and may be wrong. There is no great harm in a generous, or even a sanguine, recognition of an axiom as true, so long as we reserve to ourselves the power of testing its truth in particular applications. The harm only begins when, in order to apply the axiom for the sake of drawing a conclusion from it, we slur over the question whether as so applied it is true. And our argument about the logical axioms is not affected by the superiority of the mathematical ones—if they are superior¹—in resisting this misuse. We have seen, anyhow, that the misuse does occur in logic, and also that the logical axioms (when applied in the only possible way) are downright misleading. Their defects do not require to be spoken of as vague future possibilities, but are well within the range of any one's understanding at the present day. The only excuse I can suggest for the common habit of ignoring them lies in the real difficulty of recognising consistently the distinction between a rule considered as a major premiss and a rule merely 'up in the air'. One reason why it is so often overlooked may perhaps be found in the fact that the same verbal statement of a rule is naturally made to serve both purposes, so that it is difficult to see that the two different interpretations of the same form of words amount in effect to two different rules,—or to a rule on the one hand and the ghost of a rule on the other. And the difficulty is increased by the fact that there is no pair of contrasted names in general use by which to

¹ It may, for instance, be held that where a mathematical law has been applied to a subject which does not admit of mathematical treatment, there we have an abstract law applied to the given whole instead of to an abstracted part only of the particular case. I gather that Mr. Schiller would in some such way regard mathematical and logical axioms as alike in providing the same temptation to misuse.

mark the difference and keep it in view. The distinction is not simply (*e.g.*) that between 'abstract' rules and others. The least inappropriate¹ pair of names seems to be 'applicable' and 'inapplicable'. By an applicable rule we should thus mean any rule that is not afraid to run the risk of application in minor premisses; or any rule which is not interpreted so as to beg the question it professes to answer about what may be inferred (supposing it true) in particular cases of a specified kind. When it says that all A are B, it refuses to shelter behind the explanation that any case of apparent A which is *not* B thereby stands condemned as not properly a case of A at all. For example, the Law of Contradiction taken in the abstract does no more than define the word 'contradiction,' by saying that if two apparently contradictory statements do not cancel each other they are not really 'contradictory' statements; while on the other hand the applicable Law of Contradiction ventures beyond this verbal explanation, and therefore can claim only a *prima facie* truth which may in any case mislead those who trust it simply.

If our party are right in this matter, and if no relevant objection can be brought against our view, what we must expect, of course, is to be told that it is so obviously true that it need not have been expressly stated. That is the usual fate, in controversy, of philosophical views when it becomes clear, in the end, that they have to be admitted. It was for this reason that I began by quoting an instance where our view (in spite of its being so obvious) was actually not understood. That is however only one instance out of many. Nearly everybody, I believe, except the Pragmatists, is still disposed to put unquestioning faith in general rules which are only not misleading when they lead nowhere, which are only undeniable when they beg the question they profess to answer when applied. The common platitudes or truisms of every-day thought get their whole inspiration from this practice. Formal Logic is entirely built upon it. And the more ambitious any system of Metaphysics is, the more it seems driven in the same direction. We may admit quite easily that it is only in the interests of practice that there is any fault to be found with attempts to soar above all practical considerations. But then the whole dispute seems to lie be-

¹ Inappropriate because I admit that an inapplicable rule is too much honoured by being called a rule at all. But this objection cannot pretend to be more than literary.

tween those who think they can understand what is meant by a claim to Truth which is above all considerations of practice and exempt from all scrutiny, and those who assert that no such attitude is intelligible. To the latter it seems that to make this claim for any theory is a covert way of confessing its instinctive dread of facing a theory's only test.