THE
HORSE-POWER OF THE PETROL MOTOR
IN ITS RELATION TO
BORE, STROKE AND WEIGHT.

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§ 1. INTRODUCTORY.

THE foundation argument of the present paper is based on *Dimensional Theory*.

It has for many years been a surprise to me how little the theory of dimensions is known, much less employed, by engineers, and if in the present paper I can bring it home to some few here that there is a valuable tool lying idle which is worthy of frequent employment, the time taken in the preliminary discussion of *method* will have been in no sense wasted.

The past neglect of dimensional theory by the engineer seems all the more inexplicable when we remember how in other matters discovery and method are taken "red hot" from the physical laboratory and adapted post haste by the modern engineer to meet the insatiable demands of western civilization.

§ 2. THE THEORY OF DIMENSIONS.

The theory of dimensions may be said to be founded upon the very simple and obvious fact that a *time* can never be equal to a *volume* or *space*, or an *area* to a *linear* quantity, or generally one physical quantity to another involving different fundamental quantities.

The question of what constitutes a *fundamental quantity* touches on the ultimate definition of our conceptions, but it is customary to recognise *three fundamental quantities*, and three only. These are :— *Length*, *Mass* and *Time*, or, for short, L., M. and T., into which all

other physical quantities can be resolved. The expression denoting the constitution of any derived quantity is said to give the *dimensions* of that quantity. Thus, velocity is a length divided by time and has the dimensions $\frac{L}{T}$; *acceleration* is velocity generated per unit time, or $\frac{V}{T}$, ultimately $\frac{L}{T^2}$, and so on. The *dimensions* of most of the physical quantities employed by the mechanical engineer are given for reference in Appendix (1).

It is a necessary property of all equations representing physical quantities that the *dimensions* of the two sides are in agreement. We often find an expression that apparently falsifies the above statement, but such expression or equation is always *incomplete*—there is something *left to be understood*, such as a physical quantity *included in a constant*. Thus, if I say that the acceleration produced on a motor vehicle is proportional to the force that produces it, I may write the statement in the form of an expression: $f=cF$, and where f is acceleration, F , force, and c is a constant. Now this seems a perfectly clear statement properly expressed, but we have dimensionally:—

$$\frac{L}{T^2} = \frac{ML}{T^2}$$

and the dimensions do not balance; *there is an M on the right hand side and none on the left.*

Now, what does this M mean? It means that my equation is not complete; I have missed out the mass of something on which the acceleration depends. Ah! we have it. The acceleration is produced on a given motor vehicle. It is not *any* motor vehicle—a general expression that covers everything from an omnibus to a basket perambulator; it is a *particular* motor vehicle, and dimensional theory says: “I want to know *how much that vehicle weighs.*” You see it does not want to know the make of the tyres or the cylinder measurements. It tells us precisely what is missing to make the expression complete; it wants to know the *mass*.

An engineer should acquaint himself with the *dimensions* of the physical quantities with which he has to deal, and should make a habit of checking his equations whenever in doubt. Mistakes may easily occur owing to the employment of an *incomplete* expression in cases where the *complete* expression is necessary. The method of *dimensions* will unfailingly point out a missing factor or a mistaken quantity, if such exist.

§ 3. THE CONDITION OF SIMILARITY OF FIGURE AND ITS CONSEQUENCE.

Before proceeding to the main discussion there is a further preliminary matter to which it is necessary to devote some attention, *i.e.*, the question of *geometrical similarity* in bodies of varying size.

Let us take for discussion the case of a rectangular brick. We know that the volume of such a body, taking volume as a quantity we wish to investigate, may be expressed as a function of its three linear measurements, no matter what the proportions or size of the brick may be. If we know these measurements we can at once calculate the volume.

Now, owing to the simplicity of form of a brick we lose sight of an important point. We can define a brick by three measurements, but we cannot so define the volume of an irregular form. There is, however, another way of expressing the volume of a brick; we might take any one measurement l as representing the *size* of the brick and a co-efficient c as proper to the shape, thus:—

$$\text{Volume} = l^3 \times c.$$

Then the co-efficient depends upon the *shape* of the brick and is constant for bricks of similar geometrical form.

We thus have the volume expressed as a *function* of the *linear size* and the *geometrical shape*, and in this form the expression is applicable to any and every form of body from an Atlantic liner to a vegetable marrow.

Now, so long as c is constant, that is to say, so long as we are dealing with bodies of geometrically similar form, we know without elaborate measurement that the surfaces of the different bodies are in the relation of their values of l^2 because they are *surfaces*, and the dimensions of $\text{surface} = L^2$; likewise we know that the volumes are as l^3 because they are *volumes*, and the dimensions of $\text{volume} = L^3$.

Further, if the *distribution of matter* in the bodies is geometrically similar, that is to say, if the bodies are *homomorphous*, as when all are of the same density, or being of not uniform density are of like density in like parts; the masses will be in the proportion of ρl^3 for density, (ρ) is of the dimensions $\frac{M}{L^3}$, and $\frac{M}{L^3} \times L^3 = M$.

Thus we know that if two machines, say petrol motors, be built part for part alike, but differing in *scale*, their weights are as their respective *linear measurements cubed*.

Now this is an ordinary everyday fact with which every engineer is acquainted, and may be looked upon as a mere matter of

common sense, but, like many other matters of common sense, it is only one step removed from the abstruse.

§ 4. HORSE POWER AS A FUNCTION OF LINEAR MEASUREMENT.

The maximum power capacity of an engine is likewise a function of its geometrical form and its linear dimension. Taking as before geometrical form as constant, that is, assuming geometrical similarity, let us investigate *power* as a function of linear measurement, l .

Now we know that one of the determining factors in the power developed by an engine is the revolution speed, and we must first prescribe the general conditions that determine the maximum revolutions at which an engine can be run. We know that, given adequate port areas, the limit of speed is determined as in structures of all kinds by the *strength of materials*. We are not justified in stressing the material in one machine any higher than the similarly situated part in another; our limiting condition is therefore *stress*, $\sigma = \text{constant}$. If we assume the same materials to be used in different machines we have ρ also constant. There is no need, however, to remove these factors from the equation; we might want to employ different materials having different strengths and densities. Let us take ρ , σ and l as the variables on which the h.p. depends and deduce the general expression.

We have—

$$\text{H.P.} = l^p \times \rho^q \times \sigma^r \times \text{constant},$$

or dimensionally,

$$\frac{ML^2}{T^3} = L^p \times \frac{M^q}{L^{3q}} \times \frac{M^r}{L^r T^{2r}}$$

Whence,

$$q + r = 1$$

$$2r = 3$$

$$p - 3q - r = 2$$

from which we have—

$$r = 1.5$$

$$q = -.5$$

$$p = 2$$

so that the full expression is—

$$\text{H.P.} = \frac{\sigma^{1.5}}{\rho^{.5}} \cdot l^2 \times \text{constant}.$$

(1)

We will proceed to discuss the consequences of this result.

§ 5. IMMEDIATE DEDUCTIONS FROM H.P. EQUATION FOR ENGINES OF SIMILAR FORM.

In the first place we will suppose that the same material be used in different cases for like parts; we then have ρ and σ constant and:—

(a) The power varies simply as the *square of the linear dimension*. Such linear dimension may be the *stroke* or the *cylinder diameter* or any other datum, provided the same similarly situated measurement be taken in every case.

(b) If the basis of measurement be two linear measurements (such as bore and stroke), the rating measurement may be the simple product of such measurements or the *product of their fractional powers* provided the *sum of their indices* = 2, that is to say, so that the product shall be of the *dimensions* L^2 . Thus, if D be the diameter and S the stroke, the rating measurement might be D.S or $D^{1.5}.S^{.5}$ or $D^{1.1}.S^{.9}$ etc., or generally— $D^n.S^{2-n}$. (2)

Corollary, that for engines of other than similar geometrical proportions the same law applies, the values of the respective indices being chosen according to principles not so far investigated, for we may regard the similarly proportioned engine as a special case to which the general law must conform.

(c) That the *weight per H.P.* of similarly designed engines varies *inversely as the linear measurement*. A most important consequence, as pointing out the essential saving of weight resulting from small units and multi-cylinder engines. Example:—A four-cylinder engine with 5" $\times$ 5" cylinders is (fly-wheel apart) of but one-half the weight of a proportionately designed single-cylinder engine of the same power, *i.e.*, one of 10" diameter and stroke.

(d) With certain reservations that will be discussed later, no horse power rating rule is defensible that does not conform to the L^2 law; that is to say, being based on linear quantities, the sum of the indices must equal 2.

§ 6. UNIVERSAL CHARACTER OF THE L^2 LAW.

It is a remarkable fact that the H.P. varies as L^2 law here deduced, seems to be almost if not quite universal. Thus the H.P. capacity of an animal, other things being equal, depends upon the oxygen supply, that is to say it is proportional to the *area of lung membrane*, and in turn to the *cross sectional area of the aorta*, &c. Again, in the case of the railway locomotive the limitations to the

H.P. are, the grate *area*, the heating *surface*, the sectional *area* of the steam pipe, &c., &c. It is never a *volume*, as the *volume* of the fire-box, or the *volume* of the boiler.

At one time I was disposed to attribute this generalisation to the fact that the *dimensions* of *power* involve L^2 thus, $\frac{M \cdot L^2}{T^3}$, but any such explanation is inadequate, for *energy* also involves the same linear power, and energy is always measured by the *mass*, or under certain restrictions the *volume* of the determining factor. Thus the heat energy released by the combustion of a quantity of coal depends upon its *mass*, but the power developed is proportional to the rate of combustion, and so, other things being equal, to the total *surface* of the incandescent lumps of fuel.

It would seem probable that the considerations that apply in the case of a machine producing or transmitting mechanical energy, as herein demonstrated, where the conditions are regulated by the factors stress and density, also obtain over a much wider range of conditions, including the production or transference of energy in other forms, such as the production of heat by combustion; this aspect of the subject is outside the scope of the present paper.

§ 7. EXTENSION TO INCLUDE CHANGES OF GEOMETRICAL PROPORTION.

The comparison of the power capacity of different engines when the condition of similarity is infringed is in reality a highly complex problem, for the reason that we now have to deal with H.P. as a function of *geometrical form*, and the possible forms of change are multiply infinite.

If the problem were presented in its widest mathematical sense, as supposing engines *proportioned by pure chance*, then the position might be considered hopeless. In reality the issue is brought within very narrow limits by the assumption of a certain degree of skill on the part of the designer, and other considerations of a practical nature.

There are many factors that may independently limit the H.P. of an engine, on the principle that the strength of a chain is its weakest link. For example, a restricted port or valve area may limit the H.P. to vastly less than that legitimately due to the cylinder measurements. By common consent it is agreed that the cylinder measurements form the most suitable basis for rating purposes. Whatever basis be chosen we shall expect the *rating rule* to have some influence towards *distorting* the design as due to natural

evolution ; for instance, if we were to take the valve or port area as basis, the designer would be tempted to cut this down as much as possible, and a highly undesirable kind of *distortion* would come into vogue, resulting in great loss of efficiency. In choosing the

FIG. 1.

cylinder measurements the tendency is in the opposite direction, *i.e.*, the ports are made as free as possible, which, inasmuch as it tends to efficiency, may be looked upon as a fault on the right side. It must be understood that there is no *strictly scientific* reason why the cylinder measurements alone should be taken as the basis, it is a matter of convenience or common sense ; the tacit assumption is made that the other determining factors are so dealt with that their effect is negligible,—it is for the designer to see that this tacit assumption is complied with.

Having determined on cylinder measurement as a basis, it remains for us to investigate the conditions controlling the index values in the expression $D^n \times S^{2-n}$ so that the designer under the rating rule cannot gain by adopting extreme proportions, that is to say, so that the rule adopted will not result in any distortion of the design in respect of the proportions of stroke to diameter.

§ 8. APPROPRIATE VALUE OF THE INDEX "n."

When we discuss the relative importance of bore and stroke as determining the H.P. of an engine, we have again to consider the question of engine speed as limited by inertia and the strength of the materials employed. We may either suppose the maximum to be reached when the material is stressed up to its elastic limit by the inertia of the reciprocating parts, or at some stated point short of this extreme. In any case the first step in the solution of the problem is the determination of the *mass* of the reciprocating parts in terms of the bore and stroke, in order that the conditions of σ (stress) = constant may be ascertained.

Fig. 1 is a drawing of a piston and connecting rod. Figs. 2 and 3 are drawings in which the portions are functionally dissected. It is evident that to support a given pressure the piston end (Fig. 2 a) will require to be of a thickness proportional to its diameter, this applies whether the form is flat as shown, or conical, or whatever its shape; its mass will therefore vary as D^3 . The piston ring, with its containing "junk" (Fig. 2 b), is also a part that evidently should preserve its geometrical proportion, its mass is in any case independent of the stroke, and therefore varies as D^3 . The small and large ends of the connecting rod (Fig. 3) are also unaffected by the stroke, and their linear dimension requires to increase directly as D , the same applies to the gudgeon pin and the lugs and belt of metal by which the connecting rod stresses are taken by the piston (Fig. 2 c); the mass of all these parts therefore varies as D^3 . The only remaining portions are,—the shank of the rod, which evidently varies as $D^2 \cdot S$; and the "shell" of the piston (Fig. 2 d), which is a rather doubtful quantity. If we assume that the length of the piston varies in the direct ratio of its diameter, then this also falls into line with the majority, and varies as D^3 ; this probably is not exactly true, but very nearly so.

We thus have the shank portion of the rod varying as $D^2 \cdot S$ and the remainder of the major reciprocating parts as D^3 , and as the

shank only constitutes about $\cdot 15$ of the total mass,* I propose to ignore the exception and take the mass of the reciprocating parts as a function of D only, and as proportional to D^3 . The error will be small; such as it is it will favour the short stroke.

FIG. 2.

FIG. 3.

