than water. It would, therefore, increase both the sensitive-ness of the calorimeter in measuring small quantities of heat and the range of temperatures throughout which an investigation might be carried.

This calorimeter, as now made, enables the experimenter of ordinary skill in manipulation to obtain results which are comparable in accuracy and consistency with those obtained by any calorimetric method in use ; ordinary care in weighing and in reading the thermometers used being the most important considerations to be observed. For these reasons it is well calculated for student use as well as for research-work, for which its wide range of application makes it especially suitable.

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XXXVII. *The Viscosity of Mixed Gases.*

By WILLIAM SUTHERLAND.*

NEARLY fifty years ago Graham, in his beautiful experiments on the motion of gases, encountered the fact that by mixing a little hydrogen with carbonic dioxide the mixture can be rendered more viscous than the carbonic dioxide, although the hydrogen is less viscous than it. The fact seems to have arrested his attention rather strongly ; for he made a large number of experiments on the transpiration of mixed gases through capillary tubes, with the result that, if t_1 is the time of transpiration of a gas 1 under certain conditions as to pressure and temperature, and t_2 the time for a gas 2 under the same conditions, then for a mixture of n_1 volumes of 1 with n_2 volumes of 2 the time t is in most cases given with considerable accuracy by the empirical formula

$$(n_1 + n_2)t = n_1t_1 + n_2t_2 ;$$

but when hydrogen is one of the ingredients, the formula fails entirely to represent facts. As the viscosity is proportional to the time of transpiration, a similar empirical formula holds for the viscosity of most mixtures, but fails for those containing hydrogen. This peculiar experimental fact also arrested Maxwell's attention ; for in his Bakerian lecture in 1866, on the viscosity of gases, he mentions it as a remarkable thing that while hydrogen is only half as viscous as air, a mixture of equal volumes of hydrogen and air is 15/16 as viscous as air. Maxwell's measurements of viscosity were made with

* Communicated by the Author.

his oscillating-disk apparatus, and were therefore specially suitable for controlling Graham's results. But in 1879 Pulfj went into the subject more thoroughly by carrying out with an oscillating-disk apparatus a large number of measurements of the viscosity of mixtures of carbonic dioxide and hydrogen, which had proved to be the most exceptional of all with Graham : his results were completely confirmatory of Graham's.

But to my knowledge the theory of this interesting phenomenon has not hitherto been undertaken, though apparently so suited to serve as a test-case for the kinetic theory. Doubtless the general principles of the kinetic theory are now so well established as to hardly require further confirmation ; but still, apart from the interest in the problem for its own sake, it will be advantageous to consider the theory of it, because in pushing on with investigations of the properties of molecules it is very desirable at the present juncture to ascertain how far the simplest hypotheses will carry us and where they cease to be sufficient. Moreover an inquiry into the theory of the viscosity of mixed gases offers an additional advantage in throwing light on the theory of the viscosity of a single gas.

In the theory of the viscosity of a gas we suppose the gas to be confined between two parallel solid planes, one of which is at rest and the other moving in a fixed direction in its own plane with velocity w . It has been proved experimentally that the slipping which takes place at either solid plane can be neglected, unless the distance between the planes is only a small multiple of the mean free path of the molecules of the gas. Neglecting the slipping then, and denoting the distance between the planes by D , the velocity of the gas in a layer at distance x from the fixed plane is $w x/D$, and on the average each molecule in the layer has this velocity in addition to its velocity of agitation. The viscous action between the two parts of the gas on opposite sides of the parallel plane at distance x from the fixed one, is due to the velocities of agitation carrying velocities greater than $w x/D$ from distances greater than x to distances less than x , and *vice versa*. To follow the conversion of the molar energy due to the velocities $w x/D$ into molecular energy, that is, to follow the conversion of the molar energy into heat by the action of the gaseous friction, we have to consider the collisions taking place in the layer at x between molecules coming from its opposite sides, and these may be said to come on the average from distances $x+l$ and $x-l$: consequently the velocity of the centre of gravity of each colliding pair both before and after collision is $w x/D$, which is the permanent velocity characteristic of the

