MR. COOPER.—“Is that an ejector in the exhaust pipe, with pipe dropping into the shaft cistern, and what purpose does it serve?”

MR. HENDERSON.—“That is a small ejector operated by the final effort of the exhausting steam. Should the fluid rise in the crank case above where it is wanted, it will also rise in the dip-pipe well, and seal the mouth of the dip-pipe, and be thrown out with the exhaust. At all other times the blast of the exhaust across the ejector will free the crank case from heated vapors and maintain it in a cool condition. Up to this time all the crank-case engines are running with cases heated up above 212° . The crank case in this engine will be cold.”

MR. COOPER.—“Allow me to thank Mr. Henderson for the care he has taken in the preparation of the paper and drawings illustrating his new engine, and for presenting them so clearly for the benefit of members of the INSTITUTE.”

LOSS OF HEAD IN HYDRAULICS.

BY IRVING P. CHURCH, C.E.

It is the purpose of the present article to offer some comment on Mr. Frizell's paper, in the October number, 1886, entitled “Coefficient of Efflux from an Orifice Furnished with a Short Pipe,” in which there seem to be evidences of misconception, or at least a very unusual view, of a fundamental point in hydraulics, viz., “loss of head.” It is true that the final result in Mr. Frizell's paper is correct, but the error as to “loss of head” in the first part of his work, is annulled by the subsequent introduction of what, from his standpoint, must be regarded as a “gain of head,” an idea which, in the case under treatment, would controvert the principle of the conservation of energy.

Mr. Frizell seems to proceed on the supposition that a loss of velocity (and, therefore, of velocity head $\frac{v^2}{2g}$), in the steady flow of water, implies a “loss of head” in the technical sense (*i. e.*, implying a corresponding loss of energy). It is worth while inquiring, then, what constitutes a “loss of head” in hydraulics, in the sense which implies a loss of energy. That there may be a

loss of velocity without any "loss of head," is immediately apparent when we examine that fundamental formula of hydraulics, viz., Bernoulli's theorem, for the steady flow of water without friction in (motionless) rigid vessels and pipes, as demonstrated and illustrated in all text books on hydraulics (see also Eq. (1) and context in Section 26 of the article, "Hydro-mechanics," *Encyclopædia Britannica*.) This theorem may be thus written:

$$\frac{v_m^2}{2g} + \frac{p_m}{\gamma} + z_m = \frac{v_n^2}{2g} + \frac{p_n}{\gamma} + z_n \quad (B)$$

in which m and n are any two positions (fixed in space) in the same stream line, while v_m and v_n denote the velocities of the water passing these respective points; p_m and p_n the internal pressure at m and n , and z_m and z_n the respective heights of m and n from an arbitrary horizontal datum plane situated below both m and n . γ denotes the weight of a unit of volume of water, and g the acceleration of gravity. Naming the terms in Eq. (B), each one of which is in quality a height, or "head," we may say, therefore, that in steady flow without friction, *the sum of the velocity head, pressure head and potential head* at any position (m) in a stream line, is equal to the sum of the corresponding heads at any other position (n) of the same stream line.

If each term in Eq. (B) is multiplied by W , the weight of water flowing per unit-time, it becomes an equation of energy, announcing that without friction the energy present at m , viz., the sum of the three forms, *kinetic energy, pressure energy*, and potential energy, is equal to the corresponding sum at n . [Rankine, p. 100, *Steam Engine*, uses the term *actual energy* for kinetic, and potential energy for the sum of what are here called pressure and potential energy.] It is, therefore, quite evident that if m and n are at the same level, and the kinetic energy at n is greater than that at m by a certain amount, then the pressure energy at m exceeds the pressure energy at n by an equal amount. (See p. 467, of "Hydro-mechanics," *Ency. Britt.*, Figs. 33 and 34, for experiments by Mr. Froude, illustrating this fact.)

If, however, friction cannot be avoided (either *external*, against the vessel walls, or *internal*, through eddying, cross-currents, etc.), or if a motor of proper construction be interposed between m and n , m being down-stream from n , we find that in steady flow the

sum of the three heads at m is always less than that at n , and the deficiency is properly called a "loss of head" (Rankine) or "height of resistance" (Weisbach), and the product of this loss of head by the weight of flow per unit-time gives the energy spent by the water in friction (uselessly) or upon the motor (usefully) per unit-time (*i. e.*, the power). In the case of friction, or internal disturbance, loss of head is peculiarly well-named (as the lost energy cannot be regained in any manner), and is found to be so nearly proportional to the square of the velocity and independent of the pressure, that it is usually expressed in the form

$$\zeta \frac{v^2}{2g},$$

where ζ is a coefficient, found by experiment, and is approximately constant for a given fitting, such as an elbow, bend, short section of pipe having a sudden enlargement, etc., and v is the velocity where the disturbance occurs (or just down stream from it, if there is a change of section). This velocity may be an unknown quantity in some problems, but can usually be expressed in terms of the velocity at the discharging end of pipe.