Let R = revolutions per second, then :—

$$\text{H.P. varies as } D^2 \cdot S \cdot R, \quad (3)$$
 and maximum inertia force *varies as* $m \cdot R^2 \cdot S$, but this force is sustained

* Compare Appendices II. and V.

by an area (the area of the rod), which *varies as* D^2 or stress *varies as* $\frac{m R^2 S}{D^3}$, but m *varies as* D^3 , so that stress *varies as* $D \cdot R^2 \cdot S$ which by the conditions is constant. We can thus eliminate R in H.P. equation (3) by dividing by the square root of the constant $D \cdot R^2 \cdot S$, and we get—

$$\begin{aligned} \text{H.P. varies as } & \frac{D^3 \cdot S}{\sqrt{D} \cdot \sqrt{S}} \\ \text{or,} \quad \text{H.P.} = & c \cdot D^{1.5} S^{.5} \text{ or, } c \cdot D \cdot \sqrt{D} \cdot \sqrt{S} \quad (4) \\ & \text{where } c \text{ is a constant.} \end{aligned}$$

In order to show the possible magnitude of error introduced by the neglect of the shank of the connecting rod, I have also worked the above calculation on the basis of m varies as $D^{2.5} S^2$, which seems a sufficiently liberal allowance; we have

$$\begin{aligned} \text{Stress varies as } & D^{.8} R^2 S^{1.2} \\ \text{H.P.} \quad \text{,,} \quad \text{,,} \quad & \frac{D^3 \cdot S}{D^{.4} S^{.6}} = D^{1.6} S^{.4} \end{aligned}$$

so the influence of our assumption is not, at most, very serious. As a rating rule I should feel disposed to support any values lying between $n=1.5$ and $n=1.6$ in the L^2 law of § 5. Any value between these limits would not leave much opening for a “special design.” For the further purposes of the present paper I employ the form $D^{1.5} \cdot S^{.5}$.

§ 9. CONDITIONS OF LEAST WEIGHT.

It is evident that for any given *type* of design, there must be some particular proportion of stroke to diameter appropriate to the condition of *least weight*. By the term *type of design* as here used it is intended to convey the condition that, in different engines, the arrangement and function of the parts is identical, the parts compared each to each are situated in the same relation to each other, and are alike functionally.

The *proportion of least weight* may be different in engines designed on different plans, for instance, one engine with an open and another with a closed crank chamber; it may also be different where different materials are used for like parts of different density or thickness.

In order to determine the proportion of least weight for any given *type*, a trial design is first made, *preferably* about the proportions that are known to be near the mark. This design is then

dissected into its functional components after the manner in which we treated the piston and connecting rod in Figs. 1, 2, 3, and the weight of each component is separately considered as a variable, and expressed as a function of D and S . The results are then classified, all functions of like indices being lumped together, and the total weight calculated from the trial design is added up under each heading.

Either D or S is then eliminated (preferably the latter), by dividing by the appropriate power of the H.P. factor $D^{1.5}.S^{-5}$ which is of course constant, and so we have the weights of the different functional components expressed as functions of D alone, the total weight being the sum of the expression.

The next step is to differentiate in respect of D , and solve the resulting expression in the usual manner for zero value; the value of D so obtained will be the value appropriate to *least weight*. The corresponding value of S will then be obtained from the constant $D^{1.5}.S^{-5}$. The final design for least weight can then be made.

I have worked out an actual case for a motor which is admittedly somewhat rudimentary in its components. Fig. 4 is the trial design, purposely made of extreme proportion. Fig. 5 is the design of least weight. The calculations are given in Appendix II.

§ 10. INFLUENCE OF CHANGES IN THE DENSITY AND STRESS ON THE H.P. DEVELOPED.

We will now revert to the general expression (1) of § 4—

$$\text{H.P.} = \frac{\sigma^{1.5}}{\rho^5} . l^2 \times \text{constant}$$
—and discuss the influence of changes in the physical attributes of the materials employed, *i.e.*, variations of σ and ρ .

Translated into ordinary language the expression (1) shows—

That in similarly designed engines the H.P. varies as the 1.5th power, that is, as the cube of the square root of the stress, and as the square root of the density of the materials employed.

Now it is evident that the weight of the engine also will depend upon the variables ρ and l , and for the conditions of geometrical similarity the form of this expression is $W = \rho . l^3 \times \text{constant}$; (5) so that the H.P. per unit weight, which is the quantity which is of most interest to us— $\frac{\text{H.P.}}{W}$ —will be:—

$$\frac{\sigma^{1.5}}{\rho \times \rho^{.5}} \times \frac{l^2}{l^3} = \frac{\left(\frac{\sigma}{\rho}\right)^{1.5}}{l} (\times \text{constant}).$$

Let us denote the quantity $\frac{\sigma}{\rho}$ by the symbol ϕ , and term it the "specific strength" of the material; then we have—

$$\frac{\text{H.P.}}{W.} = \frac{\phi^{1.5}}{l} \quad (6)$$

We have now the question of weight-saving in a nut-shell. The above expression shows that to which I have already drawn your attention—the importance of sub-dividing the power unit by employing a multiplicity of cylinders of individually small size, for we have the *h.p. per unit weight inversely as the linear dimension*, the latter, l , being the denominator in the above expression. We can also see at once the importance of employing materials of high *specific strength*; the form of the expression shows that if we can, by employing *all round* a higher grade of material, say of 10 per cent. greater specific strength, we shall effect a saving of weight of approximately 15 per cent.*

Of course, it is not always possible to effect an improvement in the quality of the material in every part of a machine, and it is of considerable interest to us to ascertain where and how the saving in weight is most usefully effected.

§ 11. WEIGHT SAVING CONSIDERED IN DETAIL.

Let us, to fix our ideas, suppose that we have at our command two kinds of material, one of which has just four times the *specific strength* of the other; and let two carefully designed engines be built to the same specification, one from each kind of material. Now it is evident that, part for part, the one engine can be built one-fourth the weight of the other. There may be some slight difficulties in design, owing to the slenderness of some of the parts, but we can brush this difficulty to one side by supposing the difference of *specific strength* to be wholly due to a 4 : 1 *difference of density*; that is, σ remains constant.

So far we have accounted for the $\frac{\text{H.P.}}{W.}$ varying in the direct

* In this approximation, as in others subsequently given, the proportion stated is that of the differential coefficient, only strictly true for infinitesimal variations.

ratio of ρ only, but the one engine will not only be lighter than the other but it will develop more power, for its reciprocating parts will give rise to less inertia and the revolution speed can be increased. The extent to which the revolution speed can be increased is in the inverse ratio of the square root of the weight of the parts, or in the case in point the revolution speed can be doubled. Thus the H.P. of the lighter engine will become twice as

great as the heavier one, or its $\frac{\text{H.P.}}{W}$ will be 4×2 ; that is, eight times as great, which is $4^{1.5}$ in accordance with the equation.

We thus see that on the former supposition of a 10 per cent. improvement in the material, producing (approximately) a 15 per cent. improvement in the power-weight factor, 10 per cent. of this improvement is due to the direct lightening of the engine and 5 per cent. to the increased power derived from the higher revolution speed rendered possible.

It is thus evident that by far the greater importance attaches, relatively speaking, to the quality of the material employed in the pistons and connecting rods, for these reciprocating parts do not usually exceed 10 per cent. of the total weight of the engine, and attention given to this 10 per cent. is of as much effect as similar attention devoted to any other 50 per cent. of the engine. It is thus found advantageous to adopt the very highest class of material for pistons and connecting rods. For some years past I have employed a high grade of nickel steel both for the connecting rod stampings and for the blanks from which the pistons are turned, and I believe that the results would justify even more attention still being paid to the reduction of weight in these organs.

§ 12. A SECONDARY EFFECT.

There is a secondary effect which must not be lost sight of, and which results in a saving of weight which is not obvious from a mere inspection of equation (6).

We have seen that the change in the power-weight factor as due to $\phi^{1.5}$, takes the form of a saving of weight in the direct ratio of ϕ , and in an increase of power in the relation $\phi^{.5}$. But we may not want increased power, it is usually some stated power that is required, so that by equation (1) l^2 will require to vary inversely as $\phi^{.5}$, that is l varies as $\frac{1}{\phi^{.25}}$, substituting in equation (6) we have:—

$$\frac{\text{H. P.}}{W} \text{ varies as } \phi^{1.75}$$

under the conditions of stated H.P., that is to say, H.P. = constant. This may be alternatively expressed by saying that for a given H.P. (for an engine of given number of cylinders), the weight varies *inversely* as $\phi^{1.75}$.

The equation (1) may be written in the form :—

$$\text{H.P.} = \sigma \cdot \phi^{.5} \cdot l^2 \times \text{constant.} \quad (7)$$

In this form the σ relates to the stress in the working fluid, that is the cylinder pressure; taking this as constant we have h.p. varies as $\phi^{.5} l^2$, and when H.P. is constant we have $\phi^{.5} \cdot l^2 = \text{constant}$, or, l varies as $\frac{1}{\phi^{.25}}$, which substituted in equation (6) gives

the same result as before, *i.e.*, W varies as $\frac{1}{\phi^{1.75}}$.

We thus see that the saving of weight to be effected by employing high grade material is even more than we had hitherto concluded, so that a 10 per cent. higher specific strength would give about 17.5 per cent., instead of 15 per cent., as previously concluded. The earlier figure was perfectly correct so long as the linear dimension of the engine is the constant, instead of the H.P.

§ 13. CONDITIONS OF LEAST WEIGHT.—FURTHER DISCUSSION.

The present method of investigating the proportions of *least weight* is comprehensive, and includes such appurtenances as ignition mechanism, fly-wheel, &c., and may even be extended to include the whole power transmission and change speed gear.

Under these circumstances, it is evident that there are certain factors that may be ignored as being independent of the variables D and S , for example, the ignition organs, the valve heads, portions of the valve boxes, &c; that is, (a) parts that are *essentially constant* (example, *sparking plugs*), and (b) parts that are constant in respect of H.P., that is, vary as $D\sqrt{D}\sqrt{S}$ (example, *valve heads*), and which consequently would vanish automatically in the process of calculation (§ 9).

As a general rule, in practice the whole of the valve and ignition gear may be thrown out at the start, though there may be exceptional cases in which parts of the valve gear require to be taken into account.

§ 14. INFLUENCE OF FLY-WHEEL ON D/S RATIO.

It frequently happens that there are unnatural restrictions to the size of fly-wheel permissible. Such restrictions usually take the form of a diametrical limit which cannot be exceeded.

When there is no such artificial restriction, the limiting factor is the safe stress of the material of which the rim is composed, due to centrifugal force, the rim being *in tension*. It is well known that this results in there being a *safe limiting velocity* which depends solely on the material employed.

When the limiting condition is that of safe velocity, there being no artificial restriction, it is evident that, for a given percentage speed variation, the weight of the wheel will require to be directly proportional to the cylinder volume, that is, *varies as* D^2S .

When the size of the wheel is settled by a diametrical limit the problem becomes more complex, and the expression is derived as follows:—

Rim velocity *varies as* R , that is as $\frac{1}{\sqrt{S}}$;

therefore, *energy per unit mass varies as* $\frac{1}{S}$;

or, *weight per unit energy varies as* S .

But, *energy per explosion varies as* D^2S ;

or, *weight of rim varies as* D^2S^2 .

Thus, the form of expression for minimum value differs, and the resulting D/S proportions for least weight will be different according to the circumstances. *When there is a diametrical limit on the fly-wheel the least weight proportions will result in a shorter stroke than when there is no such limit.*

§ 15. PROPORTIONS OF LEAST WEIGHT AS AFFECTED BY THE NUMBER OF CYLINDERS.

In the foregoing investigation it has been supposed that the *number of cylinders* is a matter settled in advance. It has been shown that on the basis of *geometrical similarity* the smaller the units the less the weight, so that any attempt to solve the problem of least weight on the broad basis would give the number of cylinders as infinite, or in practice the greatest number that the circumstances allow.

Under actual conditions, a limit is reached in the number of cylinders that can be employed, even from the point of view of

weight saving alone. Not only do certain organs, such as those of ignition, constitute a *constant per cylinder* that sooner or later neutralises any gain that would otherwise accrue, but the available methods of manufacture are not perfectly elastic, and it is not commercially possible otherwise to maintain the condition of *similarity*; the minimum thickness, for example, that metals will flow in casting, is in some cases a limiting factor.

By an elaboration of method it would doubtless be possible to take account of all these incidental factors, but the complication would hardly be justified. In the absence of any complete solution certain general conclusions may be drawn.

The greater the number of cylinders for any given h.p. the less energy storage required, and therefore the less fly-wheel weight.

Now we have either—

(a) No restriction as to diameter and fly-wheel weight *varies as $D^2 \cdot S$* or for H.P. constant, *varies as $S^{\frac{1}{2}}$* .

(b) With diametrical restriction, weight *varies as $D^2 \cdot S^2$* or for H.P. constant *varies as $S^{\frac{1}{2}}$* .

In either case an increase in the number of cylinders results in a diminution in the importance of a quantity tending to shorten the relative stroke, consequently the proportions of least weight will vary in some degree with the number of cylinders, and the greater the number of cylinders the longer will the stroke become in proportion to the diameter.

The increase in the relative length of stroke will be greater when there is a restriction as to diameter than when there is no restriction.

§ 16. THE RATING RULE.

One of the collateral objects of the present paper is the investigation of the *rating rule*, a somewhat vexed question amongst those concerned with the practical trial of motor vehicles and the organising of competitions.

The difficulty of framing a *rating rule* is not a matter peculiar to the automobile, it is a problem of no less difficulty in the yachting world, and the experience of yacht rating is instructive and, though not immediately to the point, there is much that is analogous in the two dissimilar sets of conditions.

In the early days of yacht racing, when boats were built for actual utility, for cruising or pleasure purposes, the framing of a

rating rule, to appropriately handicap the larger craft, presented no difficulty. The similarity of purpose resulted in a general similarity of form, and almost any measurement properly employed would serve as a basis. When, however, owners commenced to have boats built specially for racing, at the expense of other sea-going qualities, designers were not slow to take advantage of any weak points in the system of measurement, and the shape of the hull became fearfully and wonderfully *distorted*, and every new rating rule that is invented results in some *new variety of distortion*.

