61. A Transformation in Elementary Geometry

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Thus, by summation, since $f(\infty) = \Gamma(n)$,

$$\log \Gamma(n) - \log f(m) < \frac{n^2+n}{2m} \text{ and } > \frac{n^2+n}{2(m+1)}.$$

But $\log \left(1 + \frac{n+1}{2m}\right)^n$ lies between these same limits, and hence when $n < 1$ the logarithms of $\Gamma(n)$ and of $\left(m + \frac{n+1}{2}\right)^n \frac{n!}{n(n+1) \dots (n+m)}$ differ by less than $\frac{n^2+n}{2m^2}$.

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61. A Transformation in Elementary Geometry.

1. ABC is any triangle; CI the bisector of the vertical angle.

Let each point p in the plane of the triangle be transformed by the following rule into another point p' :

Invert p from C , taking as radius of inversion a mean proportional to the sides CA , CB , and let this inverse be q . Invert q with respect to the line CI , that is, take its geometrical reflection p' in this line. This double inversion converts p into p' .

2. It is clear from the above that any straight line passing through C transforms into a straight line also through C ; and that two straight lines which thus transform, the one into the other, make equal angles with the sides AC , BC . Again, that any circle becomes a straight line or circle according as it passes, or does not pass through C .

3. In particular it will be found that:

- (i.) The circum-circle of ABC transforms into the base AB , and *vice versa*.
- (ii.) The in-circle (I) into a circle touching CA , CB , its chord of contact with the sides passing through the centre I' of the ex-circle (I').

(iii.) The ex-circle (I') into a circle touching CA , CB , its chord of contact passing through I .

4. As an application we have at once a theorem of Mannheim:

"Two circles touch CA , CB and the circum-circle, their chords of contact with these sides passing through I and I' respectively."

5. If we take as the square of the radius of inversion $\frac{1}{2}CA \cdot CB$,

A transforms into the middle point of BC ,
 B " " " " CA .

The circumcentre becomes the foot of the perpendicular from C on AB . Thus the nine-points circle is changed into the circle through the ends of the base and circumcentre, and the ex-circle (I') becomes a circle touching CA , CB at P , Q , where PQ bisects CI at right angles.

This gives a solution of (230), since it is known that the nine-points circle touches (I').

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