(The only objects of introducing the $2g$ are that then, $v^2 \div 2g$ being a height, ζ must be an abstract number, and hence the same for any system of units, while $1 \div 2g$ occurs as a factor in other terms, rendering algebraic reduction somewhat simpler. Perhaps this will be satisfactory to M. Cazin. See article by him in *Van Nostrand's Mag.*, for Sept., '86.)

Hence, we may write Bernoulli's theorem, with friction, as follows (m being the down-stream position):

$$\frac{v_m^2}{2g} + \frac{p_m}{\gamma} + z_m = \frac{v_n^2}{2g} + \frac{p_n}{\gamma} + z_n - \sum_n \zeta \frac{v^2}{2g} \quad (\text{BF})$$

(As is well known, for a long pipe

$$\zeta = 4f \frac{l}{d},$$

where f ranges from .0050 to .0100, or more, and l = length and d = diameter of pipe.)

Let us apply eq. (BF) to the case of flow into the air through a circular "orifice in thin plate" in the vertical side of a large tank. Let h = vertical height of still surface (exposed to air) above centre

of orifice. Call the (practically) still surface position n , and take position m in the contracted section of jet where the filaments are parallel and consequently under atmospheric pressure. Also, let the datum plane pass through m . Then we must substitute

$$v_n = 0, \frac{p_n}{\gamma} = \text{height of water barometer} = b, z_n = h;$$

$$\frac{p_m}{\gamma} = b, z_m = 0,$$

while from experiment v_m has been found to be

$$= \phi \sqrt{2gh}$$

(ϕ being the "coefficient of velocity" for this case), so that

$$v_m^2 = \phi^2 2gh.$$

It is required to find ζ for the loss of head

$$\zeta \frac{v_m^2}{2g},$$

due to friction around the edges of orifice and contiguous portions of vessel wall. Hence, we obtain

$$\phi^2 h + b + 0 = 0 + b + h - \zeta \phi^2 h \quad (1)$$

whence

$$\zeta = \frac{1}{\phi^2} - 1 \quad (2)$$

Again, *Fig. 1*, let us consider a horizontal short pipe in the vertical side of the large tank, the case treated by Mr. Frizell (p. 287, JOURNAL OF THE FRANKLIN INSTITUTE, Oct., '86).

As before, take n in water surface, and m in the jet at outlet of pipe, where the filaments are parallel and pressure atmospheric; for m , however, let us write 2 to assume Mr. Frizell's notation. Call the contracted section 1 and denote by C the ratio which its

sectional area F_1 bears to that of the pipe F_2 ; *i. e.*, $F_1 = CF_2$, C being a "coefficient of contraction." Hence, $v_1 = v_2 \div C$. The loss of head between n and 1 is

$$\zeta \frac{v_1^2}{2g},$$

(where ζ has probably the same value as just found in eq. (2), though in the present case the pressure at 1 is much less than atmospheric,) and that lost between 1 and 2 (the *only* loss, begging Mr. Frizell's pardon, if we neglect the slight skin friction in pipe) is

$$\frac{(v_1 - v_2)^2}{2g}$$

by Borda's formula, for a sudden enlargement, *i. e.*,

$$\left(\frac{1}{C} - 1\right)^2 \frac{v_2^2}{2g}.$$

With these two losses of head, then, equation (BF) gives

$$\frac{v_2^2}{2g} + b + 0 = 0 + b + h - \zeta \frac{1}{C^2} \frac{v_2^2}{2g} - \left(\frac{1}{C} - 1\right)^2 \frac{v_2^2}{2g} \quad (3)$$

From (3), after substituting for ζ from eq. (2), we have

$$v_2 = \frac{1}{\sqrt{1 + \frac{1}{(\phi C)^2} - \frac{1}{C^2} + \left(\frac{1}{C} - 1\right)^2}} \sqrt{2gh} \quad (4)$$

If now we assume that C is equal to the coefficient of contraction for discharge through the circular orifice in thin plate into the air, then $\phi C = \mu$ = the coefficient of efflux for that orifice and eq. (4) reduces to Mr. Frizell's result. If with Prof. Unwin (*Encyc. Britann.*) we disregard the loss of head between n and 1, (4) reduces to

$$v_2 = \frac{1}{\sqrt{1 + \left(\frac{1}{C} - 1\right)^2}} \sqrt{2gh} \quad (5)$$

which is Prof. Unwin's form.