The same conditions obtain with certain variations in the case of the automobile. It is the manufacturer that wants to win races and competitions instead of the owner, and it is the manufacturer who tries to cheat the rating rule; the conditions are otherwise very similar.

Now, it is highly undesirable that the design of the petrol car should be pulled about by changing fashions in the way of rating rules, and it, therefore, behoves us as automobile engineers and designers to thresh this matter out once and for all; it is not a matter in which perfection is attainable; any rating rule whatsoever is an *addition to the conditions of the problem of automobile design*, and as such must result in *distortion*; all that we can secure by intelligent forethought is that the rating rule is of such a nature that the *distortion* is of a beneficial kind, or at least that it shall be harmless.

§ 17. THE RATING RULE—*Continued*.

The question of the effect of a rating rule in producing *distortion* in design has already been touched upon in § 7, and the possibility of undesirable kinds of distortion has been pointed out. The employment of the cylinder measurements as a basis tends to make the designer give very free ingress and egress to the mixture and exhaust gases, a most desirable feature as tending to a reduction of petrol consumption.

It has been proved on the hypothesis assumed (*i.e.*, that the h.p. depends upon the variables l , ρ and σ) that:—

(1) The H.P. rating rule must obey the L^2 law—that is to say, must be based on a quantity of the dimensions $= L^2$.

(2) That if based upon the two linear quantities, diameter D and stroke S , it must be of the form $D^n \cdot S^{2-n}$, where the appropriate

value of n deduced from mechanical considerations is from 1.5 to about 1.6.*

Departures from (1) lead to *distortion* in the direction of an increase or decrease, as the case may be, in the number of cylinders; where the index is in excess of 2 it pays to increase the number of cylinders to the highest possible limit, and *vice versa*.

Departures from condition (2) lead to a distortion in the proportions of bore and stroke. When the value of n is increased it pays to build long-stroke engines; when the value of n is decreased the short-stroke engine has the advantage. So long as the value of n is chosen between the limits given, it is doubtful whether a designer could obtain any material advantage by adopting extreme proportions.

Taking everything into account, the simpler form of expression† appears the more desirable, *i.e.* :—

$$\text{H.P. varies as } D \cdot \sqrt{D} \cdot \sqrt{S} \text{ or } \sqrt{D^3 S}.$$

§ 18. THE RATING RULE (*continued*).—THE H.P. CONSTANT.

We have so far discussed the *form* of the expression on which the maximum power output depends; we have now to discuss the question of the *constant* C which will be appropriate to the conditions, so that :—

$$\text{H.P.} = C \cdot \sqrt{D^3 S}.$$

The expression, so far as we have gone, is independent of the *cycle* of the engine, in fact it might apply to a steam or other fluid pressure engine, the constant C being adjusted to suit.

In the case of the Otto cycle, at present almost universal, where D and S are given in inches C may be taken as .4, giving an ordinary maximum B.H.P. value *per cylinder*. For other forms of internal combustion engine appropriate values of C may be obtained based upon the cylinder pressures ordinarily attained in the particular cycle employed and the frequency of the impulses.

I do not think that it is expedient to attempt to run steam cars, and those propelled by the internal combustion engine, *in one class*, especially in hill-climbing contests, and though the proposed rating

* There is no theoretical objection to the value of n changing from point to point along the H.P. : D/S curve so long as at every point the sum of the indices is = 2. (See Appendix II.)

† Compare Appendix V.

rule may be applied to the steam car, the constant being made proportional to the boiler pressure, such rating should be only employed for the purpose of comparison with other steam cars.

The *constant* should be looked upon as a quantity liable to alteration to meet new conditions such as might arise from developments either in design or in the *cycle*. It is evident that, even if no departure be made from the Otto cycle, it may be found possible to greatly enhance the power obtainable from a given size or *rating* of cylinder by some artificial means, such as that proposed by Daimler in one of his very early patents, *i.e.*, super-compression by surfeiting the cylinder at the end of the suction stroke. Under these conditions, some check may have to be applied by the responsible authorities, which will take the form of a *schedule* or *tariff* prescribing appropriate constants.

It is well known that certain compact forms of combustion space tend to higher mean pressures and greater economy, and without doubt such forms will give an advantage to competitors who adopt them. So long as no corresponding disadvantage exists such variations should be encouraged rather than otherwise, for that which tends towards higher mean pressure for a given composition of charge also tends to higher thermal efficiency. If, however, it should be found *by experience* that in the race for high pressures some undesirable forms of valve gear come into vogue, such as might be found to possess grave disadvantages in the hands of the public, it might become necessary to adopt special legislation, *by scheduling a special value of constant*.

The above is given as an example of the manner in which a rating rule of *correct form* may be adapted to meet all contingencies; it is not a very likely example but one that illustrates the point. If the rating rule is not of the *correct form*, no reasonable alteration of the constant will make it serve.

§ 19. SOME UNACCOUNTED FACTORS.

In equation (7), § 12, H.P. *varies as* $\sigma \phi^{.5} l^2$, it has been pointed out that the symbol σ relates specifically to the stress or compression in the working fluid, and if this vary independently *as a function of* l we may find an infraction of the L^2 law.

Such an infraction does not constitute a violation of the dimensional theory, for in supposing σ to become a function of l independently of its *dimensions* as stress $\frac{M}{L.T^2}$ we tacitly invoke

some further physical quantity or quantities, such, perhaps, as *viscosity*, not represented in our dimensional equation.

Mr. Dugald Clerk has shown that a difference in linear size of about $2\frac{1}{2} : 1$ gives a difference of mean pressure of about 9 per cent.,* or we have σ varies as l^{-1} approximately. There is some doubt whether this is always the case, or whether in the particular series of experiments some inexactitude in the conditions of similarity may have had some influence; in any case it is generally admitted that the larger the cylinder the higher the pressure obtainable for mixture of given composition, and if we accept the above figures and embody σ as a function of l in equation (7) we have:—

H.P. varies as $\phi^{\cdot 5} l^{2\cdot 1}$ an incomplete expression giving the extent of the probable departure from the L^2 law.

If we modify expression (2) to correspond we have:—

$$\text{H.P. varies as } D^n S^{2\cdot 1-n}$$

In view of the doubt that at present exists as to the extent of the influence of size of cylinder on mean pressure, and the fact that on the most reliable information at present available, the magnitude of this influence is small, it is dubious whether anything material is gained by introducing this factor into the rating expression. It would seem safer at least for the time being to accept the L^2 law implicitly.

Another unaccounted factor is that introduced from considerations of *mechanical efficiency*. In general, we are accustomed to expect the mechanical efficiency of a large engine to be better than that of one of small size; it is doubtful whether this is well founded. It is quite probable that the higher efficiency of a large engine is an actual fact, but that this is in part due to the greater care in design and construction that is bestowed on works of importance. On the other hand, there is every probability of a real effect of sensible magnitude, owing to the viscosity of the lubricant, and that of the working fluid. This, for engines, *geometrically similar* will be detrimental to the efficiency of the engine of smaller size, and for engines of different D/S ratio, it will be more detrimental to the efficiency of the engine of relatively short stroke.

Under all circumstances, the short stroke engine is at some slight

* "Limits of Thermal Efficiency in Internal Combustion Motors," Proc. Inst. C. E., 1907. (See Table, p. 11.)

disadvantage on the score of mechanical efficiency, for its main crank bearings and connecting rod heads consume about the same energy per revolution for a given revolution speed as a longer stroke engine of the same cylinder diameter. This is probably one reason why the stroke is proportionately so much longer in the gas-engine, where economy is the all-important factor, and weight is of secondary importance.

APPENDIX I.

Length, L. Mass, M. Time, T.

Area, $L \times L$	=	L^2
Volume, $L \times L \times L$	=	L^3
Velocity	=	$\frac{L}{T}$
Acceleration, $\frac{V}{T}$	=	$\frac{L}{T^2}$
Force, $\frac{M V}{T}$	=	$\frac{M L}{T^2}$
Stress or Pressure (<i>Force per unit area</i>)	=	$\frac{M}{L.T^2}$
Energy = <i>Force</i> $\times$ <i>Length</i>	=	$\frac{M.L^2}{T^2}$
Power = <i>Energy per unit time</i>	=	$\frac{M L^2}{T^3}$
Density = <i>Mass per unit volume</i>	=	$\frac{M}{L^3}$
Viscosity	=	$\frac{M}{L.T}$

APPENDIX II.

CALCULATION FOR LEAST WEIGHT.

BASIS : H.P. varies as $\sqrt{D^3S}$.

Sample proportions chosen for trial design, 4 inch diameter $\times$ 6 inch stroke.

Trial design given in Fig. 4.

The flywheel is designed as suitable for a single cylinder engine such as to permit of a 5 per cent. speed fluctuation at full load at 600 revolutions per minute, that is, at about half maximum speed.

The functional parts of the engine are dissected out as in the following table, and classified according to the manner in which their weight varies in relation to the variables D and S, thus :—

TABLE OF VARIATION OF PARTS.

<i>Varies as D³.</i>					<i>Weight.</i>
Cylinder end	.	.	.	.	1.73
Water jacket end	.	.	.	.	1.84
Water jacket floor	.	.	.	.	.80
Cylinder "foot"	.	.	.	.	3.90
Piston end	.	.	.	.	.58
Piston "junk"	.	.	.	.	1.70
Piston belt	}				3.00
Piston pin					
Connecting rod, small end					
Crank pin	.	.	.	.	3.50
Total	.	.	.	.	17.05
<i>Varies as D²S.</i>					<i>Weight.</i>
Cylinder shell	.	.	.	.	10.8
Water jacket shell	.	.	.	.	8.6
Frames and bolts	.	.	.	.	8.2
Connecting rod shank	.	.	.	.	2.6
Crank webs (portion only)	.	.	.	.	2.5
Flywheel rim	.	.	.	.	90.0
Flywheel boss	.	.	.	.	6.0
Flywheel spokes	.	.	.	.	14.0
Total	.	.	.	.	142.7

<i>Varies as $D^{\frac{8}{3}} S^{\frac{1}{3}}$.</i>	<i>Weight.</i>
Crank neck and main bearings	6.0
? Piston shell	.8
Total	6.8

The small number of groups is largely due to the simplicity of the design, but even in a much more fully developed design it is

FIG. 4.—Trial Design.

FIG. 5.—Least Weight.

surprising how great is the percentage that falls into one or other of the first two groups.

The basis on which the index values for the last group are founded is that of making the diameter of the journal appropriate to the torque, and the projected area of the bearing surface proportional to D^2 .

The piston shell (refer Fig. 2) is a quantity whose variation is very difficult to rationalise; it is evidently something between D^3 and D^2S , and it is included under the same heading as the crank journal as a matter of convenience.

When in doubt as to the relations of any part the designer can

make two or three designs for that part and test his indices of variation, or establish such indices empirically.

In the following tabulated calculation the totals from the analysis of variation are first expressed in terms of the values of D^3 , D^2S , &c., to which they are proportional; these are then divided by powers of the h.p. constant $\sqrt{D^3S}$ and multiplied by the *numerical values* of the same quantity, calculated from the data of the trial design. The powers of $\sqrt{D^3S}$ chosen are such as will eliminate S from the expression, and we then have the weight of each group of parts expressed in terms of D alone. The remainder of the calculation then consists of solving the sum of the expressions so obtained for minimum value.

$$\begin{array}{lcl}
 D^3=64 & \left| \frac{17.05}{64} = .266D^3 \right. & . \quad . \quad .266D^3 \\
 D^2S=96 & \left| \frac{142.7}{96} = 1.479D^2S \times \frac{384}{D^3S} = \frac{568}{D} \right. & \\
 D^{\frac{3}{2}}S^{\frac{1}{2}}=73.3 & \left| \frac{6.8}{73.3} = .0928 D^{\frac{3}{2}}S^{\frac{1}{2}} \times \frac{7.27}{DS^{\frac{1}{2}}} = .674 D^{\frac{1}{2}} \right. &
 \end{array}$$

Solve for minimum value—

$$W = .266 D^3 + .674 D^{\frac{1}{2}} + \frac{568}{D} \quad (1)$$

Differentiating, we have—

$$\frac{dW}{dD} = .798 D^2 + 1.122 D^{\frac{1}{2}} - \frac{568}{D^2} = 0 \quad (2)$$

From which we obtain by trial or plotting—

$$D = 4.97$$

But we know that H.P. constant

$$\sqrt{D^3S} = 19.6$$

Whence—

$$S = 3.13$$

Or approximately— $D = 5 \quad S = 3\frac{1}{4}$

The re-design of engine to comply with these conditions is given in Fig. 5.

It will be found, when re-designing an engine, to comply with the conditions of least weight, that impossibilities sometimes arise; parts that are presumed in their variations to be independent are found to interpenetrate or “foul.” Examples of this occur in Fig. 5. Note the piston-junk and belt that coalesce in this re-design. It is for this reason that it is important to select reasonable proportions for the trial design, for the nearer the trial is to the final design the less trouble arises from this cause. If the

first trial is found to be wide of the mark, a second trial design may be found necessary.

An inspection of the components of the groups of parts discloses the fact that the fly-wheel is a very serious factor in the particular type chosen; it accounts for 110 lbs. out of a total of 142 lbs. under the D²S heading.

Now, in a multi-cylinder engine, the fly-wheel weight may be reduced, and the fly-wheel *weight per cylinder* very much reduced. Let us take the limiting case, and presume the number of cylinders so great that the fly-wheel may be virtually discarded; our expression becomes—

$$W = .266 D^3 + .674 D^{\frac{3}{2}} + \frac{115.6}{D} \quad (3)$$

Differentiating for minimum value, we have—

$$\frac{dW}{dD} = .798 D^2 + 1.122 D^{\frac{1}{2}} - \frac{115.6}{D^2} = 0 \quad (4)$$

Whence—

$$D = 3.4 \\ S = 9.77$$

A most striking difference.

