XLIV.—*On the Laws that Govern the Rounding of Particles of Sand.* By WM. MACKIE, M.A., M.D. (Plates XVIII. and XIX.).

(Read 29th April 1897.)

IN my inquiry into the origin of the sandstones of Eastern Moray I have had frequent occasion to note the various degrees of rounding presented by the various mineral particles that enter into their composition. During the last three years I have examined a large number of specimens from all the sandstone formations represented in the area, and from all important divisions of these formations. Amid much that was variable in the highest degree, it soon became evident that so far as the rounding of the constituent particles was concerned, a certain amount of uniformity prevailed, and though from a very early period of the inquiry it was no less evident that quite a number of factors entered into and co-operated to produce the results observed, it was nevertheless soon recognised that these results were fairly constant, and in the majority of cases strictly comparable from specimen to specimen. In order if possible to discover guiding principles, the better to compare past with present conditions, and generally it might be to illustrate both, I examined the sands of the rivers of the adjoining land area, the sands of the shore of the Moray Firth, and the sands of the sand dunes of Culbin. All these, though often in less degree, appeared to illustrate the operation of the same general principles. It was very early noted that the size of the particles within certain fairly defined limits at either end had a decided bearing on their degree of rounding. Over and over again it was observed that the larger grains of a sand or sandstone are usually the more rounded. Only once, or at most twice, has an exception been noted in beds at the very base of the Upper Old Red series. In many cases, even in the same specimen, in passing from the smaller to the larger grains, the transition from angular to rounded forms was found to take place quite suddenly. A very slight increase in the diameter of the particles often made all the difference between roundness and angularity. The degree of rounding was therefore apparently not in the simple ratio of the diameters of the grains. Size naturally suggested

LAWS THAT GOVERN THE ROUNDING OF PARTICLES OF SAND. 299

weight as an element in the case, and when particles of approximately equal size but of different specific gravities were examined side by side in the same specimen, it was found to be the general rule that the particle of higher specific gravity was the more rounded. Thus, of fragments of quartz and zircon of nearly equal size, the zircons would in nearly all cases be out of all comparison the more rounded of the two groups. Then, again, it was observed that where the size as well as the specific gravities of the particles were practically equal, but an appreciable difference existed in their degrees of hardness, it was generally found that the particles of the mineral with the lower degree of hardness were in the vast majority of instances the more rounded of the two. Thus, where all the particles of a sand or sandstone exhibited a certain degree of rounding, the feldspars would almost always show more rounded forms than the quartz grains. Here it will be observed that the amount of rounding is in the indirect ratio of the hardness of the mineral. Another factor in the results, and one that naturally suggests itself, is the distance over which the different particles have travelled. Of this, of course, in most cases there was little chance of judging; but where the origin of particles could be determined with more or less exactness, the greater distance of travel coincided in a large number of instances with the greater degree of rounding. Exceptions occurred. Thus it was observed in the case of a particular class of grains—the quartz grains of the Netherly diorite near Rothes—that in L.O.R. beds, fourteen miles in direct line from the parent rock, they were always found perfectly angular, while in the U.O.R. beds, only five miles away, they were almost always beautifully rounded, and in still another series of undetermined age, but in microscopical characters agreeing most closely with the L.O.R. beds—I mean in the *Asterolepis* beds, Nairn—at twenty-five miles distance they were present in fine grains, conspicuous by their roundness among the generally angular debris of that rock. Then, again, the constituent particles of sandstones of undoubted L.O.R. age were found to be invariably angular, of the U.O.R. series to be of mixed character, rounded and angular grains occurring in varying proportions in the several divisions of the group, and even in different beds of the same division, of the 'Reptiliferous' beds, to be all of them, small and large alike, rounded to a degree to which no parallel could be found among recent sands, while in the small portion of Oolitic sandstone falling within the area, they were again angular in the highest degree. These observations suggested differences in the agents of transport as well as variations in the conditions of deposit, and when in relation to the rounding process the action of the

agencies known to us came to be investigated, it was at once seen that the sands of Culbin, which owe their form largely to the action of wind, were, after a transit of only a few miles, much more rounded than sands of the same degree of fineness after many miles of water carriage, and much more rounded even than the same sand after prolonged churning by the waves in the tide-way of the sea-shore. It was thus evident that at least five factors were concerned in the result:—

- (1) The size of the particles.
- (2) Their specific gravity.
- (3) Their hardness.
- (4) The distance over which they had travelled.
- (5) The agent by which they had been transported.