(Mr. Frizell has not quoted Prof. Unwin with strict accuracy, having replaced the latter's coefficient of contraction by μ , the coefficient of efflux for the thin-plate orifice.)

Since the only difference between Prof. Unwin's and Mr.

Frizell's treatment of this case (as far as the final result is concerned) consists in the consideration, by the latter, of the entrance friction, which the former does not introduce into his analysis, though he refers to it in the context; and since this friction is not a matter "of impact and momentum," like the resistance due to sudden enlargement further on, it is difficult to see that Mr. Frizell has presented the matter in any new light, except in advancing the novel idea that loss of head (or loss of energy as he calls it) depends only on loss of velocity, but that this idea is erroneous is made evident not only by examining Bernoulli's theorem, as already noted, but also in a striking manner by working a simple numerical example as follows: Let $h = 16$ feet in *Fig. 1*, then we compute v_2 to be $\cdot 815 \sqrt{2gh} = 26\cdot16$ feet per second. Taking 2 as datum plane, we see that the total head at n is composed of a velocity head of zero, a pressure-head of 34 feet, and a potential head of 16 feet; while at 2 we have a velocity-head of $(26\cdot16)^2 \div 2g = 10\cdot62$ feet, a pressure head of 34 feet, and a potential head of zero. The total loss of head then is $(0 + 34 + 16) - (10\cdot62 + 34 + 0) = 5\cdot38$ feet. In other words, each pound of water flowing has lost 5·38 foot-pounds of energy between n and 2, and no more, and this loss is mainly due to the sudden enlargement between 1 and 2.

But Mr. Frizell asserts that between positions 1 and 2 alone a loss of head has occurred of an amount

$$= \frac{v_1^2}{2g} - \frac{v_2^2}{2g} = \frac{v_2^2}{2g} \left(\frac{1}{C^2} - 1 \right),$$

which if C be taken equal to 0·64 reduces to 15·30 feet, *almost the entire 16 feet head (= h) under which efflux takes place.*

Aside from the facts already referred to on p. 467 of "Hydro-mechanics," *Encyc. Britann.*, a simple experiment with a common flexible rubber tube will convince any one that the loss of velocity occurring between the narrow and full section of a *gradual* enlargement, when the difference of sectional areas is not extreme, does not imply a corresponding loss of head, the loss of kinetic energy being made up by an equal gain of pressure energy. Suppose the tube attached at one end to a faucet and a jet at about 45° with the horizontal to be issuing from the other extremity, the flow steady. It will now be observed that a very considerable reduction of sec-

tion may be effected at any part of the tube (not so near the jet as to affect the area of its section) by compressing it between the thumb and finger, without appreciably affecting the range of the jet. [See also p. 885, Coxe's *Weisbach*.]

Another instance in point is the "Whirlpool Chamber," invented by Prof. James Thomson, and forming a special feature in the centrifugal pumps manufactured by Williamson Brothers, Kendal, Eng.

This chamber surrounding the centrifugal pump, by furnishing a gradual enlargement of passage way, gives opportunity for the kinetic energy stored in the water, where it leaves the pump blades with considerable velocity, to be gradually transformed into pressure energy, instead of being wasted in internal friction (*i. e.*, eddying), as with other kinds of centrifugal pumps.

As practical examples of the utilization of water-power in each of the three forms just mentioned, it is hardly necessary, perhaps, to call to mind the following familiar and well-marked cases:

(1.) The *vertical water wheel*, using energy in the potential or gravity form; *i. e.*, in passing through the wheel the water experiences no notable change, either in pressure or velocity, but simply in vertical position.

(2.) The *water-pressure engine*, which receives the liquid under pressure and rejects it under a considerably smaller pressure (less than atmospheric if a "draft-tube" is employed), no considerable change in velocity or vertical positions having occurred; *i. e.*, the motor uses the power in the pressure form.

(3.) The turbine of free deviation (and the Pelton "Hurdy Gurdy"), where the water has the same pressure and same vertical position at exit as at entrance, while its absolute velocity has been greatly reduced; *i. e.*, energy has been utilized in the kinetic form.

Cornell University, April 4, 1887.