We are unable to accept this result as final, owing to the fact that the great length of stroke invalidates our assumption as to the negligibility of the weight of the connecting-rod shank, and the investigation will be subsequently continued on a revised h.p. basis.

Before proceeding to the further investigation it is of interest to refer to Fig. 6, in which plottings are given of the weight as a variable dependent upon D, as in equations (1) and (3) of the present Appendix.

Lines are given showing not only the variations of total weight, but the individual variations of the components. The minimum points on the two curves, with and without fly-wheel, are given, as determined by differentiation. It will be seen that if a variation of 10 per cent. from minimum value of W is permitted, an engine of 4-in. cylinder diameter by 6-in. stroke is such as the "trial design" fulfils the conditions for either the single or multi-cylinder. We may now proceed to show the revision of the h.p. factor results in much less variation of proportion being actually required than might be supposed from the foregoing computation.

In Fig. 7 we have the weight of the reciprocating parts represented for *variation of stroke*, D being supposed constant. The curve is composed of two portions, one of which, i.e., parts that

vary as D^3 , is constant, and the other, which varies directly as the stroke, *i.e.*, the shank of the rod $\propto D^2S$.

Least Weight.

FIG. 6.—Weight plotted as function of Cylinder Diameter.

Abscissæ = Diameter.

Ordinates = Weight.

FIG. 7.—Weight of Reciprocating Parts.

D = Constant.

S = Abscissæ.

W = Ordinates.

The proper index values of D and S have been calculated for this "curve," and are given in the figure for values of S equal to $\cdot 5D$, $\cdot 75D$, D , $1\cdot 25D$, &c. It will be seen that the departure from the

simple expression $\sqrt{D \cdot S}$ becomes greater the longer the relative stroke, and though the simple expression is probably good enough for a Rating Rule, it is insufficiently accurate for the determination of the proportions of least weight.

With the values given in Fig. 7 it is possible by repetition of the process to approximate as closely as may be desired to the truth. When the results of the first computation give a D/S ratio different to that to which the h.p. indices (the n of expression $D^n \cdot S^{2-n}$) relates, a new index value should be taken in harmony with the ascertained D/S ratio and the calculation repeated. This may be done as often as deemed necessary.

The revision of the h.p. indices results in the case of the engine without fly-wheel in a much shorter relative stroke than given by equation 4. A first approximation on the basis $H.P. \propto D^{1.65} \cdot S^{.35}$ is found to give—

$$D = 4.4 \qquad S = 3.87.$$

APPENDIX III.

HISTORICAL NOTE.

THE investigation that forms the subject of the present communication was roughed out in the early part of 1905, but put on one side owing to pressure of other work.

In June of 1905 I was asked by the Secretary of the Midland Automobile Club to suggest an h.p. rating formula. I refrained at first from giving my views, not being at the time prepared to publish the then unfinished investigation. On having it represented to me that the matter would not become the subject of debate, and that a sound basis of computation was urgently required, I was induced to deal with the matter in a letter to the Chairman of Committee as follows, in which the matter is dealt with tersely, and substantially embodies the conclusion of the present paper :—

53, Hagley Road, Edgbaston,
Birmingham.

June 15th, 1905.

HAMILTON BARNESLEY, Esq.,
Messrs. The Lanchester Motor Co.,
Montgomery Street, Sparkbrook.

DEAR BARNESLEY,

As requested, I give you below my formula for rating cars for hill-climbing.

I think all are agreed that it is undesirable to employ a formula that *favours unduly* any extreme proportions of *diameter stroke* ratio. The two formulæ that have been used recently are both open to serious objection on this ground :—

The basis of *cylinder volume* is bad owing : *Firstly*, to the fact that it unduly favours small-sized cylinders. *Secondly*, it unduly favours the short-stroke quick revolution engine ; and *thirdly*, it is fundamentally wrong on purely scientific grounds as being a *cubic* or *volumetric* quantity, instead of being a *square* or *superficial*

quantity. In scientific phraseology the formula is of the wrong "dimension."

The basis of *piston area* is wrong, as favouring the long-stroke engine. It is assumed in this rating that the *piston speed* is the practical limiting factor. This is a rough and ready basis sometimes used by engineers for purposes of comparison, but is only applicable where the proportion of the engines do not differ seriously. The real limit is *piston inertia*, and it is susceptible of proof that the correct basis for the formula should be $D \times \sqrt{D} \times \sqrt{S}$,

where D is diameter in inches.

„ S is stroke in inches.

This quantity should be divided by a constant, to give an h.p. rating if such is desired, a suitable constant is 2 to give about the *outside* maximum possible, or say 3 to give the *normal* maximum power.

The result, of course, requires to be multiplied by the number of cylinders.

A few examples are given as an illustration :—

Diam. D.	Stroke S.	H.P. Outside Max.		H.P. Normal Max.	
		1 Cyl.	4 Cyl.	1 Cyl.	4 Cyl.
4 "	4 "	8	32	5.33	21.33
4 "	3 "	6.94	27.7	4.62	18.5
4 "	6 "	9.8	39.2	6.54	26.2

F. W. L.

Unfortunately the assurances I had received were not justified, for immediately the formula was proposed, certain members of committee objected, although they had not previously shown signs of having any pronounced views on the subject, and instead of my formula being accepted, as I had understood was morally certain to be the case, copies of my letter were taken and discussed by third parties.

This procedure was not, however, without some advantage, for it elicited the following communication from Mr. Remington, of the Wolseley Company, which by the courtesy of the Hon. Sec. of the M. A. C. I am able to publish, and to which, having had no previous opportunity, I take the present occasion of replying. I have numbered the paragraphs of Mr. Remington's letter to avoid necessity for repetition.

A.A.R./C.

Dated: *July 1st, 1905.*

Copied: *July 7th, 1905.*

1. As discussed on Thursday night, I consider that for obtaining the probable actual b.h.p. of a petrol engine neither Mr. Sturmeys or Mr. Lanchester's formulas are satisfactory. The former gives values which are too low for a long-stroke engine, and too high for a short-stroke, thus, in a handicap, unduly favouring engines having a long stroke.

2. On the other hand, Mr. Lanchester's formula gives too high a value for long strokes, and too low a value for short strokes, thus unduly favouring short-stroke engines.

3. It is evident that Mr. Lanchester based his formula on a small cylinder engine having equal bore and stroke, and considered that the piston speed would vary directly as the inertia of moving parts, *i.e.*, assuming that the weight of the moving parts remain constant for any length of stroke, providing the bore remains the same. He also made too great an allowance for the loss of efficiency in larger engines due to various well-known causes.

4. In the light of this investigation, I have gone further into the matter, and find that undoubtedly for all sizes made with petrol engines the h.p. will vary directly as the square of the diameter, and also as some function of the stroke. As the weight of the moving masses increases with the length of stroke, the square root is too high a function to decide the variation of power by varying the stroke. I find a much nearer value is the cube root of the stroke, that possibly errs on the side of showing slightly too much power for a very long stroke. Having decided the variables, it is necessary to adopt some constant to bring the relative value to an actual one to suit the existing engines.

5. I find that a suitable constant is 5.

6. I append a table giving the values for various sized engines working on this formula. I have used two constants for the formula I propose, the one having a value of 4.75 I have put in as it shows the relation between the three formulas, giving the same h.p. for an engine having 4 ins. by 4 ins. cylinders. This shows the relative proportions of power given for various sized engines for the three proposed formulas.

7. I consider that the formula under column D. is the most satisfactory, and nearest correct.

8. A striking instance of the inaccuracy of the formula under column A. is shown by the second and third items.

9. It is evident that if two cylinders 4 in. by 4 in. give 10·66 h.p. as shown, one cylinder of the same stroke having exactly twice the area, *i.e.*, 5·66 in., must, in the nature of things, give twice the h.p.

10. Column A. shows that this formula does not do so, and that the factor $D \times \sqrt{D}$ is not satisfactory for the determination of power due to cylinder area. On the other hand, items 5 and 6 show the shortcomings of the formula under column B. It is evident that an engine having twice the stroke of another must give more h.p., but as previously shown, not so great a difference as shown under column A.

11. Taking into account these points, and also considering them on a theoretical basis, I consider that the formula under column D. is the most satisfactory, and also it gives results, as far as my experience goes, that are well in accordance with practice.

	Bore.	Stroke.	No.	A.	B.	C.	D.
1	4	4	1	5·33	5·33	5·33	5·06
2	4	4	2	10·66	10·66	10·66	10·12
3	5·66	4	1	9	10·66	10·66	10·12
4	4	4	4	21·33	21·33	21·33	20·26
5	4	3	4	18·5	21·33	19·4	18·43
6	4	6	4	26·2	21·33	24·4	23·18
7	7½	6	4	62	68	78	74·1
8	7½	8	4	72	68	86	81·7

A. A. R.

7/7/05.

Mr. Remington's objection, p. 2, is dealt with in the present communication. There is theoretically some slight ground for this objection, but only of the magnitude indicated.

It is doubtful whether this would not in practice be more than compensated by the lower mechanical efficiency of the short-stroke engine. In any case I am not greatly adverse to some slight adjustment of the indices as in sect. 8 and Appendix II., except on the grounds of complication.

3. Mr. Remington is wrong in supposing my formula to have been based in the manner stated. Also I have made no allowance for loss of efficiency in large engines due to "well-known causes."

4. Mr. Remington's formula H.P. varies, as $D^2.S^{\frac{1}{2}}$ does not con-

form to the L^2 law; the expression $D^2.S^{\frac{1}{2}}$ is of the dimensions $L^{\frac{3}{2}}$, which is higher even than can be justified on Mr. Clerk's measurements of cylinder pressure (compare par. 19), and therefore favours a multiplicity of cylinders.

8. Mr. Remington, referring to the table appended to his letter, refers to what he terms a "striking instance of inaccuracy" in my formula. He supports his contention in par. 9. His train of reasoning is plausible but unsound, for the mass of the larger piston will increase in greater ratio than its area, and it will not be possible to run an engine of 5.66 in. cylinder diameter at the same revolution speed as one 4 in. diameter, the stroke being equal. This is the assumption tacitly made: *Engine No. 2* would give more h.p. than engine *No. 3* (other things being equal), and the fact that Mr. Remington's formula does not show this difference alone suffices for its condemnation. The first conclusion in par. 10 is consequently in error. The second conclusion in par. 10 is a condemnation of the D^2 formula. In this I thoroughly agree with Mr. Remington. Par. 11 appeals to experience in support of Mr. Remington's formula, but there is room for a variety of opinion.

APPENDIX IV.

VARIOUS PROPOSED H.P. RATING FORMULÆ.

DURING the past few years, a variety of propositions have been made with a view to the solution of the rating difficulty.

Certain of these, though commendable in themselves, are difficult of practical achievement. As such may be cited the proposal to base the measure of h.p. on the consumption of fuel.

Confining ourselves to proposals based on cylinder measurement, we have the *volumetric rating* frequently adopted abroad in motor cycle competitions—the *quart d'un litre* class, for example; such a rating has more recently been advocated for larger vehicles by some correspondents. It has been shown in the present paper any such rule favours a multiplicity of small cylinders.

There is something to be said in favour of *volumetric rating* as a possible agency in the future development of the automobile engine.

We know that the cost of running a motor car is largely proportional to its weight, and, with otherwise equal durability, a saving in weight is a saving in running cost. Any rule, therefore, that encourages extreme lightness, not merely a *cutting of weight* but a *legitimate reduction of weight*, may exercise a potent influence on the design of the future.

Now a *volumetric rating* encourages a multiplicity of cylinders that is conducive to weight saving, and there may, therefore, be some future for a special class of competition run under volumetric rules.

This would be nearly equivalent to a *weight rating* in which designers are handicapped according to the *weight of their engine*, it being assumed that weight means h.p.

Such competitions would doubtless be won by “freak” machines, but in the development of such “freak” machines much valuable experience would be obtained in the direction of weight saving.

In any case competition under *volumetric rating rules* should be a thing apart from ordinary trials.

More recently the D^2 basis has been advocated and subsequently adopted by the Automobile Club, and known as the "A C rating."

This is of the right "*dimensions*," and would serve if all engines were of the same D/S proportion, but is manifestly absurd as an attempt to properly represent the h.p. This fact is clearly recognised in a circular issued recently to the trade suggesting that manufacturers should adopt the A C rating for purposes of catalogue description. It is ingenuously stated in this circular that the formula is *not* intended to be a scientific statement of H.P.

A more serious attempt to evolve a rational rule was made by Professor J. B. Henderson in the "Autocar" of December 15th, 1906.

This is based on the theory of similar structures, and Professor Henderson's conclusions are rather similar to my own, but no organised attempt seems to have been made to properly determine the value of the index n .

It would appear that Professor Henderson makes the tacit assumption that the measurement of the cylinder on its three co-ordinate axes are of symmetrical or equal importance, for his final expression is $(D^2 \times S)^{\frac{2}{3}}$ or $D^{1\frac{1}{3}} \cdot S^{\frac{2}{3}}$.

This is most assuredly an error, and arises from an assumption made in taking as a linear datum quantity the cube root of the piston displacement volume.

A warning may be added as to the "practical" testing of cylinder rating rules. Quite commonly lists are made of some of the cars entered for competition, and these are worked out according to the proposed rule and as to performance or by some other rule, or, perhaps, are figured according to the maker's catalogue, &c. Now a rule would require to be egregiously bad to show up under these circumstances, for the real values are not known usually to within 10 per cent. or so, and in general the D/S variation between one make and another is not great.

A rule that would carry as a logical consequence a diameter of 3 inches and a 12 inch stroke would not seem materially at fault under these circumstances; such "tests" are entirely delusive.

APPENDIX V.

FIG. 7, to which reference is made in Appendix II., shows the mass of the reciprocating parts to vary as follows:—

$$m \text{ varies as } D^3 + c D^2.S \quad (a)$$

In this expression from trial design I find that the constant c is about .3.