layer at x . Of course the average effect of the collision is largely to destroy the two small relative velocities lx/D and $-lx/D$, and to convert the energy associated with them into energy of agitation. This is easily proved in the theory of viscosity, and then it is assumed that on the average a single collision suffices to bring a pair of colliding molecules both into the permanent state characteristic of the layer in which they collide. But, as will be shown below, a single collision does not on the average suffice to bring about this state; and the difficulty of defining in a simple manner the state of the molecules in any layer at any instant, makes the relation between viscosity and the constants of the molecules not so definitely specifiable as in the numerical relations hitherto given. What appears to me a better statement of the motions of the molecules in each layer will be given below, though the matter is really of slight importance in the case of a single gas; but when it comes to a question of a mixed gas, then a correct specification of the state of each layer becomes the essence of the problem.

In the theory of the viscosity of a single gas, then, it is customary to regard only those molecules which collide in a layer as being characteristic of that layer, those molecules which are passing through it without collision being taken account of in some other layer where they do collide. An approximate expression for the viscosity of a single gas can be very easily established; for if λ is the mean free path, then, as the mean paths have their directions evenly distributed in space, the mean projection of the free paths on the axis of x is the mean distance of a hemispherical surface of radius λ from the diametral plane, which is $\lambda/2$. Thus all the molecules which cross our plane at x may be said on the average to have experienced their last encounter at distances $\lambda/2$ and $-\lambda/2$ from it, so that each molecule that crosses from a distance greater than x carries on the average momentum of amount $nmv\lambda/2D$. If there are n molecules in unit volume with average velocity v , then in unit time $nv/6$ may be taken as the number of molecules crossing unit area; so that the positive momentum carried in unit time across unit area in our plane from distances greater than x is $nmv\lambda/12D$, and the same amount of negative momentum is carried in the opposite direction across unit area, so that the total effect is $nmv\lambda/6D$, in which $nmv\lambda/6$ is the viscosity η .

The constant $1/6$ is not exact because of the rough and ready approximations; but, for the purpose in hand, it may be regarded as not very different from the more accurate $\cdot 064$, which takes its place as the result of the elaborate

solution (see, for instance, Tait, *Trans. Roy. Soc. Edinb.* xxxiii. and xxxv.). But the exact value of the constant does not concern us in our present problem, which is to determine the viscosity of a mixture of gases in terms of the viscosities of the pure gases and not absolutely. Accordingly we can avail ourselves of the above easy and convenient method of evaluating the viscosity of a mixture. With v the mean velocity of the molecules of the single gas, and ν the average number of collisions experienced by a molecule in a second, then $\lambda = v/\nu$, and our result for a single gas becomes

$$\eta = nmv^2/6\nu. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now according to the kinetic theory for forceless, smooth, perfectly restitutional, spherical molecules of radius a , and mean square velocity $3\kappa^2/2$ (see, for instance, Tait, *Trans. Roy. Soc. Edinb.* xxxiii.),

$$\nu = 2n(a+a)^2\pi^{\frac{1}{2}}(\kappa^2 + \kappa^2)^{\frac{1}{2}}; \quad . \quad . \quad . \quad . \quad (2)$$

and by these two equations the viscosity is expressed solely in terms of the given mechanical constants of the system of molecules.

In the case of a mixture of two gases, 1 and 2, let the proportions be n_1 molecules of 1 to n_2 molecules of 2 in unit volume; then the number of encounters that a molecule of 1 experiences in unit time with molecules 2 may be denoted by ${}_1\nu_2$, and

$${}_1\nu_2 = 2n_2(a_1+a_2)^2\pi^{\frac{1}{2}}(\kappa_1^2 + \kappa_2^2)^{\frac{1}{2}}; \quad . \quad . \quad . \quad . \quad (3)$$

(see Tait, *Trans. Roy. Soc. Edinb.* xxxiii.); and the number of encounters per second of a molecule of 1 with other molecules of 1 is

$${}_1\nu_1 = 2n_1(a_1+a_1)^2\pi^{\frac{1}{2}}(\kappa_1^2 + \kappa_1^2)^{\frac{1}{2}}. \quad . \quad . \quad . \quad . \quad (4)$$

