EFFECT OF DRAWING ON THE ELASTICITY OF COPPER WIRE.

BY J. R. BENTON.

WHEN different specimens of the same substance are used for determining elastic moduli, the results vary considerably, so that the values obtained for any one specimen describe the behavior of that specimen only, and do not hold universally for the substance in question. This discordance is explained by the assumption of differences in the physical condition of the various specimens. Such differences are produced in the case of metals by rolling, hammering or drawing, or by varying the manner of cooling from high temperatures.

Determinations of the elastic moduli of metals are most often made on wires. It is therefore especially important to know how the elasticity of wires is affected by the processes used in their manufacture; of these, drawing is the most important.

This article describes some experiments on the changes in elasticity which the process of drawing produces in copper wires. I shall first give a statement of the results obtained from the experiments, and leave till afterwards the discussion of the methods of observation, as they will be of less interest to the general reader.

STATEMENT OF RESULTS.

The following are the principal facts that have come to light in the experiments:

1. If a copper wire is first annealed by heating electrically¹ to redness and is then drawn, its modulus of rigidity decreases, and Young's modulus increases.²

¹It has been shown that the passage of an electric current through copper wire does not influence Young's modulus, excepting in so far as it heats the wire. It is probably safe to assume further that the current has no specific effect on the modulus of rigidity. See M. C. Noyes, *PHYS. REV.*, 3, 432-447, 1896.

²The increase of Young's modulus in drawing copper wire was observed by Wertheim, *Compt. rendus*, T. XV., p. 110; *Pogg. Ann. Ergänzungsband*, II., pp. 1-70.

2. Each successive drawing causes a further diminution of the modulus of rigidity, and a further increase of Young's modulus; for both moduli the amount of change becomes less with each drawing.

3. If finally the wire is annealed after having been drawn several times, the modulus of rigidity returns to a value considerably greater than its original value, and Young's modulus to one considerably less than its original value.

The numerical results for two pieces of wire taken from the same coil are given in Table I.

TABLE I.

Treatment of Wire.	Modulus of Rigidity in Dynes per sq. cm.	Young's Modulus in Dynes per sq. cm.	Diameter in cms.
Annealed and drawn } once.	$10^{11} \times$ (Wire A.) 4.017 ± 0.010	$10^{11} \times$	0.15042
Drawn 2 times. . . .	3.946 ± 0.013		0.13914
“ 3 “	3.919 ± 0.011	13.87 ± 0.14	0.13060
“ 6 “	3.876 ± 0.011	13.90 ± 0.13	0.11220
“ 9 “	3.863 ± 0.008	14.20 ± 0.12	0.09410
Annealed again . . .	4.322 ± 0.008	11.90 ± 0.12	0.09318
Annealed.	$10^{10} \times$ (Wire B.) 4.177 ± 0.012	$10^{10} \times$	0.16174
Drawn once.	4.015 ± 0.011	13.21 ± 0.14	0.15082
Drawn 3 times . . .	3.961 ± 0.010	12.90 ± 0.13	0.13662
“ 5 “	3.948 ± 0.008	13.26 ± 0.13	0.12298
“ 7 “	3.945 ± 0.008	13.40 ± 0.12	0.11238
“ 9 “	3.897 ± 0.008	13.32 ± 0.12	0.10012
Annealed again . . .	4.305 ± 0.008	11.09 ± 0.12	0.09856

The values given for Young's modulus hold for isothermal extension, while those given for the modulus of rigidity hold for adiabatic torsion.

The results are shown graphically in Figs. 1, 2, 3 and 4. The vertical line through each plotted point in the diagrams gives the limits of probable error for the corresponding determination.

It is probable that the difference in elastic properties between the wires annealed before drawing and those annealed after drawing is due to the fact that the latter cooled from red heat more rapidly on account of their smaller diameter, and were therefore more completely annealed.

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

It is a matter of common observation that drawn copper wires offer much greater resistance to bending than annealed ones. This would be explained by an increase in Young's modulus. But the change in Young's modulus, as seen in Table I., is not sufficiently great to account for the very considerable increase in resistance to bending. We have no right, however, to assume that the change in Young's modulus is uniformly distributed over the cross-section of the wire. If, for example, it should happen that the modulus increased for the outer part of the cross-section, but decreased for the inner part, we might have no difference in the elasticity of extension, but would nevertheless have an increase in the elasticity of bending.

These considerations seemed to make it worth while to determine Young's modulus for the same wires by the method of bending. The results are given in Table II. Contrary to expectation the values obtained by flexure were considerably less than those obtained by extension. This would seem to indicate (though it would by no means prove) that Young's modulus increases more in the interior of the wires than near the surface. It is to be noticed that the values obtained by the two methods very nearly agree for the annealed wire. The greater stiffness of drawn wires as compared with annealed wires must then be due simply to the increase in the *limit* of elasticity. The flexural elasticity itself increases only slightly in drawing.