The amount of rounding of a particle varies directly as (1) (2) (4); and indirectly as (3). We may therefore express the total amount of rounding (R) as proportional to or varying with the size of the particle multiplied by its specific gravity, multiplied by the distance it has travelled, the whole divided by its hardness, thus—

$$R \propto \frac{\text{Size} \times \text{specific gravity} \times \text{distance}}{\text{Hardness}}$$

Size and specific gravity come together as weight or mass. Thus the weight of a cube of which the side is x is expressed by $x^3 \text{ spg.}$

Distance of travel is more accurately expressed as the number of times the particle has turned on its axis in covering the stated distance (d). This is easily expressed, as the distance divided by the rolling circumference of the particle. It is to be supposed, of course, that in each revolution the same amount of work towards the rounding process is done. This, however, is only approximately true, as the weight of our hypothetical particle is gradually lessening, and its rolling circumference slowly changing in outline as well as lessening in amount. But as these two effects tend to balance one another,

$\frac{\text{distance travelled,}}{\text{rolling circumference of particle}}$ will fairly represent the number of revolutions of particle in covering the distance d . For our hypothetical cube this will be $\frac{d}{4x}$. Substituting these

$$\text{quantities we get } R \propto \frac{x^3 \text{ spg.} \cdot \frac{d}{4x}}{h} = \frac{x^2 \text{ spg.} d}{4h} \text{ (or generally } \frac{x^2 \text{ spg.} d}{mh}$$

where m varies with outline of figure).

LAWS THAT GOVERN THE ROUNDING OF PARTICLES OF SAND. 301

This is the result for a cube. For other figures the result is much the same, only the coefficient of h varying. For a sphere the coefficient of h would of course be 3.1416, which is the lower limit of variation. For all symmetrical figures the coefficient of h could be easily determined, but as the particles with which we have to deal are for the most part of very irregular, of very different and indeterminate shapes, the coefficient of h may be neglected altogether. It is only in the case of idiomorphic particles that it need be considered at all.

The above result shows that for particles of the same substance of the same form but of different sizes, distance remaining constant, R varies as the squares of their diameters. For particles of different substances, size, shape, and distance being constant, R varies as

$$\frac{\text{the specific gravity of the particles}}{\text{their hardness}} = \frac{spg}{h}.$$

This gives a different value for each mineral, but one which is practically constant for the same mineral. Such value may be called the coefficient of roundability,¹ or the specific "roundability" of the particular mineral. The values for a few of the commoner substances observed in sands have been calculated

out in the table given below. The above formula $\frac{spg}{h}$ holds for

air. For water a slightly different formula obtains. All bodies lose weight in water by an equal volume of water. The weight of our hypothetical cube in water would therefore be $w^3(spg - 1)$, so that the coefficient of roundability becomes for water $\frac{spg - 1}{h}$.

This, when calculated out, gives of course a series of lower values for the different minerals.

In calculating out the following table the average of all the specific gravities and the average of all the hardness values given opposite the different minerals in Dana's System of Mineralogy, 5th edition, have been used.

¹ This is a terrible hybrid. A classical friend to whom I applied suggests "*Strongylidity*" (from *στρογγύλος*, round, or *στρογγυλίζω*, I make round.) This word has a *strong* Saxon aroma which would be likely to be suggestive of qualities of a different order. My own inclination is to "*psephicity*" (from *ψμφος*, a pebble), naturally meaning *facility for being made into pebbles*.

Table of Relative "Psephicities."

Mineral.	In Air. $\frac{Spgr.}{h}$	In Water. $\frac{Spgr. - 1}{h}$
1. Quartz,	.38	.23
2. Orthoclase,	.4 [Kaolinite 1.4]	.29
3. Andalusite,	.42	[Compact varieties : Impure much less].
4. Tourmaline,	.43	.3
5. Labradorite,	.45	.29
6. Staurolite,	.5	.36
7. Garnet,	.53	.39
8. Hornblende,	.57	.39
9. Zircon,	.59	.45
10. Kyanite,	.6	.43
11. Sphene,	.66	.7
12. Rutile,	.68	.51
13. Magnetite (as type of iron ores),	.86	.7
14. Biotite	1.05	.7

Observation over a comparatively large number of examples generally confirms the theoretical order given above as the order which approximately obtains in nature. It is of course impossible to eliminate differences in the distances of travel of the different particles or even to make any but the roughest allowance for relative size, but when this has been done some exceptions to the theoretical order are found to exist. Such exceptions are frequently traceable to some peculiarity of the particular mineral, oftener perhaps to errors of specific determination in the mineral groups.

Among some of the peculiarities of minerals that give rise to exceptions, the following may be mentioned :—

(1) *Susceptibility to chemical change.* This usually operates in the direction of lowering their hardness, and it is possible that if the appropriate correction in this respect could be made they would still be found to fall into line. Felspar when highly kaolinised, as it is for most part in river sands, gets rounded too quickly and is an exception coming under this heading.