Then v_1 being the average velocity of a molecule of 1, its mean free path in the mixture is

$$\lambda_1 = v_1/({}_1\nu_1 + {}_1\nu_2). \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Then if, as in the case of a single gas, we assume that a single collision even between molecules of 1 and 2 suffices on the average to bring them to the characteristic state of the layer in which they collide, we can take the viscous action due to the molecules of 1 as given by the above simple theory of a single gas, namely $n_1m_1v_1^2w/6({}_1\nu_1 + {}_1\nu_2)D$. But of course this assumption is quite unjustifiable, except in the case where the molecule of 1 has the same mass as that of 2, in which case it becomes just as allowable as in that of a single gas, neither more nor less. When m_1 is different from m_2 , then if the two solid planes and the mixed gas between them were at

rest, the effect of collisions between the two sets of molecules would be to keep $m_1\kappa_1^2 = m_2\kappa_2^2$. If one of the solid planes is set in motion with velocity w , then on the average a molecule of 1 and a molecule of 2 coming from opposite sides of the layer at x to collide in it will, in addition to the velocities of agitation, have velocities $(x + \lambda_1/2)w/D$ and $(x - \lambda_2/2)w/D$. And the velocity of the centre of mass of the two molecules is not $w x/D$, characteristic of the layer at x ; so that to assume that a single collision between a 1 and a 2 suffices to bring both to the state characteristic of the place where the collision occurs is much less justifiable than in the case where 1 and 2 are identical. It is true that, corresponding to the pair we have just considered, there is another pair whose relative positions are the reverse, so that the velocity of the centre of mass of the four molecules both before and after their collisions is the $w x/D$ characteristic of the layer where they collide; but it is quite evident that there must be further collisions before the four molecules can be treated as typical.

It simplifies the course of the work if for the moment we ignore this difficulty and push on for an expression for the viscosity of the mixture on the assumption that all collisions are as effective as in a single gas: the result will be correct for a mixture of gases of equal molecular masses, and will be suitable for adaptation to the case where the molecular masses are different. For the rest of the viscosity due to the molecules of 2 we can write a similar expression to that just given, and get

$$\eta = \frac{n_1 m_1 v_1^2}{6(\nu_1 + \nu_2)} + \frac{n_2 m_2 v_2^2}{6(\nu_2 + \nu_1)}; \quad \dots \quad (6)$$

while for each ingredient of the mixture we have

$$\eta_1 = \frac{n_1 m_1 v_1^2}{6\nu_1}, \quad \eta_2 = \frac{n_2 m_2 v_2^2}{6\nu_2}. \quad \dots \quad (7)$$

But $m_1 v_1^2 = m_2 v_2^2$ and $m_1 \kappa_1^2 = m_2 \kappa_2^2$, and the equations given suffice for the elimination of everything but the η 's, m 's, and n 's; thus

$$\begin{aligned} \eta = & \frac{\eta_1}{1 + \frac{n_2}{4n_1} \left\{ 1 + (\eta_1 m_2^{\frac{1}{2}} / \eta_2 m_1^{\frac{1}{2}})^{\frac{1}{2}} \right\}^2 \left(\frac{m_1 + m_2}{2m_2} \right)^{\frac{1}{2}}} \\ & + \frac{\eta_2}{1 + \frac{n_1}{4n_2} \left\{ 1 + (\eta_2 m_1^{\frac{1}{2}} / \eta_1 m_2^{\frac{1}{2}})^{\frac{1}{2}} \right\}^2 \left(\frac{m_1 + m_2}{2m_1} \right)^{\frac{1}{2}}} \quad (8) \end{aligned}$$