If we assume this form of expression, and proceed as in § 8 of the paper, we have the following:—

$$\text{Stress varies as } \frac{m.R^2 S}{D^2}$$

Substituting for m (as in (a)), we have—

$$\text{Stress varies as } R^2 S (D + c S) \quad (b)$$

which is constant. (For maximum permissible stress is constant.)

Therefore, by (3), § 8, we have—

$$\text{H.P. varies as } \frac{D^2 S}{\sqrt{S (D + c S)}} = \frac{D^2}{\sqrt{\left(\frac{D}{S} + c\right)}}$$

Substituting for the constant c the numerical value above given, we have—

$$\text{H.P. varies as } \frac{D^2}{\sqrt{\left(\frac{D}{S} + .3\right)}}$$

It now only remains to insert a constant appropriate to the horsepower unit, which I find for maximum H.P. to be about $\frac{1}{2}$, and we have—

$$\text{H.P.} = \frac{1}{2} \times \frac{D^2}{\sqrt{\left(\frac{D}{S} + .3\right)}}$$

This constitutes a rating rule that includes all quantities that are worth including, and which covers the whole range of D/S variation, and comprehends the changes of index value given in Fig. 7.

When writing the paper I considered that the full expression as here presented would be too complicated, but I find that for rating purposes such is not the case; it is, in fact, quite simple. For the purposes of least weight calculation the new form of expression is too inconvenient and results in mathematical complications. It is best for least weight computation to adhere to the procedure given in the text, and repeat as often as necessary. If the present expression could be employed, it would give the final result at once; but I have not found it reasonably possible.

Inspection of the complete expression is of considerable interest. If we suppose the stroke very short so that D/S is very great, and c relatively unimportant, we evidently approximate to the condition of c being negligible, and the expression becomes $D \sqrt{D} \sqrt{S}$. If we suppose that the stroke is relatively long so that D/S becomes negligible, then we have the A. C. form of expression, D^2 , divided by a constant. Both these results are in harmony with the conclusions reached in the body of the paper and in Appendix II.

THE DISCUSSION.

The PRESIDENT (Colonel R. E. Crompton): I need hardly call upon you for a vote of thanks to the author of the paper, it has been of such great interest to us, and as time is so pressing I will at once ask Mr. Russell Clarke to open the discussion.

MR. RUSSELL CLARKE: I rise with great diffidence to open the discussion. I have not had nearly enough time to thoroughly study this paper. It is a paper which gives one more to think about than any paper I have heard read for a long time. I certainly agree with our President that we are very much indebted to Mr. Lanchester for the able way in which he has dealt with this question. There are one or two points, however, that to my mind need elucidation. I will not put any definite questions for the moment, but what I would like to ask the author presently about is the assumption that is made on p. 158 in the paper. In that assumption the author has assumed that the horse-power of an engine can be expressed in a formula of the product of the length raised to a certain power, multiplied by density of the materials raised to a certain power, multiplied by the co-efficient of strength of materials raised to a certain power. Now I feel very doubtful about that assumption. Unless that assumption is right, of course, a very large proportion of the argument in this paper falls to the ground. I want to make a remark or two about the methods adopted by the author in this paper before I deal with that assumption, and before tackling that assumption mathematically. I suppose every member of this Institution who has ever done any calculations at all, has at times found himself with an equation at the end of his sheet of paper, and he wonders whether it is right or not. He says, "I will try it with dimensions," and thinks to himself, this is the velocity $\frac{L}{T}$, and checks it off with dimensions, and says, probably, "My equation is all right." This theory of dimensions is a very valuable instrument in testing whether a calculation is right or not. It is usual for anyone to do his mathematics first; he generally works his problem out, and then tests it with dimensions; but in this case I feel doubtful whether the author has fairly done his mathematics. What he has done is this. He has made an assumption, and he has put in certain assigned dimensions of the various

quantities, and he has used those dimensions as means for solving his equations. He has put down first what he has assumed in the form of an equation:—H.P. = $l^p \times \rho^q \times \sigma^r \times \text{constant}$.

He says to himself, "The horse-power is

$$\frac{ML^2}{T^3} = L^p \times \frac{M^q}{L^q T^{2q}} \times \frac{M^r}{L^r T^{2r}}."$$

He says, "My ρ is density, so it must be $\frac{M^q}{L^q T^{2q}}$, and my σ is stress,

that is $\frac{M^r}{L^r T^{2r}}$," and from that he arrives at the interesting result,

which I entirely agree with in the main, that the horse-power varies as the square of the linear dimensions. That result is to my mind very largely correct, and it also agrees with the rating rule formulated by the Automobile Club, which is, of course, that the horse-power varies or is determined actually by the square of the diameter of the piston multiplied by the number of cylinders, divided by a co-efficient which is sometimes 2, sometimes 2.5, sometimes 3. I want to criticise this assumption a little more. The author could arrive at that result without any further complications if he had simply said horse-power varies as l^2 , or some power of $L = \frac{ML^2}{T^3}$.

He would have got $P = 2$, that is, he would have got that the horse-power varies as L^2 . Of course, all this has been done in order to avoid dealing with the revolutions of the engine. I quite sympathise with anyone who wants to get rid of the revolutions, because they vary. I do not know whether it is not a better thing to assume a definite piston velocity than to do without revolutions. You know exactly where you are, and arrive at a rating rule something like that of the Automobile Club. It is better to do this than to try to avoid your mathematical difficulties. I am anxious not to be rude.

MR. LANCHESTER: I don't mind.

MR. RUSSELL CLARKE: I know I shall have a bad time afterwards when Mr. Lanchester makes his reply. But about this assumption, first of all I quite agree that the author is entitled to assume that the horse-power varies as some power of length. Let me deal with σ ; σ is the stress, which is lbs. per square inch or something of that sort. There is nothing in the author's operations on that afterwards to tell us what that stress is. It might as well be pressure at the end of the stroke as anything else. If he had said,

"I will determine the horse-power of the engine with a proper formula," and if he had said that σ represents pressure at the end of the piston stroke, he would have been perfectly right in going through with dimensional work to get out the same formula he has here. If you adopt these methods you may let l^p represent the length of the steering column, and ρ^a might represent the volts of the accumulator. With regard to σ I cannot think of any really complicated dimensions, but I do not want to be flippant. σ might be used to represent the price of the driver's clothes if that can be expressed physically in dimensions. I have seen a book, "The Electron Theory," by Fournier d'Albe, in which every possible conception is expressed in dimensions. I am certain the author of that book could express the price of clothes in dimensions. I am anxious to hear how the author can possibly say that this assumption that he has made is justified, and how when you have got an assumption like that you can treat it as if it were gospel truth, and apply to it the very delicate mathematical method that he has used. To my mind this method is only useful for testing whether what you have got as a result of mathematical calculations is right or not.

I pass from that point to the rest of the paper. The rest of the paper is extremely logical and most interesting. I feel that all that has been said in this paper about the lowest weight really applies perfectly well, whether what I have said in regard to the method adopted is correct or not. However, if you assume the L^2 law, that law can be arrived at in a much more straightforward way, by going at it "bald-headed" and saying, I will assume a definite piston velocity. In practice it is found, if you go through the best classes of petrol engines, that the piston velocity never varies more than about 10 per cent. from 1,000 feet a minute. That gives you something to go on, and when you have something to go on you have made an assumption you can understand. You know exactly where you are. I have read the paper with great care, and I am forced to ask what has become of the revolutions? These have been got rid of, and to my mind they have been got rid of by scamping the mathematics of this very difficult question. Perhaps I ought to withdraw that expression, but I do not mean it in any derogatory sense.

Mr. LANCHESTER: I don't mind, I shall have my turn presently.

Mr. RUSSELL CLARKE: Before sitting down, I will again join in

thanking Mr. Lanchester for putting the matter before us in such an interesting and original manner.*

Professor H. L. CALLENDAR: I think we may regard it as somewhat a sign of the times when one of our most scientific and leading designers adopts the theory of dimensions for working out a problem of this kind. Of course, those of us who are students of physical science know what a useful tool this theory of dimensions is, and that it will always give correct results for the variations of any physical quantity, provided only that the premises in the equations with which we start are carefully introduced, and none of the necessary factors are omitted. I believe that Mr. Lanchester has done that, and has introduced them in a simple and direct manner, with a result familiar to most engineers; viz., that horse-power varies as the square of the linear dimensions. He has given other interesting points, such as that the ratio of power to weight varies inversely as the linear dimensions, which follows as a corollary from the first statement. That is an interesting and important point in the design, not only of motors but of a great many other machines, particularly of flying machines, where the condition of minimum weight is all-important. It is shown that you can reduce the weight upon the principle of using a number of small units instead of a single big unit. By this practice power in proportion to weight is gained. As an example, we may take a four-cylinder engine of 5-inch bore. Such an engine is more than twice as powerful in proportion to its weight as a single cylinder of equal power. That is an important matter from some points of view; and if designers of flying machines were to realise that fundamental and important fact, we should get flying machines a great many years earlier than we are likely to get them. Most of the designs one sees seem to ignore this absolutely fundamental principle. I should like to say a good deal more about the interesting points raised at the end of Mr. Lanchester's paper, especially where he speaks of some unaccounted factors which may influence the rating rule—the variation of power with the size of the engine. But as I have promised to read a paper on a closely related subject next month, I think, as the time is rather late, I should like to defer any remarks I have to make on that point until next month. I feel grateful to Mr. Lanchester for the admirable way in which he has

* Some additional observations by Mr. Russell Clarke will be found on page 211.

put this question before us, and for the lucid manner in which he has enunciated the theory of dimensions.

Dr. H. S. HELE-SHAW: I do not know which to admire most, the courage of Mr. Lanchester in treating this subject in the light of the theory of dimensions—it is the first time I have seen a mechanical engineering problem treated that way, although it is quite familiar to physicists in dealing with electrical problems—or the way in which Mr. Russell Clarke attacks this particular application of the theory. Mr. Russell Clarke does not appear to understand the way in which the author gets at his argument. To me the author's reasoning on page 158 is quite clear; it quite satisfies me. However, I will leave Mr. Lanchester to deal with Mr. Russell Clarke himself, and also to explain where the disappearing T has gone to as a factor in the equation. I should like to remark, even at the risk of reiterating what has been said before, that we ought to be grateful to Mr. Lanchester for reminding us of the application of the theory of dimensions to engineering problems, and especially for bringing home to the younger members of the Institution the value of that theory. Had it been always appreciated it would have saved a great many people much trouble, especially in the early days of engineering, when the text-books published might have been freer from mistakes than they were. I remember receiving a prize on one occasion, a book, the author of which is dead, so I will leave his ashes in peace, but it was a text-book in which the data for engine calculations were proposed to be obtained by getting the details of engines of all kinds and working them out into various formulæ, without the slightest reference whatever to the general theory of dimensions. The result was that most extraordinary results were obtained, simply because the author did not understand the basis of sound calculations. It was facetiously remarked in regard to these results that the horse-power of a certain steamboat was correctly represented by the height of the funnel multiplied by the number of doors in the stokehole. We do not get text-books like that nowadays, but we do see outrageous things stated, I am sorry to say, in technical papers; things which would be absolutely avoided if the dimensional theory were recognised and acted upon. Thus we sometimes see such statements as “lbs. of coal per horse-power”; “knots per hour”; or “acceleration per second,” and so on. This sort of thing should not be, and I hope this Institution will lead us always to be careful, in discussing matters connected

at any rate with motor engineering, and to keep to correct methods of statement. I am sure that a paper like this is extremely valuable, if only from that point of view. I entirely agree with Mr. Russell Clarke, that the dimensional theory may be useful as a check. It further diminishes difficulties in dealing with different systems of units. We need not, if we correctly apply this theory, be so much troubled about changing from one system of units to another in daily work, or be in a desperate hurry to adopt the metric system. We need not change our system of units, if we only understand what our units are and what the theory of dimensions is.

I will now come to the immediate subject of the paper. It is extremely interesting to see the way in which the Automobile Club arrive at their formula, which is exactly the same in principle as Mr. Lanchester's. It is $C \times D^2 \times$ the number of cylinders. Mr. Lanchester's is practically the same, as long as he assumes that engines are SIMILAR in form. In regard to the Automobile Club's formula, I was curious to look at some of the results of the working out of that formula, and was struck with the extraordinarily close way in which the Automobile Club formula agrees with the actual results of engines as worked out upon this formula. I need not read these results, though I am not sure that they were ever published; but they show that, taking about thirty leading engines, there is not more than from 10 to 12 per cent. variation, which shows that engineers in practice arrive at fairly good results in rating their own engines. The Automobile Club concluded that they would ignore T because the average number of revolutions was found to be about 1,000 per minute, or (assuming a 6 in. stroke) the piston speed 1,000 ft. per minute. That is the reason given by the Automobile Club for ignoring T . I am leaving Mr. Lanchester himself to explain why he does not include T in his own results.

The result of rating by this formula (and this is the last remark I have to make) should be, as Mr. Lanchester shows, to improve the engine. It may lead to some alterations, but they should be all in the right direction. We all know that the rating rules for yachts, which was to multiply the sail area by the length of the water line, worked well for a time until they were abused. In the present case I hold with Mr. Lanchester in his treatment of the subject, and, if you will allow me to say so, it indicates that touch of genius in Mr. Lanchester's work which is so characteristic of all he does. He has seized upon one of the only mechanical

engineering problems in which he could apply the reasoning of dimensions satisfactorily, viz., the automobile engine. Here you can safely say that it pays you to use the very best material for the working parts. Under the rating rule, whether you use Mr. Lanchester's formula or that of the Automobile Club, any alteration will be in the direction of using better material and getting more horse-power, because, not having to bring in the question of velocity, and only introducing the diameter of the cylinder, the higher the number of revolutions the more higher the result you get for a given rating. Thus the man who uses the better steel (chrome or nickel steel, or the very best he can get) for his piston rod, and so on, can afford to run his engine at a higher speed than the man who uses more inferior material. He can afford to reduce the weight of his engine. If he uses automatic lubrication he can afford to run at a higher speed and get more actual result out of his engine. This applies both to touring and to racing cars, because here are two cars rated equally either by Mr. Lanchester's rule or by the Automobile Club rule, and the man who is doing the best work, who is fitting the best parts, who is going to the expense of hardened steel bearings, and so forth, can afford to run his engine at a high speed; and if he automatically lubricates his engine, uses the best steel he can get for his piston and other parts, he is able to reduce the weight of his engine and the weight of his car, and is able to win races, and also able to supply a better touring car than others who do not take these matters into consideration; so that in every way we may be thankful to think that the rating of the Royal Automobile Club, and also the rating rule laid down by Mr. Lanchester himself, and arrived at by a totally different method of reasoning, agree, as they should do, and both equally will tend to the improvement of the automobile. I will now sit down, thanking Mr. Lanchester for the way in which he has treated this subject, raising it, as he has done, to a higher plane than that upon which it is generally discussed.