TABLE II.

Young's modulus determined by bending. (Wire B.)

Treatment of Wire.	Young's Modulus in Dynes per sq. cm.
Annealed and drawn once.	$10^{11} \times 11.23 \pm 0.12$
Drawn 3 times.	11.79 ± 0.12
“ 5 “	11.88 ± 0.12
“ 7 “	11.83 ± 0.12
“ 9 “	12.08 ± 0.12
Annealed again.	10.82 ± 0.12

DETERMINATION OF THE MODULUS OF RIGIDITY.

In determining the modulus of rigidity the following method was used: A wire (α) about 235 cm. long was fastened at the top

to a firm bracket (*b*), the end of the wire being tightly clamped between steel plates. On the lower end of the wire was clamped a rod (*c*), the ends of which were fastened so as to prevent torsion of the lower end of the wire. At the middle of the rod was a hook, from which weights (*d*) could be suspended so as to make the wire hang straight. Exactly half way between *b* and *c* a brass disk (*e*), 15 cm. in diameter, and weighing about 1 kg., was fastened. On the brass disk two lead cylinders (*f, f*) (4.7 cm. in diameter; 2.5 cm. in height) could be laid. These cylinders were always laid in exactly the same position; this was attained by means of small pegs (*g*) which projected from the disk *e*, each cylinder being brought into contact with the proper pegs.

Fig. 5.

Fig. 6.

The method of observation was as follows: the period of torsional vibration was determined when the lead cylinders were in place; then the lead cylinders were replaced by hollow brass cylinders of the same dimensions externally, and the period was again determined. The purpose of the brass cylinders was simply to insure that the resistance of the air to the motion of the disk should be the same for both determinations. The period of swing in the two cases, and the difference in the moment of inertia, together with the dimensions of the wire, give the necessary data for determining the modulus of rigidity.

The modulus of rigidity F can be calculated from the formula

$$F = \frac{4\pi l' K}{r^4(t_2^2 - t_1^2)}, \quad (1)$$

where l' is the length between the disk and either end of the wire, K is the difference in moment of inertia due to substituting the lead cylinders for the brass ones, r is the radius of the wire, and t_2 , t_1 , are the periods with the lead cylinders and the brass cylinders, respectively.

If we put $\sqrt{\frac{m}{\pi l \sigma}}$ in place of r , where m is the mass of

wire, l is its total length, and σ its density, the formula becomes

$$F = \frac{4\pi^3 l^2 \sigma^2 K l'}{m^2(t_2^2 - t_1^2)}. \quad (2)$$

Of the quantities in this formula, l and l' were determined by direct measurement with a meter scale; their errors were taken as not greater than 0.2 cm., and 0.1 cm. respectively, or about 0.1 % in each case. σ was determined by weighing the wire in air and in water. One determination was made for each wire experimented on, and the mean was used, being 8.849 for the annealed wires, and 8.840 for the drawn wires, the probable error being in each case ± 0.005 , or about 0.06 %. If any change in density took place after the first drawing of the wire, it was within the limit of the experimental errors.¹ K was calculated from the dimensions and masses of the two lead cylinders and the two brass cylinders. Its value in C.G.S. units was 21210 with a probable error not greater than 17, or about 0.08 %. m was determined by weighing the wire; its probable error was taken as not greater than 0.0003 gr., or about 0.001 %. t_1 and t_2 were determined by observing the time when a mark on the disk swung past its position of rest at every fifth swing for about 250 swings; by properly combining these observations the period of the disk could be determined with a probable error of 0.0006 sec., or 0.01 % to 0.03 %. This gives for $(t_2^2 - t_1^2)$ a probable error not greater than 0.1 %. In determining the period an ordinary watch was used, with which it was possible, after practice, to read to the nearest fifth of a second. The combination of these errors gives the probable error of F between 0.2 % and 0.4 %.

The formula used assumes (1) constancy of external conditions, (2) homogeneity of the wire, (3) perfect firmness of the supports, and (4) that the wire has the shape of a circular cylinder. The possible imperfect fulfillment of these assumptions leads to the consideration of the following possibilities of systematic error:

¹ Marchand and Scheerer state that the density of copper wires is increased by drawing. The wire for which they found this result had density between 8.895 and 8.896, and must therefore have been of different copper from the wires I used. Moreover, the changes due to drawing appear only in the third decimal place, and are so irregular that their conclusions appear hardly justifiable. Marchand and Scheerer, *Journal für praktische Chemie*, 27, p. 193. 1842.