So far our equations have related to forceless smooth perfectly restitutional spherical molecules; but as it has been

shown (Phil. Mag., Dec. 1893) that the effect of molecular attraction in the viscosities of gases and allied phenomena is important, we must take account of it here. It was also shown (Phil. Mag., Dec. 1893, and July 1894) that molecular attraction can be taken account of by replacing $(\alpha_1 + \alpha_2)^2$ and $(\alpha_1 + \alpha_1)^2$ in such equations as (3) and (4) by

$$(\alpha_1 + \alpha_2)^2(1 + {}_1C_2/T) \text{ and } (\alpha_1 + \alpha_1)^2(1 + {}_1C_1/T),$$

where ${}_1C_2$ depends only on the mutual potential energy of a 1 and a 2 in contact, and ${}_1C_1$ on the potential energy of a 1 and a 1, and T is the absolute temperature. On introducing the factors $1 + {}_1C_2/T$, $1 + {}_1C_1/T$, and $1 + {}_2C_2/T$ into equations (3) and (4) and eliminating as before, we obtain the equations

$$\left. \begin{aligned} \eta &= \frac{\eta_1}{1 + \alpha_1 n_2/n_1} + \frac{\eta_2}{1 + \alpha_2 n_1/n_2}, \text{ where } \alpha_1 \text{ is similar to } \alpha_2, \\ \text{and} \\ \alpha_1 &= \frac{1}{4} \left\{ 1 + \left(\frac{\eta_1 m_2^{\frac{1}{2}} (1 + {}_1C_1/T)}{\eta_2 m_1^{\frac{1}{2}} (1 + {}_2C_2/T)} \right)^{\frac{1}{2}} \right\}^2 \left(\frac{m_1 + m_2}{2m_2} \right)^{\frac{1}{2}} \frac{1 + {}_1C_2/T}{1 + {}_1C_1/T} \end{aligned} \right\} \quad (9)$$

to replace (8) when molecular attraction is taken account of. If α_1 and α_2 were each unity this equation would reduce to $(n_1 + n_2)\eta = n_1\eta_1 + n_2\eta_2$, which Graham found empirically to be true within the limits of experimental error for mixtures of N_2 and O_2 , CO and O_2 , and CH_4 and O_2 , but not for mixtures containing H_2 . Thus we see that the simplicity of Graham's result for these pairs of gases is the result of what may be called an accidental simplification of the true relation, brought about by a certain accidental approximate relation between certain physical constants of the two gases of each pair.

Let us write the last formula in the form

$$\eta = \frac{\eta_1}{1 + {}_1\nu_2/{}_1\nu_1} + \frac{\eta_2}{1 + {}_2\nu_1/{}_2\nu_2}; \quad \dots \quad (10)$$

and we can see that, as regards the general case of any two gases, its defect consists in making each of the ${}_1\nu_2$ encounters just as effective in producing viscosity as each of the ${}_1\nu_1$ or ${}_2\nu_2$. According to the ordinary mode of presenting the theory of viscosity of a single gas, which has already been sketched, it is not easy to see exactly how serious the defect may be or how it is to be remedied. But there is another method of regarding viscosity according to which we get more at the heart of the process.

In a single gas the average distance normal to the solid planes travelled by a molecule is $\lambda/2$, so that the molecules which collide in any thin layer may be considered as coming

from distances $\lambda/2$ parallel to the axis of x on opposite sides of the place of collision with velocities $w\lambda/2D$ and $-w\lambda/2D$ relative to the layer at x which moves with its permanent velocity xv/D . Now the total number of encounters occurring in unit time in unit volume of the layer is $nv/2$, of which half will occur between the molecules of the two sets, and the other half between the members of each set amongst themselves. The average value of the momentum communicated to one another by the molecules colliding with opposite velocities α and α is $2(\alpha + \alpha)mm/3(m + m)$, so that while a molecule brings in momentum $m\alpha$ to the layer, it loses on the average at the first collision only $2m\alpha/3$; thus, after one collision between each pair of molecules the distribution of velocities may be represented by evenly distributed radii of a sphere to represent the evenly distributed average velocities v , then with half of these velocities taken at random compound a velocity $w\lambda/6D$ in the direction of w , and with the other half at random compound a velocity $-w\lambda/6D$, so that the original spherical surface is resolved into two having their centres displaced in opposite directions by a distance representing $w\lambda/6D$. According to the usual method of presenting the theory of the viscosity of a gas, this distribution is assumed to be identical with that represented by the undisplaced sphere, and the assumption is plausible when it is remembered that the small variations introduced by compounding the small velocities $w\lambda/12D$ and $-w\lambda/12D$ with the average velocity v may be merged in the much larger variations represented by Maxwell's law of the velocities which we have ignored in considering only average velocities. And yet an assumption which makes the effect of an encounter practically independent of the amount of momentum communicated by the colliding molecules to one another must be erroneous, and must affect the usual expressions for viscosity with an error appearing in the value of the numerical coefficient. As each collision is only two thirds as effective in communicating momentum as it is assumed to be in the ordinary theory, we ought, in the usual expressions for the viscosity, to introduce an additional numerical factor $2/3$, which would cause the value of the mean free path of a gas as calculated from its experimental viscosity to be $3/2$ of the value usually given.