MR. DUGALD CLERK: Mr. Lanchester is a very old friend of mine of nearly twenty years' standing. When he was a young engineer I speedily came to the conclusion that he had a brilliant way of treating engineering and scientific subjects, a way which has always shown a touch of genius. Coming along from dinner to-night with the members of the Council I was very much distressed by Mr. Lanchester asking me if I had read his paper. I was sorry

to say I had not done so. He was desirous that I should say something about it. Then shortly afterwards, referring to a paper of mine, I asked him if he had read my paper, and he said he had not, so that on that point we are now even. So far as this paper is concerned I wish I had read it before I came to the meeting, but I have listened to it with all the greater interest, perhaps, on account of hearing it fresh, and I must say I admire the treatment of the subject very much. I admire it all the more because Mr. Lanchester is dealing with a problem which I have had to deal with myself to a considerable extent. I believe that to some extent I was responsible for the rating rule which has been adopted by the Automobile Club. I may say, however, that in regard to that responsibility I have always been at variance with my friend Mr. Lanchester, who thinks that rating rule is all wrong. In some ways undoubtedly it is. In framing that rating rule, as far as I was concerned, what I did was something like what Mr. Russell Clarke described as going at it bald-headed. I took a constant piston speed. For this purpose I took a number of engines in actual practice, some of which I tested myself and others I got tested for me by other engineers, and on the results of these tests I came to the conclusion that even with considerable variation in stroke and diameter all of them might be taken to run—I am talking of petrol engines—at very nearly a thousand feet per minute. I therefore assumed all engines to have a constant piston speed. The next matter to go into in order to get something approximate in a rating rule is to find the mean pressure. This variable pressure I have found by actual experiments, tests of various kinds, brake tests, and so on, does not vary very much with good petrol engines. Any good petrol engine should give a mean available pressure at the crank pin of not less than 70 lbs. per square inch. Taking that as a basis—you cannot get a scientific rule in a simple way, but you can get a workable rule—I proposed 2.4 as the co-efficient; the club have taken 2.5 for the present rating rule, and I am very glad to hear that it gives satisfactory results. I determined it very carefully from the engines I tested. I believe it could be improved by such a modification as Mr. Lanchester suggests, because, undoubtedly, other things being equal, with a long piston stroke you can get higher piston speed. Therefore, if you can determine the law of it you are justified in including it in formula. That is the beautiful part of Mr. Lanchester's in-

vestigation. He has gone to work in a very bold and original way, taking fundamental propositions and deducing a general law which is broadly true under many varying circumstances, indeed all sorts of varying circumstances. Under that particular method of treatment he has come to certain conclusions as to the effect of piston stroke and all the other matters that we had to go into, and has given that conclusion in a very neat and beautiful formula. There is only one point which Mr. Lanchester asks my opinion upon. He refers to some experiments made by me, or, rather, on my engines, by the Institution of Civil Engineers. There were three engines, the diameter of the cylinder of the smallest engine being $5\frac{1}{2}$ in. and the diameter of the other two 9 in. and 14 in. respectively. From these three engines a number of experiments were made to determine the power obtainable, using the same mixture, the same compression, the same piston speeds as nearly as possible. As a result of these experiments we found to my surprise that the variation between the smallest and the largest engine was exceedingly small; the small engine only gave five horse-power while the largest engine developed sixty horse-power, and yet the difference in thermal efficiency was only something like 9 per cent. or 10 per cent. between the largest engine and the smallest. I agree with Mr. Lanchester for the purpose of argument that until other experiments are made and until other reasons are given—which I hope we may have from Professor Callender in the paper he has promised us—we cannot deal more accurately with this matter. Until we can deal with it more accurately and get more accurate information we can leave it out of the formula. Another reason for leaving it out is that the relative dimensions of petrol engines vary but little—3 in. to 4 in. or 5 in. or thereabouts, though in gas engines there are larger variations. I have listened to Mr. Lanchester's paper with very great pleasure, and I thank him for the very interesting evening he has enabled us all to spend. The paper has been a most valuable one for all of us, I am sure.

Professor W. C. UNWIN: I only received the paper to-day, through the kindness of the President, and I am not in a position to criticise it. The paper has interested me immensely, although there are one or two points on which one feels one would like to have a little more information, particularly in regard to the assumptions made. I am very much inclined to think that it is entirely satisfactory as a general solution of the problem Mr. Lanchester has put before him.

I hardly think he has earned one or two of the criticisms which have been raised. At the same time, at this late hour I will not venture to attempt any small criticism on the points which seem to be doubtful.

Mr. MERVYN O'GORMAN: I wish to publicly congratulate Mr. Lanchester on the paper he has given us; and not only on the paper itself, but on the fact that he has raised the discussion of this question to a higher plane than it has usually occupied, in which the whole thing is often dealt with as an ordinary poky works job aluminium-where-brass-was kind of spirit. It is a high-class paper, and Mr. Lanchester has treated us to some high-class thinking. He cannot have his head always thinking in this way, surely. He must have in some corner of his brain a department for this high thinking. Men engaged in business are not accustomed to thinking in this way. I admire also the straightforward way in which Mr. Russell Clarke leads the attack. That is also refreshing. It seems to me clear, however, that the piston speed being limited we may get rid of T. No one would alter the stroke for any purpose except to encourage the increase of piston speed. We take a rating which would lead to that encouragement. The great benefit of the D^2 formula is that it has done good work, and I am amused to see that even Mr. Lanchester uses the Club formula when he wants to do a little work, as on page 159. At this late hour I must not waste the time of the meeting, but I cannot entirely omit to express the pleasure at being given an opportunity of publicly thanking Mr. Lanchester for the work which he has done in the general interest of automobilism as evidenced by the results of this paper, both in his calculations for minimum engine weight and correct engine rating.

Mr. ARCHIBALD SHARP: I should like to join in the congratulations of the Institution on having this paper read and being published in its proceedings. The most brilliant generalisations have been brought out, and I venture to think that this paper will prove to be classic, and will be quoted in all the text-books dealing with this part of the subject. I could have wished the author could have taken a little more time and space to have discussed more fully the particular stress in any particular type of engine that actually limits the speed of the engine. I take it that in the one-cylinder, big fly-wheel type of engine the limiting factor will be the bursting stress in the fly-wheel; but in the multi-cylinder type of engine, with a comparatively small fly-wheel, I should have liked to

have heard more as to what is the actual limiting factor in actual work. Is it the size of the port openings? Is it the stresses on the connecting rod, due to high speeds, or is it the bearing pressure on the crank pin or the gudgeon pin? I think this paper is unique in so far as raising an excited discussion amongst some of the members before the actual meeting. The formula, which is the basis of the paper, expressing horse-power as a function of length and other physical quantities, is certainly very startling. I have been discussing with some members how horse-power can be stated without any reference to time. I think if one would take a simple analogy, the horse-power which can be got out of an engine surely is dependent on size. It is analogous to a definite integral, and we all know that a definite integral is a constant, although it may be deduced from an expression which does involve a variable.

Coming back to the fly-wheel, I think the fly-wheel effect of the piston and the connecting-rod masses is of great importance, if it is wanted to push the speed of an engine up to the utmost maximum. The fly-wheel effect of the piston masses is so entirely different according to the number of cranks used. In the one, two, and four cylinder types of motor-car engines, that effect is wholly detrimental. Twice in each revolution the piston masses have to be wholly brought to rest, the kinetic energy has to be reduced to zero, and that energy has to go to the rotating shaft. In the three-crank engine, which, of course, includes the six-cylinder engine, that effect is not quite so bad, but has still a disturbing effect. If you take the average kinetic energy of the pistons in the three-crank type of engine, the variation is about 20 per cent. above and 20 per cent. below the average. If you take the four-crank engine with cranks set at 90 degrees—that includes the eight-cylinder type of engine used on cars and motor boats—the variation is only 1 per cent. above and below; and if ever a five-crank engine, with the cranks set at equal angles, is used, there the fly-wheel effect would have a variation of less than one-tenth of 1 per cent.; so that in these different types of engines it is possible that the necessity of reducing the weight of the piston and connecting rods might not be quite so great as it is in the four-cylinder type of engine.

I must join in the thanks to the author for the instruction we have received.

The PRESIDENT, before calling upon Mr. LANCHESTER to reply, said: I regret it is one of the pains of sitting in the chair that I

cannot use my mathematical knowledge to dissect the paper which has been presented to us. On page 172 the author says he could not class steam cars in contests with petrol cars, but he gave an improper reason why steam cars should not so compete. He said practically, that he would debar them from competing with petrol cars because of the cumulative effects of the steam pressure. Of course, he ought to know that steam cars only came into competition since they lost their cumulative effects—since they generated their power stroke by stroke, as the internal combustion engine does.

Several other gentlemen have sent in their names as desirous of taking part in this discussion, but I must ask them to send in their remarks to the Secretary in writing. I hope Professor Unwin will see his way to add to our proceedings by sending in his remarks.

Mr. F. W. LANCHESTER, in replying on the discussion, said: I propose to deal with some of the remarks of the more recent speakers first, whilst I have the points raised fresh in my mind.

Professor Sharp has asked me the more precise meaning of the symbol σ , or stress; in particular, whether it applies to the bursting stress of the fly-wheel, or to what it relates. To this I would reply that σ applies to the stresses in every part of the engine. The whole problem is one in which the assumption is made of proportional design, and the stresses concerned are those not only of the fly-wheel but of every other part of the engine that is liable to stress. Whenever the dimensional method is employed in the solution of a problem of this nature, the *law of similarity* is assumed in relation to any physical quantity wherever it may occur. In equation (7) I have succeeded in splitting the index of σ , as given in the general expression (equation 1), and thus have indicated the extent to which the stress is related to the pressure of the working fluid.

Professor Sharp also discussed the question of piston and connecting-rod inertia. The points he raised are principally important in relation to the question of balancing an engine, and as such they are not within the scope of the present discussion.

Whether the engine be one of three, four, or any other number of cylinders does not affect the application of the theory set forth in the present paper; the difference pointed out by Professor Sharp does not concern the stresses that arise in each individual cylinder, the conditions of the present problem are not altered by alterations of "phase" to which the different cranks may be subjected. The

changes in fly-wheel velocity due to the interchange of kinetic energy between the fly-wheel and reciprocating parts are small in comparison with that due to the energy of impulse, and it has been supposed in the example given that the weight of the fly-wheel is determined by the latter consideration. In any case the point is one that is covered by the general method of treatment.

Mr. O'Gorman's remarks being mostly appreciative call for little in the nature of reply. He has honoured me by saying that I have, on p. 159 (§ 5), made use of the formula of the Automobile Club, and that this fact has been the cause of some amusement to him. Now this is plausible, but it is scarcely an adequate statement of fact. It happens that, in the passage in question, I am dealing with engines of *similar proportions*, not with engines whose D/S ratio is variable, but with engines of *constant geometrical form*. So long as this condition is rigidly applied, *any linear quantity squared* will serve as a basis of rating; even the diameter of a split cotter in some remote part of the valve gear would suffice. As it happens, in the particular case in question, Mr. O'Gorman will find that the Automobile Club rating gives precisely the same value as the $D \cdot \sqrt{D} \cdot \sqrt{S}$, or even $D^n S^{2-n}$, whatever the value of "*n*." *For engines of equal bore and stroke my rating expression and that of the Royal Automobile Club are identical.*

Mr. Russell Clarke, in opening the discussion, subjected the *method* of investigation employed in the present paper to the most scathing criticism. If Mr. Russell Clarke's remarks are justified, then the method is no method at all.

It is difficult, when we get on ground relating to a method of work which is established and used by several of the greatest physicists of the day, to go into the question of whether the method is right or wrong.

I can merely quote parallel cases of the employment of dimensional theory, such as have been accepted by the highest authorities. To cite such a case, Prof. H. S. Allen, in a paper on the "Motion of a Sphere in a Viscous Fluid," accepted and read before the Royal Society, employed precisely the method of application of dimensional theory here used. He deduced in precisely the same manner, as set forth in § 4 (p. 158), the general expression for the resistance of a body in terms of *area, velocity, viscosity and density*, showing, amongst other useful results, that if the resistance varies exactly as velocity squared it must be independent of viscosity, a result which is generally accepted under the conditions of the

stated hypothesis. There are many similar applications of the method.

So far, then, as the method employed in the present paper is concerned, it is conducted on established lines. We have a physical quantity which is dependent upon other physical quantities, and we deduce the relation of the one quantity to those on which it depends by a direct application of the theory of dimensions.

It is open for Mr. Russell Clarke to disagree with me as to the quantities comprised in the hypothesis; he may say, for instance, that I have not included the effect of the viscosity of the lubricant, or of the working fluid, or he may question whether the speed at which an engine can be safely run is limited by the stresses of its working parts, but he may not call into question an established method of physical research—it is outside the province of the present Institution.

I have but a few further remarks to offer on Mr. Russell Clarke's observations. He somewhat sarcastically suggests that, while I was about it, why should I have troubled about ρ and σ at all; why not write simply horse-power varies as L^2 , that is, dimensionally $\frac{ML^2}{T^3} = L^2$? He says: "He would have got $p = 2$, that is, he would have got that the horse-power varies as L^2 ." In reply to this, I can only say that *if* in any problem it could be shown that horse-power depends upon a linear quantity, *and upon nothing else*, it certainly would vary as L^2 , and the suggested procedure would be perfectly regular. There are, however, in the present problem other variables, and it would be wrong to leave out the other variables; the fact that the result would be the same in respect of L is merely a coincidence.