1. As regards external conditions : (a) The temperature at which the different determinations were made varied between 19° and 23° C. The variation of the rigidity modulus between these temperatures is within the limits of error of my determinations, as shown by the experiments of Kohlrausch and Loomis,¹ of Pisati,² and of Katzenelsohn.³ (b) The weights hung on the wire were not always the same, so that the condition as regards longitudinal stress varied between 4.8×10^8 and 11.2×10^8 dynes per square cm. The variation in modulus of rigidity due to this cause is also within the limits of experimental error, as shown by my work on the dependence of modulus of torsion on tension.⁴ The condition of the wire as regards torsion of course varied during each swing, but this had no appreciable effect, as shown by the fact that the swings were sensibly isochronous. The amplitude was always kept less than 10° .

2. Homogeneity of the wires can not be assumed, so that the values obtained for F must be taken as integrated values for the cross section of each wire. Probably the copper itself was in fact not homogeneous ; besides that there was always a film of oxide on the surface of the wire, which could not be entirely gotten rid of. After annealing, the surface was cleaned with HCl, which was then washed off with water, leaving the surface perfectly clean ; but it soon became tarnished by exposure to the air.

3. Suppose the body to which the wire is clamped at the bottom not to be perfectly firm, the top being perfectly firm ; then the motion of the body at the bottom will approximate to that of a body of very great moment of inertia, hung on to the bottom of the wire, and free to twist with the wire. The motion of such a system will be given by the equations

$$\left. \begin{aligned} \frac{d^2\theta}{dt^2} &= -\frac{M}{K_1} [\theta + (\theta - \varphi)] = \frac{M}{K_1} (\varphi - 2\theta) \\ \frac{d^2\varphi}{dt^2} &= +\frac{M}{K_2} (\theta - \varphi) \end{aligned} \right\} \quad (3)$$

where M is the direction-force of the wire, K_1 and K_2 are the moments

¹ Kohlrausch and Loomis, Pogg. Ann., 141, 1870.

² Pisati, Nuovo Cimento, V., pp. 145-150.

³ Katzenelsohn, Beiblätter der Physik, 12, p. 307.

⁴ PHYS. REV., Vol. XII., pp. 100-114, 1901.

of inertia of the disk and the body at the bottom respectively, and θ and φ are the respective angular displacements. The elimination of φ gives

$$\frac{d^4\theta}{dt^2} + \left(\frac{M}{K_2} + 2\right) \frac{d^2\theta}{dt^2} + \frac{M}{K_1 K_2} \theta = 0. \quad (4)$$

If initial conditions $\theta = 0$, $\varphi = 0$, $\frac{d\theta}{dt} = \omega$, and $\frac{d\varphi}{dt} = 0$, are assumed, the solution of this equation takes the form

$$\theta = c_1 \sin c_2 t + c_3 \sin c_4 t \quad (5)$$

where c_1 , c_2 , c_3 and c_4 are constants depending on M , K_1 , K_2 and ω , and c_3 is very much smaller than c_1 if K_2 is very much larger than K_1 . This shows that the average period taken for a large number of swings will not be affected by a slight lack of rigidity of the support; for $c_3 \sin c_4 t$ will be negative as often as it is positive, and the disk will therefore swing through its position of rest too soon as often as it will do so too late.

4. As regards the shape of the wire: (a) It is safe to assume that its cross section is the same at all points of its length, as the die-plate through which the wire is drawn is of very much harder material than the wire, and would not wear away appreciably during the drawing of two meters of wire. (b) The wire is always more or less bent, so that its axis not a straight line, but a curve of double curvature. That such deviation from straightness had no appreciable effect in my experiments, I proved empirically by purposely bending and kinking a wire and then determining the period of torsional swing, which was found not to differ from that of the straight wire. (c) It is possible that the cross section may not be perfectly circular. If this is the case, then as a first approximation it can be considered elliptical. Then $\frac{1}{r^4}$ of formula (1) must be replaced by $\frac{1}{2} \frac{1}{ab} \left(\frac{1}{a^2} + \frac{1}{b^2} \right)$, where a and b are the semi-axes of the ellipse. If we set $a = r + \delta$ and $b = r - \delta$, we find that the difference between $\frac{1}{r^4}$ and $\frac{1}{2} \frac{1}{ab} \left(\frac{1}{a^2} + \frac{1}{b^2} \right)$ is of the order of δ^2 . The error in assuming that the

cross section is circular will be less than 0.1 per cent. if δ is less than $\frac{1}{32}r$; and δ was less than $\frac{1}{32}r$ for all the wires I used, as shown by micrometer calipers; so that this source of error need not be feared.

DETERMINATION OF YOUNG'S MODULUS.