With this principle, that the effective value of a collision is proportional to the momentum exchanged by the colliding molecules, we can adapt our equation (9) to the general case of mixed gases by replacing ${}_1\nu_2/{}_1\nu_1$ by $({}_1\nu_2/{}_1\nu_1)({}_1\mu_2/{}_1\mu_1)$, where ${}_1\mu_1$ and ${}_1\mu_2$ are the average momenta exchanged when 1

collides with 1 and with 2; and we have now to obtain expressions for these momenta. I have not succeeded in doing this from first principles, but by the study of Graham's experimental data have been led to a simple result allowing of very simple interpretation, namely, that if in expression (9) the coefficients α_1 and α_2 are multiplied respectively by

$$\{2m_2/(m_1+m_2)\}^{\frac{1}{2}} \quad \text{and} \quad \{2m_1/(m_1+m_2)\}^{\frac{1}{2}},$$

all the experimental results can be represented by the formula so derived, namely,

$$\left. \begin{aligned} \eta &= \frac{\eta_1}{1 + \beta_1 n_2/n_1} + \frac{\eta_2}{1 + \beta_2 n_1/n_2}, \quad \text{where } \beta_2 \text{ is similar to } \beta_1, \\ \text{and} \\ \beta_1 &= \frac{1}{4} \left\{ 1 + \left(\frac{\eta_1 m_2^{\frac{1}{2}} (1 + {}_1C_1/T)}{\eta_2 m_1^{\frac{1}{2}} (1 + {}_2C_2/T)} \right)^{\frac{1}{2}} \right\}^2 \left(\frac{2m_2}{m_1+m_2} \right)^{\frac{1}{2}} \frac{1 + {}_1C_2/T}{1 + {}_1C_1/T} \end{aligned} \right\} \quad (11)$$

Thus the ratio ${}_1\mu_2/{}_1\mu_1$ is represented by $\{2m_2/(m_1+m_2)\}^{\frac{1}{2}}$, and we can easily trace the origin of the form of this; for the momentum exchanged in an average collision between an m_1 and an m_2 with velocities α_1 and α_2 being

$$2(\alpha_1 + \alpha_2)m_1m_2/3(m_1+m_2),$$

and between an m_1 and an m_1 being $2m_1\alpha_1/3$, the ratio of the two is $\{2m_2/(m_1+m_2)\}\{(\alpha_1+\alpha_2)/2\alpha_1\}$; and if this is equal to $\{2m_2/(m_1+m_2)\}^{\frac{1}{2}}$, then $(\alpha_1+\alpha_2)/2\alpha_1 = \{2m_2/(m_1+m_2)\}^{\frac{1}{2}}$, but if ${}_1\overline{V}_2^2$ and ${}_1\overline{V}_1^2$ are the average squared relative velocities of a 1 and a 2, and of a 1 and a 1, then

$${}_1\overline{V}_2^2/{}_1\overline{V}_1^2 = (\kappa_1^2 + \kappa_2^2)/2\kappa_1^2 = (m_1+m_2)/2m_2,$$

$$\therefore (\alpha_1 + \alpha_2)^2 \{{}_1\overline{V}_2^2\}^{\frac{1}{2}} = (2\alpha_1)^2 \{{}_1\overline{V}_1^2\}^{\frac{1}{2}};$$

so that if $(\alpha_1 + \alpha_2)^2$ and $(2\alpha_1)^2$ are regarded as quantities which are being carried across the mixed gas with average velocities proportional to ${}_1\overline{V}_2$ and ${}_1\overline{V}_1$, then the final meaning of our result is that they are carried at the same rate.