The suggestion that we should go at it "bald-headed" and *assume* a definite piston velocity would, it is true, be quite simple, but unless one can usurp the functions of the Creator, it would be impossible, in the case of a short stroke engine, to limit the stresses to an amount that could be considered safe. When we exceed the stress that the material will stand the machine comes to pieces, we cannot avoid it; *stress* is the limiting factor, not piston speed.

The subsequent speakers in general expressed themselves so very favourably to the paper, that it is difficult to find points on which to reply.

Professor Callendar points out the importance of the least weight problem, and some of the conclusions reached in connection with

the flying machine. That aspect of the subject is not one that I have neglected; the subject of aerial flight has for many years been present in my mind; indeed, my interest in the problem of flight was largely responsible for my initially interesting myself in the petrol engine and automobile design. It is some twelve years or more since I first had to do with the study of aerial flight; at the present time I have a work in the press on the subject—on the dynamics of flight—in which in several ways I make use of the principles I have laid down. I fully concur with Professor Callendar's remarks; when flying machines become a matter of course, the flying machine designer will have to know how to design for least weight, just as the ordinary engineer has now to design so as not to exceed the strength of his materials. The different parts of a machine will be tabulated in the text-book of the future, as functions of *diameter* and *stroke* or such other factors as may be essential, so that the designer will merely have to take out his quantities, classify as tabulated, and solve for minimum value.

Answering Dr. Hele-Shaw's remarks, I do not agree with him that the Automobile Club formula amounts to the same thing as my own. It is perfectly true that under certain restricted conditions the two give the same results, but the A. C. rating is based implicitly on the assumption of *constant piston speed*.

Perhaps I may deal with this point most effectively by reading an extract from one of the appendices to the paper which I unfortunately have not had time to read in full: "More recently the D^2 basis has been advocated and subsequently adopted by the Automobile Club, and known as the A. C. rating. This is of the right *dimensions*, and would serve if all engines were of the same D/S proportion, but it is manifestly absurd as an attempt to represent the horse-power under actual conditions. This fact is clearly recognised in a circular issued recently to the trade suggesting that manufacturers should adopt the A. C. rating for purposes of catalogue description. It is ingenuously stated in this circular that the formula is not intended to be a scientific statement of horse-power."

Having gone wide of the mark it is most convenient to explain that the shot was not aimed at anything in particular.

There is one other passage I should like to read that has immediate bearing on the discussion:—

"A warning may be added as to the *practical* testing of cylinder rating rules. Quite commonly lists are made of some of the

cars entered for competition, and these are worked out according to the proposed rule, and as to actual performance or by some other rule, or, perhaps, are figured according to the maker's catalogue. Now a rule would require to be egregiously bad to show up under these circumstances, for the real values are not usually known to within 10 per cent. or so, and in general the D/S variation between one make and another is not great.

"A rule that would carry as a logical consequence a diameter of 3 in. and a 12-in. stroke would not seem materially at fault under these circumstances; such 'tests' are entirely delusive."

At present we are in the position of the early yachtsman; the rating rule has not been in operation long enough to result in the "special design." That is to say, we have not yet developed the "rating rule cheat." It would be wrong to retain a rating formula that has now been demonstrated tends to a distortion of design, and which leaves a loophole for the horse-power thief.

I have to thank Mr. Dugald Clerk for his very kind remarks, and in particular for his reply to my query raised in connection with the "unaccounted factors," § 19; I am glad to hear that he concurs with the views therein expressed.

Some of the observations made by Mr. Clerk on the origin of the A. C. formula touch the topic dealt with in the last passage quoted from Appendix IV.

I do not consider the exact magnitude of the *constant* or co-efficient used, whether it be 2.4 or 2.5, a matter of any great moment. This factor can have no influence towards the *distortion* of the design; so long as the expression is of the right *form*, the value of the constant is of secondary importance.

I do not think that Professor Unwin in his remarks raised any point to which my reply is needed.

The PRESIDENT: Your hearty acclamation shows that you all feel that we have enjoyed our evening, and that this paper, and the discussion upon it, make Professor Callendar's task still more difficult than it would otherwise have been. Our papers have been on a gradually ascending scale; each paper has been more interesting than the one immediately preceding it. Mr. Lanchester has set us a new figure, and we have to keep up to it.

WRITTEN CONTRIBUTIONS.

Since the reading of the paper the following contributions have been received :—

Professor J. B. HENDERSON : Mr. Lanchester has done me the honour of referring in an appendix to his paper to a paper of mine on the H.P. rating, published a few months ago as a protest against the A.C. rating. Mr. Lanchester's method and mine both use the laws of similar structures, and we both arrive at the same results so long as we deal with *similar* engines.

We are both agreed that the inertia stresses limit the frequency of revolution, and that in similar engines this frequency is inversely proportional to the linear dimensions, that is, the engines have the same piston speed ; but when we come to deal with engines which are *dissimilar*, then our methods and results differ.

Dissimilarity affects the inertia stresses and also the average pressure, and when writing my paper it seemed to me unjustifiable to deal with the former unless I also dealt with the latter, for I considered the latter the more important. I therefore neglected both, and looked for some dimension in dissimilar engines which could be taken as a measure of the linear dimensions, and decided on adopting the cube root of the piston displacement volume.

Since the publication of my paper, however, Mr. Dugald Clerk has published some results of experiments on gas engines, showing that the variation of size of engine on the average pressure is very much lower than most engineers anticipated, the difference being only 12 per cent. for a variation of cylinder diameter from 5 in. to 14 in. Mr. Lanchester has, therefore, on this authority decided to neglect the variation of average pressure, and has boldly analysed the inertia stresses due to the piston and connecting rod. His result is $H.P. = C D^{\frac{2}{3}} S^{\frac{1}{3}}$, mine is $H.P. = C D^{\frac{2}{3}} S^{\frac{1}{3}}$, and the A. C. formula $H.P. = C D^2$. The A. C. formula is based on the fallacy that *all* engines may run at the same piston speed, instead of all *similar* engines.

There is no doubt in my mind that Mr. Lanchester's formula is the correct one to use if it is proved in the future, that in petrol engines the variations in average pressure, due to variations in cylinder dimensions, are negligible ; and the whole decision hangs on this point. It is evident that the greater is the ratio of wall surface to cylinder volume, the greater will be the heat losses and the greater the fall in average pressure. There will be one parti-

cular ratio of stroke to bore which will make this loss a minimum; and very short stroke engines, alike with very long stroke ones, will suffer from diminished average pressure, but the amounts of these diminutions can only be obtained by experiment. It should be remembered that, in Mr. Dugald Clerk's tests on gas engines, the smallest engine cylinder used would correspond to a large one in high-speed motor car engines.

In order to compare the above three formulæ, let us consider the results which they give in, (a) a long-stroke engine; (b) a short-stroke one; and, (c) one with stroke equal to bore. The extreme cases that I know of being used in practice are $S = \frac{3}{2} D$ and $S = \frac{2}{3} D$.

The results are given in the following table:—

	A. C.	LANCHESTER.	HENDERSON.
(a) $S = \frac{3}{2} D$	$C D^2$	$C D^2 \times 1.225$	$C D^2 \times 1.310$
(b) $S = \frac{2}{3} D$	$C D^2$	$C D^2 \times 0.817$	$C D^2 \times 0.762$
(c) $S = D$	$C D^2$	$C D^2$	$C D^2$

It is at once evident that the A. C. formula greatly overestimates the power of the short-stroke motor and underestimates that of the long-stroke one.

In the long-stroke motor the Lanchester formula gives 22 per cent., and mine gives 31 per cent., more power than the A. C. formula, while in the short-stroke motor the Lanchester gives 18 per cent., and mine 24 per cent., less power than the A. C. formula. My formula gives 7 per cent. more power to the long-stroke engine and 7 per cent. less to the short-stroke engine than those given by the Lanchester formula.

I hope that, pending further investigations on the average pressure, the A. C. will adopt Mr. Lanchester's formula in place of their present one. For most practical purposes it might be modified thus:—

Since in the majority of engines the bore and stroke do not differ greatly, let us write $S = D (1 + \epsilon)$, where ϵ is a fraction less than unity. The formula then becomes—

$$\text{H.P.} = C D^2 (1 + \epsilon)^{\frac{1}{2}} = C D^2 \left(1 + \frac{1}{2} \epsilon + \text{terms containing higher powers of } \epsilon, \text{ which are negligible when } \epsilon \text{ is small}\right)$$

$$\text{H.P.} \div C D^2 (1 + \frac{1}{2} \epsilon) \div C D^2 \left(1 + \frac{S - D}{2 D}\right) \div C D \left(\frac{D + S}{2}\right)$$

Taking the same value of the constant C as is used in the A. C. formula, we have—

$$\text{H.P.} = \frac{D(D+S)}{5} \text{ if the dimensions are in inches.}$$

$$\text{H.P.} = \frac{D(D+S)}{3200} \quad \text{,,} \quad \text{,,} \quad \text{,,} \quad \text{millimetres.}$$

The difference between the power calculated by this modified form of Lanchester's formula and by the original formula is only 2 per cent. in the cases of both the long-stroke engine having $S = \frac{1}{2} D$, and of the short-stroke engine having $S = \frac{2}{3} D$, the modified form giving the greater power in both cases.

Mr. L. A. LEGROS: I agree with the author that the theory of dimensions is a most valuable aid to the engineer in checking equations and ascertaining that all the variables have been properly taken into account.

The author has given an example on page 156, showing the detection of such an error in a simple case; these errors are, however, much more likely to occur when the dimensions involve higher powers of L and T . I have occasionally met with much difficulty from this source when translating into English and English measures papers written in French, with calculations made in the metric system. An example of this occurs frequently in calculations of bending moments, where the moment of inertia I of a cross-section appears. This is the product of area by the square of a length; its dimensions are consequently L^4 , and in a particular case it must be expressed as the product of (say) square inches $\times$ square inches. The fact that the moment of inertia of a section has the dimension L^4 makes it intangible, and for this reason, probably, many engineers prefer to use the modulus Z of the cross-section, the dimensions of which are L^3 .

In the discussion the expression "per horse-power per hour" is criticised, and on looking up my notes on the constants used in conversion, I find that horse-power may be conveniently expressed in British Thermal Units per second, or the cheval vapeur in calories per second, and where quantities of heat are concerned, it is simpler to work from this basis than to convert into and back again from foot pounds or kilogrammetres per second.

In reading the author's paper I find it necessary to make some simple calculations to enable me to follow the reasoning, and I am of opinion that the paper would be more readily

understandable to most engineers if the omitted steps were given. For example, in obtaining the dimensions of stress, we have—

$$\sigma = \frac{f}{l^2} = \frac{m a}{l^2} = \frac{m}{l^2} \times \frac{l}{l^2}$$

whence stress is dimensionally $\frac{M}{L T^2}$.

It is to be hoped that the Council will publish the appendices and diagrams in full in the proceedings, and that the author may amplify the paper with the calculations of practical examples. I think that this will render the paper of still greater value as a work of reference when the time comes for automobile engineers to consider seriously the problem of the lightest possible motor in connection with mechanical flight.

Professor W. C. UNWIN: I agree so much with Mr. Lanchester that I have no remarks to make in criticism of his paper.

Mr. RUSSELL CLARKE: By the courtesy of the Secretary of this Institution, I am enabled to add a few words to the remarks I made at the discussion.

If it be possible to obtain a formula expressing the horse-power of an engine, in terms of the linear dimensions, the density of the materials used in its construction, and the stress in its parts, then given the particulars of the design of the series of engines under consideration, such formula could be obtained by the ordinary processes of calculation usually applied to such problems. Starting with the fact that horse-power varies as the product $P \cdot L \cdot A \cdot N$, where P is the pressure in the cylinder, A is the area of the piston, L is the stroke, and N is the number of revolutions, it would be necessary to obtain expressions for P and N in terms of ρ , the density of the materials, and σ , the stress in the parts. On substituting these expressions in the expression $P \cdot L \cdot A \cdot N$, we should obtain an expression for the horse-power containing l , ρ , and σ only.

Now it is well known that in certain parts of the engine the stress σ is by no means a simple function of the velocity of the parts. Thus in certain revolving parts the stress varies as N^2 , in other parts the stress varies as N , and in other parts the stress is independent of the velocity. Let us assume then that there is a certain part of the engine that is always more stressed than any other part, when the engine speed is raised to a limiting value, and that it is the consideration of the safety of this part which limits

the engine speed. Then I agree with Mr. Lanchester that the stress in that part due to velocity alone may be expressed as a power of the velocity, and if this were the only cause of the stress, it would be possible conversely to express N as a power of σ . Unfortunately, however, this is by no means the case, but the stress in that part will also be a function of the pressure in the cylinder. In fact, the stress is the sum of the stresses due to velocity and pressure. Consequently, the simplest expression possible for the stress in the part is—

$$\sigma = A + B \cdot N^x$$

where A , B , are constants. Owing, however, to the fact that the stress in the part under consideration will also depend on the stroke, the expression for σ will doubtless assume a more complicated form than this. The expression for N in terms of σ will be more complicated still.

It must now be evident that the assumption that it is possible to express horse-power as the product

$$l^p \times \rho^q \times \sigma^r$$

is quite unwarranted.

Suppose, however, the author had assumed that horse-power varied as

$$l^p \times \rho^q (A + B\sigma + C\sigma^2)$$

which is certainly less complicated than the facts warrant us in supposing. In this case the application of the dimensional theory would have led to a set of indeterminate equations. In fact, a moment's reflection will show that the assumption made by the author is the only one which leads to any result by the application of the dimensional method. Surely, however, this is not any justification for the assumption.