To determine Young's modulus the sinking of the lower end of the wire (see Fig. 5) under a known increase of load was read with a micrometer. The lower end of the wire was left free, except that it was prevented from swinging to the side by means of three strings at right angles to the wire, so arranged that they did not interfere with its vertical motion. Young's modulus was given by the formula

$$Y = \frac{sgl}{\pi r^2 e}, \quad (5)$$

where s is the mass of the weights added, g the acceleration of gravity, l the length of the wire, r its radius, and e the increase in length due to the weights added. If we put $r = \sqrt{\frac{m}{\pi l \sigma}}$, where m is the mass of wire, and σ its density, we have

$$Y = \frac{sgl^2 \sigma}{m e}. \quad (6)$$

The method of determination and the errors of l , σ , and m have already been discussed (see p. 240). s was determined within an error of 0.01 per cent. by direct weighing. To determine e , a succession of readings with the micrometer was taken as the wire was successively loaded and unloaded. The mean values were used; a correction had to be made for the sinking of the upper support, which was measured by the micrometer. The probable error of the final value of e was ± 0.0007 cm., or from 0.4 per cent. to 1 per cent. The combination of these errors gives the probable error of Y between 0.5 per cent. and 1 per cent.

The formula (6) assumes (1) constant external conditions, (2) homogeneity of the wire, (3) cylindrical shape of the wire and (4) the applicability of Hooke's law.

1. The temperature varied between 19° and 23° C. The variation of Young's modulus within these temperatures is negligible, according to the experiments of Katzenelsohn.¹ The fact that the values obtained are those for isothermal extension has already been noticed.

2. Homogeneity of the wire can not be assumed; the values of Y are to be regarded as average values for any cross section of the wire.

3. The shape of the cross section of the wire is immaterial, as that is eliminated in formula (6). The cross section must, however, be the same at all points of the length of the wire; this can be assumed to be the case with a drawn wire. It is possible that the wire may deviate from straightness; if so, it is to be expected that the wire will be partially straightened by increasing the load, so that the lower end will sink more than if only extension took place. Suppose the half length of the wire to be denoted by λ and suppose the middle of the wire to be at a distance δ from the straight line joining its extremities. If a vertical force W be applied to the end of the wire, the horizontal force at the middle tending to straighten the wire will be $W \cdot \frac{\delta}{\lambda}$. If the wire is made perfectly straight, its lower end will sink through a distance $2\delta \cdot \cos \cos^{-1} \frac{\delta}{\lambda} = 2\lambda \left(\frac{\delta}{\lambda}\right)^2$. As the wire will not be perfectly straightened by any vertical force, however great, the sinking of the lower end will be less than $2\lambda \left(\frac{\delta}{\lambda}\right)^2$ and the force acting to produce it will be $W \frac{\delta}{\lambda}$, so that the sinking produced will be of the third (or higher) order of small quantities if $\frac{\delta}{\lambda}$ is of the first order.

4. With the stresses used some slight deviation from Hooke's law is to be expected, according to the experiments of J. O. Thomson.² The limits of stress between which my determinations of Young's modulus were made, were approximately the same for all the wires, so that the results will serve for purposes of comparison.

¹ Katzenelsohn, Beiblätter der Physik., 12, 307.

² J. O. Thomson, Wied. Ann., 44, 555, 1891.

DETERMINATIONS OF YOUNG'S MODULUS BY FLEXURE.

The determinations of Young's modulus by bending were made by laying a piece of the wire across two knife-edges which were rigidly braced at a distance of 7.22 ± 0.01 cm. from each other. Exactly midway between the knife-edges a light scale-pan was suspended, on which weights were laid, and the sinking of the middle of the wire was measured with a micrometer. The weights were so arranged that the sinking always amounted to about 0.08 cm. The sinking of the supports was subtracted; and any possible rolling of the wire on the knife-edges was prevented by bending down one end of the wire and letting it rest against a nail, friction being minimized by lubrication. For these experiments the wires were made as straight as possible; but perfect straightness is extremely difficult to attain without subjecting the wire to processes (such as drawing or hammering) which would change its elastic properties. It is obvious that slight deviation from straightness will cause but slight error. No rigorous solution of the problem of the bending of a rod of any initial shape has yet been made.

Fig. 7.

The value of Young's modulus (Y_b) is calculated from the formula

$$Y_b = \frac{b^3 g P}{12 \pi r^4 h} \quad (7)$$

where b is the distance between the knife-edges, g is the acceleration of gravity, P the mass of the weights hung on, r the radius of the wire, and h the depression of the middle.

The probable errors of these quantities are as follows:

$b, \pm 0.01$ cm.	or ± 0.2 per cent.
$P, \pm 0.1$ mg.	or ± 0.0001 per cent.
$r, \pm 0.00005$ cm.	or ± 0.1 per cent.
$h, \pm 0.0006$ cm.	or ± 0.6 per cent.

giving for the probable error of the result ± 1 per cent.

PHYSICAL LABORATORY, PRINCETON UNIVERSITY.