We have now to show how closely (11) represents the experimental facts, and with that view Graham's results are reproduced in the following tables from the Phil. Trans. 1846, or his collected Chemical and Physical papers, along with the necessary data for use in equation (11) and the results given by that equation. The necessary data are the viscosities of the two pure gases which are measured by Graham at temperatures near 15°C ., and were given by him in terms of that for oxygen at the same temperature as unity, also the molecular masses referred to that of H_2 as 2, and finally the

values of the constants ${}_1C_1$, and ${}_2C_2$, and ${}_1C_2$ as given in the Phil. Mag. Dec. 1893 and July 1894, or calculated on the principles there given.

Viscosity of Mixtures.

CO₂ and H₂.

	η_1 ·751	η_2 ·450	m_1 44	m_2 2	${}_1C_1$ 277	${}_2C_2$ 53	${}_1C_2$ 115
n_1/n_2			9	3	1	1/3	1/9
η experiment			·753	·752	·734	·679	·572
η calculated			·752	·752	·736	·671	·574

CO and H₂.

	η_1 ·870	η_2 ·450	m_1 28	m_2 2	${}_1C_1$ 100	${}_2C_2$ 53	${}_1C_2$ 70
n_1/n_2			19	9	3		
η exper.			·865	·865	·843		
η calcul.			·863	·858	·835		

CH₄ and H₂.

	η_1 ·550	η_2 ·450	m_1 16	m_2 2	${}_1C_1$ 215	${}_2C_2$ 53	${}_1C_2$ 102		
n_1/n_2	39	19	9	4·7	3	1	1/3	1/9	1/19
η exper.	·550	·561	·563	·557	·559	·560	·531	·496	·480
η calcul.....	·551	·552	·552	·556	·556	·554	·528	·494	·473

Air and H₂.

	η_1 ·900	η_2 ·450	m_1 28·9	m_2 2	${}_1C_1$ 113	${}_2C_2$ 53	${}_1C_2$ 74·5	
n_1/n_2		19	9	3	1	1/3	1/9	1/19
η exper.		·896	·889	·879	·820	·719	·588	·528
η calcul.		·893	·888	·864	·801	·685	·566	·511

CO₂ and O₂.

η_1 .	η_2 .	m_1 .	m_2 .	${}_1C_1$.	${}_2C_2$.	${}_1C_2$.	
·751	1·0	44	32	277	127	187	
n_1/n_2	19	9	3	1	1/3	1/9	1/19
η exper.	·756	·768	·807	·871	·942	·976	·987
η calcul.	·762	·773	·807	·867	·931	·974	·984

O₂ and CH₄.

	η_1 1·0	η_2 ·550	m_1 32	m_2 16	${}_1C_1$ 127	${}_2C_2$ 215	${}_1C_2$ 165
n_1/n_2			1	1/3	1/9		1/19
η exper.			·808	·690	·598		·583
η calcul.			·781	·667	·597		·573

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O₂ and N₂.

	η_1 1.0	η_2 ·875	m_1 32	m_2 28	${}_1C_1$ 127	${}_2C_2$ 109	${}_1C_2$ 117
n_1/n_2			1	1/2		1/3	1/9
η exper.			·935	·914		·905	·885
η calcul.			·937	·916		·905	·887

O₂ and CO.

	η_1 1.0	η_2 ·870	m_1 32	m_2 28	${}_1C_1$ 127	${}_2C_2$ 100	${}_1C_2$ 113
n_1/n_2			3	2	1	1/2	1/3
η exper.			·967	·959	·936	·913	·904
η calcul.			·967	·957	·933	·912	·900

The only pairs of gases for which Graham's data have not been given are N₂O and H₂ and NO and H₂, but the physical constants for N₂O are almost identical with those for CO₂, and the viscosities of its mixtures are almost the same as those of mixtures of CO₂; for NO some of the data necessary for the calculation are lacking.