It is possible, however, to look at this matter more broadly. The dimensional method can only be applied to equations capable of a physical interpretation, which are *generally* true, independently of any assumption made about them. Now, in the paper, the author assumes that the horse-power depends on the safe stress allowable in a particular part of the engine. There is, however, no general physical relation between this stress and the horse-power of the engine. *Practically*, the horse-power may well be limited by a critical value of this stress, which must not be exceeded, but mathematically the horse-power cannot be expressed in this way. Consequently, the dimensional method used by the

author is inapplicable to the case, and for this reason the conclusions arrived at in the paper must not be regarded as established.

Colonel H. C. L. HOLDEN: Unfortunately for myself I was unable to be present at either the reading or the discussion of Mr. Lanchester's most interesting and able paper, but having been invited subsequently to contribute a few remarks for publication I have great pleasure in doing so.

The author of the paper, and also the speakers in the discussion which followed, appear to have omitted all reference to the practical side of the question, viz., the application of the rating system proposed, which is given on p. 167.

H. P. = $\cdot 4 \sqrt{D^3 S} \times n$. This is stated to represent "an ordinary maximum B.H.P. value." I am not quite clear what this definition means, but this is not of very great consequence, because so long as the comparative values are correct the constant can be varied and the actual figures for h.p. increased or decreased without affecting relative values.

The subjoined table shows the h.p. obtained by using Mr. Lanchester's rating formula for various well-known makes of engines in comparison with the R.A.C. rating for maximum B.H.P., viz., $\frac{D^2 n}{2}$:—

Type.	No. of Cylinders.	Bore.	Stroke.	R. A. C. rating.	Lanchester $4 \sqrt{D^3 S} \times n$	Lanchester $5\cdot6 \sqrt{D^3 S} \times n$
60 Napier	6	5"	4"	75	53·6	75
40 Napier	6	4"	4"	48	38·4	53·7
30/40 Daimler (1906) .	4	4·88	5·91	47·5	41·8	58·5
30 Daimler (1907)....	4	5·12	5·91	52·4	45·7	63·9
28 Daimler (1907)....	4	4·724	5·91	44·7	36·5	51·0
20 Lanchester	4	4	3	32·0	22·16	31·0
Engine, 4" bore	4	4	4	32·0	25·6	35·8
Do.	4	4	5	32·0	28·6	40·0
Do.	4	4	6	32·0	31·32	43·8

From this it would appear that Mr. Lanchester's formula for maximum B.H.P. certainly gives low results for the 60 h.p. Napier, and I have therefore added the last column, in which this engine is brought up to R.A.C. rating, 75 B.H.P., which is, I understand, a practically correct figure for the maximum output of this engine, and all the other figures corrected in the same proportion. An examination of these figures by anyone who is acquainted with the actual performance of the engines in question will, I think, clearly point to the fact that according to Mr. Lanchester's proposed system of rating they are credited with higher B.H.P. than they can at present give, and that the longer the stroke in comparison to the bore the worse the discrepancy.

This may be due to their design being incorrect, or it may be due to the fact that there are other factors than stress controlling the piston-speed of petrol engines. In conclusion, I may point out that the R.A.C. formula is not based on an assumption that all engines may run at the same piston speed, but upon the fact, which is readily ascertainable, that the engines of the best constructors of modern petrol cars, when developing their maximum power, do run at a piston speed which differs very little with the proportion of bore to stroke. It is not, and never has been, suggested that the R.A.C. formula is scientific or strictly accurate. All that has been claimed is that in all-round results it is not very far from the mark, which is more than can be said of many other formulæ that have been proposed.

Mr. MERVYN O'GORMAN: It would be impertinent to offer criticism on a paper full of careful and ingenious thinking without having spent at least as much thought on the subject as the author evidently has done. It is unfortunately becoming more and more customary to relegate these mathematical methods of clear thinking to such particularly learned societies as the Physical, and though I, and doubtless many here, have belonged to that society for years, and do not wish to rob it of its laurels, it is undoubtedly a good augury for the position which our Institution of Automobile Engineers is going to hold when we find it first, I believe, amongst the engineering societies to accept, as one of its ordinary papers, a lecture of a character and on the lines of Mr. Lanchester's.

The subject of rating is the portion of the paper with which I have been more particularly associated, at least in interest, though I am willing to agree that to many of us the method of calculating

the minimum weight of metal which can yield a required horse-power is of very much more practical importance and value.

In view of the hopeless slough of rating that ruled the automobile world until the publication of the R. A. C. formula a few months ago, it is worth while to consider the matter carefully.

- (1) The objects of a rating are not simple.
- (2) A formula to receive popular acceptance must be simple in the present state of the industry.

Amongst the various objects of rating two important ones may be considered first, and these must not be confused. One kind of rating concerns the individual works, and has nothing whatever to do with competitions, the other, on the contrary, is devised for races and for securing fair competition. Rating good for one purpose is not necessarily good enough for the other. As an example of the first kind, in a different branch of engineering there is the rating of accumulators, dynamos and electric motors, which, though almost fundamental to the electric manufacturing industry, cannot have any relation to a race. In this, which might be called "works rating," we find that the temperature rise of the hottest part of the dynamo, the commutator surface or the inside of the field or armature windings, after a six hours' run on load, determines what shall be called the full load, and the works rating is very often taken as the most efficient load. Here we see two ratings based respectively on temperature rise and efficiency.

In the case of petrol engines the method of works rating is modified by the fact—

- (i.) That at any speed of rotation there is a maximum rate of working beyond which the engine will not go.
- (ii.) Besides this there is a speed at which the utmost power is obtainable.

The works rating may be taken at any useful speed or at the maximum output for any speed, according to the whim or experience of the individual maker. The upshot is that all engines which attain at some speed to some H.P. are by him passed as satisfactory for use in his, say, 24 H.P. car, and the maker may be said to have rated them as suitable for sale.

It is clear enough that the rating adopted in this way has nothing to do with competitions; the manufacturer is free to make any intelligent use of the test results, and he is thereby enabled to improve his engine. No part of his assumption in rating leads him to any distortion. (It is, of course, not desirable to take a

thoroughly irrational basis such as a port or valve area, or the size of a split pin, as Mr. Lanchester jocosely suggests.)

The freedom with which the various makers have exercised their obvious right to rate their engines as they liked, though it did not affect their design, has had two drawbacks—(1) That there was no sort of comparativeness between one maker's rating and another's; (2) that the buying public had no easy means of knowing within 70 per cent. and 80 per cent. what the meaning of the mystic letters H.P. could be, although they did know that in some extraordinary way the amount of this mysterious H.P. did determine the price.

It is interesting to note that under this condition of unfettered design, bore and stroke had, for practical reasons, been made equal within about 10 per cent. Moreover, another fact disclosed itself, namely, that when the stroke was longer the revolution speed was proportionately slower in many cases. From this last it is seen that the piston speeds did not and do not as a rule show as great a deviation from one another as the length of the stroke. With piston speeds thus tending to equality, no good reason existed for withholding the publication of a rating rule on a D^3 basis, provided it was clearly stated at the same time that such a rule was not taken as a basis for competitions, and that it was not an accurate scientific expression which would give the horse-power.

If, now, we turn to the handicapping of cars for competition we have a different problem. With this in view we are in great danger of distorting design if we limit dimensions wrongly, and the *exposé* given by the author is of great value in this direction, but it by no means settles the whole question. A simple example will show this. In the development of a touring car industry a far more important work has been incidentally started—the commercial vehicle. Already commercial vehicle design is favourably reacting on touring car design, and for goods transport as well as pleasure cars the fuel economy question will have in the near future the greatest importance. Fuel economy has practically no recognition in the $D^{1.5} S^5$ formula, and as the author himself perceives his formula tends to increase engine speed and to decrease engine weight, two very desirable things in their way, but capable of grave abuse as soon as engines are designed under the formula alone, without regard to durability beyond the time of the race. Personally, I prefer fuel efficiency to any other simple form of rating, and fuel efficiency combined with $D^{1.5} S^5$ is probably a very good or

indeed a better arrangement. Nevertheless, it will be seen that we are at once getting into complications, which, however perfect on paper, are liable to be resented by the competitors. This resentment is very easily expressed by them by the non-appearance of any entrants. When this result is attained, it naturally gives satisfaction to those firms who never have had the courage of their designs, and have never attempted to show what they can do in a publicly supervised and fairly run trial.

MR. LANCHESTER'S REPLY TO THE WRITTEN CONTRIBUTIONS.

Replying to Prof. HENDERSON.

So long as *similarity* is a condition the problem is quite simple, and one on which we agree.

When any departure is made from the condition of similarity as to proportion of diameter to stroke, the difference arises. It appears to me that Prof. Henderson, in adopting the cube root of displacement volume as a linear basis has unwarrantably assumed the *symmetry* of the three co-ordinate axes. If he had followed the matter up as a *matter of dynamics* he would have avoided this error.

Prof. Henderson will note in Appendix V. to my paper I have (since the date of reading) modified the form of the rating expression to include the influence of reciprocating masses varying as D^2S .

Mr. LEGROS complains of having had to work out a few simple calculations to *verify* the dimensional values given. I think any complaint on this score is answered by the addition of Appendix I. not printed with the advance proof. I regret not having had time to furnish a greater variety of practical examples, but from the illustration given these can be worked out *ad nauseam* by any smart engineer or draughtsman, and there is more instruction to be derived from working examples out for one's self than following the work of another.

Mr. RUSSELL CLARKE comes to the attack a second time.

I think perhaps that the pith of the matter is that Mr. Russell Clarke thinks that the *dimensional method* purports to be a *mathematical method*. I do not myself think it is a *purely* mathematical method.

The "quantities" dealt with in dimensional theory are abstract physical "quantities," not integrals; *they are not quantities in the mathematical sense at all.*

The validity of the dimensional theory rests upon the fact that any relation existing between physical quantities in the abstract, deduced from their fundamental constitution, holds good for those quantities in their algebraic relations either as simple algebraic quantities, integrals or differentials of any order, provided only that the whole of the variables are properly represented. Mr. Russell Clarke wishes me to take the problem up where the power is given as in the usual engineering expression by the product of the quantities P L A N (as defined) and from this demonstrate my result in an expression containing the terms l, ρ, σ only.

If I had myself invented the theory of dimensional reasoning, or if the present paper constituted any striking departure in its application, I should not consider Mr. Russell Clarke's request for an independent demonstration unreasonable, but as things stand I feel that it is a matter he can very well attend to himself. I have given my solution in a neat and simple form, and if Mr. Russell Clarke thinks there is a better way I should be sorry to rob him of the *kudos* of presenting it himself. Perhaps it might, when duly matured, form the subject of a further paper to this Institution—one never knows.

I object on principle to employ other people's methods at their dictation. It would create a bad precedent. One might be asked on some future occasion to give alternative demonstrations, on some prescribed hypothesis, using only the methods of Euclid and Archimedes!

I have looked into Mr. Russell Clarke's alleged disproof, and I do not mind throwing out this hint—he does not take account of the far-reaching consequences of the assumption of *complete similarity* which forms an essential part of my hypothesis.

It is with great pleasure I find the name of Col. HOLDEN as a contributor to the written discussion.

I may perhaps explain to Col. Holden that the main object of my paper was not the solution of the rating difficulty, this being only dealt with as a collateral question. I consequently was induced at first to put forward an expression of the same form as that found necessary for the mathematical solution for minimum weight, that is one of the $D^n S^2 - n$ type, for rating purposes.

In the paper it was shown that the value of the index " n " is subject to change with changes in D/S , and the proposed rating expression $\sqrt{D^3 S}$ is only approximately true for a certain moderate range.

In Appendix V. (written subsequently to the reading), the question has been reconsidered purely from the rating standpoint, and a more universal and simple form of law has been deduced, of course, of L^2 dimension.

I believe this amendment (which evidently has not been placed in Col. Holden's hands) deals effectively with the only objection he raises, and in a sense shows his criticism to be well founded.

The criticism to which reference is here made is as follows :—

"An examination of these figures by anyone who is acquainted with the actual performance of the engines in question will, I think, clearly point to the fact that, according to Mr. Lanchester's proposed system of rating, they are credited with higher B.H.P. than they can at present give, and that the longer the stroke in comparison to the bore, the worse the discrepancy."

In the absence of any knowledge of my own corrected equation, I congratulate Col. Holden on having put his finger precisely on the weak spot.

Anent the remarks made by the President, which I at the time took in the light of *badinage*, but which I have subsequently learned were intended seriously.

A certain portion of my paper is quite general, and goes far beyond the title, which suggests a limitation to the internal combustion motor. My intention is, in the passage quoted (referring to the steam car), to point out the fact that the control of the cylinder pressure far above its ordinary working limit* which the driver can exercise (at least temporarily), disqualifies the steam car from appearing under any ordinary petrol engine rating.

I fear that I differ with Col. Crompton on the question of cumulative effect. I am quite aware that the "tank" boiler is substantially abandoned in favour of the "flash" type, but there is nothing to prevent it coming back to torment us if encouraged.

* 1,200 lbs. per square inch is known to have been registered on at least one occasion, and once to my knowledge an engine has been blown to pieces by the pressure employed.

Beyond this, a hot coil of pipe stores energy, and that in a most amazing quantity. Let us take, for example, 100 lbs. of iron pipe say only 100° Fah. hotter at the start of a hill climb than at the finish, we have *about* $10,000 \times .114 = 1,140$ Fah. H.U. — say 880,000 ft. lbs., or @ 10% thermal efficiency = 88,000 ft. lbs. available, or enough to lift an ordinary touring car 40 ft. vertically.

It would certainly simplify matters to neglect quantities of this magnitude, for it would be unnecessary to survey the course, or to use a telephone or a stop-watch.

The instructions could be issued thus:—

Start at the *bottom*.

Finish at the *top*.

Take the time with an *egg-boiler*, or *pocket sundial* if weather permits.

I wish to say nothing against the steam car, but at a hill-climbing competition it has a wonderful resemblance to Mr. Ah Sin at the game of "*Euchre*," "*. . . and the same with intent to deceive . . .*"