Puluj measured the viscosities of a large number of mixtures of CO₂ and H₂ by the method of the vibrating disk (*Sitzb. Akad. Wien*, lxxix.), giving the absolute values at 14°·7 C.; thus η_1 is ·000147 and η_2 is ·000089; the other constants required in the calculation remain as above, and we have the following comparison of experiment and calculation values of $10^6\eta$:—

n_1/n_2 ...	9	4	2.6	·77	·36	·19	·11	·068	·04	·025	·017
Exper....	148	149	150	148	140	131	122	111	103	99	96
Calcul....	147	147	147	142	133	122	113	106	100	96	94

The agreement between the formula (11) and experiment is not so good here as in Graham's mixtures of CO₂ and H₂, which means that there are discrepancies between Graham's and Puluj's measurements; these are probably due to slight impurity to which the viscosity of hydrogen is sensitive. But even if the agreement is not good in the details of the last comparison, the main phenomenon of the experiments is closely paralleled by the formula, for in the results both of experiment and formula the addition of one volume of hydrogen to 2.6 volumes of carbonic dioxide hardly affects the viscosity of the carbonic dioxide.

The case of CH₄ and H₂ in the previous comparison is the most interesting, as both experiment and the formula make the viscosity of a mixture of equal volumes of the two gases greater than that of pure CH₄. It must be remembered that

the formula (11) has not been obtained from purely dynamical first principles, but in addition to the usual dynamical principles of the kinetic theory involves two unproved assumptions—first that formula (10), which applies rigorously to a mixture of two sets of molecules of equal mass but unequal size, can be adapted to the case of two sets of unequal mass and size by replacing the ratios ${}_1\nu_2/{}_1\nu_1$ and ${}_2\nu_1/{}_2\nu_2$ by ${}_1\nu_2{}_1\mu_2/{}_1\nu_1{}_1\mu_1$ and ${}_2\nu_1{}_2\mu_1/{}_2\nu_2{}_2\mu_2$, where ${}_1\mu_1$ and ${}_1\mu_2$ are the average momenta exchanged when 1 collides with 1 and with 2; and second, that the relative molar velocity of collision of a 1 with a 2, namely, $\alpha_1 + \alpha_2$, is connected with the relative molar velocity of collision of a 1 with a 1 ($2\alpha_1$) by the equation

$${}_1\overline{V}_2(\alpha_1 + \alpha_2)^2 = {}_1\overline{V}_1(2\alpha_1)^2,$$

where ${}_1\overline{V}_2$ and ${}_1\overline{V}_1$ are the mean relative molecular velocities of a 1 and a 2 and of a 1 and a 1. A dynamical proof of these two assumptions is required before the foregoing dynamical theory of the viscosity of mixed gases can be considered complete.

Melbourne, March 1895.

XXXVIII. *The Thermal Unit.* By E. H. GRIFFITHS, M.A.,
F.R.S., *Sidney Sussex College, Cambridge*.*.

[Plates VI. & VII.]

[The following Paper was read at the Ipswich Meeting of the British Association. The consideration of the whole matter has now been referred to the Committee on Electrical Standards.

The Author would be glad to receive any practical suggestions with a view to their communication to that Committee.]

INTRODUCTORY NOTE.

THE majority of physicists will, I think, agree with the statement that our present system of thermal measurements rests on an unsatisfactory basis. Nevertheless the attitude of the scientific world regarding this matter appears to be one of indifference; and it is open to speculation whether this indifference arises from non-appreciation of the importance of the subject, or from a sense of the difficulties to be encountered if any action is taken.

I am glad of this opportunity of directing the attention of Section A to such an important question, for I believe the obstacles to progress are not insurmountable, provided they are attacked with the same spirit and energy which characterized the efforts of this Association to perfect our system of electrical measurements. The prevailing apathy is possibly

* Communicated by the Author.