(2) *Certain conditions of cleavage.* Some minerals seem to exfoliate along their dominant cleavage plane under the rounding agencies and thus continue to present angular outlines. Their ultimate fragments appear however to obey the law. Horn-

LAWS THAT GOVERN THE ROUNDING OF PARTICLES OF SAND. 303

blende, and to less extent felspar, are exceptions in this respect.

(3) *Vesicular habit* or the presence in a mineral of large numbers of liquid or gas pores seems to increase its tendency to angularity. This may be due to the fact that the specific gravity is thereby reduced. But it is manifestly not the whole case. Why, for instance, should garnets be found in some sands—notably the Culbin Sands—in very small and markedly angular grains? It is interesting to remember in this connection that the probable tendency of vesicular quartz to disappear was one of the results of my previous paper.

(4) *Variations of original outline.* This applies only to idiomorphic particles and is fully covered by the varying coefficient of h in the general formula. It explains why a crystal of zircon should be more speedily reduced to a rounded form than a crystal of magnetite. Zircons, however, are frequently seen in their original matrix in the crystalline rocks as rounded blebs without any trace of crystalline outline. This would explain the marked tendency of zircon to come out too high in the results which have been worked out below. Originally idiomorphic zircons take on characteristic spindle-like forms after a certain degree of rounding.

(5) *Very high degrees of hardness* probably give rise to new conditions. Particles appreciably harder than quartz, from failing to come in contact with substances as hard as or harder than themselves, would not be likely to be sensibly changed in shape after a very long journey. Thus topaz would have a coefficient of .44 in the column for air and of .31 in the column for water—that is to say, greater than quartz in both cases; while the coefficient of diamond would be .35 in air and .25 in water, or less than quartz in air and slightly greater than it in water. I have observed what I believed to be one or two microscopic topazes in the matrix of a L.O.R. conglomerate at Gollachy Mill. Their crystalline outline was perfect, but the same was true of some of the garnets that were present in abundance in the same specimen. With the behaviour of the diamond, under these conditions, I have no acquaintance; but it is certainly not to be expected that it is more susceptible in this respect than quartz. After all, the case is probably not limited to a problem in pure friction, but includes the results of direct impact as well. In that case a certain degree of rounding may be expected, even in the very hardest minerals. Reference is sometimes made in books to the frequency with which zircon and rutile occur in idiomorphic forms in sands, due, it is supposed, to their hardness. No doubt they do so occur, but so also do garnets many times their size, and the

occurrence of fragments of felspar, bounded by regular cleavage planes, is not infrequent, both in sands and sandstones. If it were the habit, and not the exception, for quartz to occur in idiomorphic form, we should no doubt frequently find it so occurring in sands. It has been suggested, on the other hand, that idiomorphic zircons and rutiles have been liberated *in situ* by the decomposition or breaking up of muscovites or biotites, in which they formerly existed as inclusions, and in which they had been carried to the position where they are now found. As micas have been repeatedly observed both in sands and sandstones, with crystals of zircon and rutile on the point of dropping out of them, while others have been seen showing empty spaces, with the regular outlines of these minerals from which they had presumably already dropped, the latter view seems at least to contain a modicum of truth.

(6) *Difficulties of specific determination* probably account for not a few anomalies. Thus tourmaline and hornblende are not always easily distinguished. With nearly spherical particles it is manifest that the occurrence of the darkest pleochroic tint, when the long axis of the polarising prism is parallel to or at right angles to the *long* axis of the fragments, has really no meaning, and a little reflection will show that, even where there is a longer and shorter diameter in our hypothetical particle, these differences need not be absolutely diagnostic of tourmaline or hornblende. Many particles are also too thick to show reliable pleochroic differences. With a little experience, however, it is usually possible to decide, with more or less accuracy, from the general characters of the respective minerals. Again, what are put down as "iron ores" in the following determinations are manifestly not all iron ores. Opacity, which has been taken as the determining characteristic, may cover fragments of organic matter or particles of minerals not transparent, unless of an excessive degree of thinness. Hence this is one reason why the iron ores, in all the specimens which have been discussed in detail, come out relatively too low.

(7) *The impurity or composite character of the mineral fragments* also accounts for some anomalies. The iron ores also come under this heading. More frequently than other minerals, these have been observed to carry fragments of the matrix of the original rock along with them. These being mostly of quartz or felspar tend to reduce the specific gravity of the composite particle, and hence bring out the result too low. They also interfere with the regularity of the rounding process, which naturally leads to a like result.

(8) *Idiosyncrasies of some minerals*. Thus magnetite, from its attracting other particles, is apt, in transmitted light, to give

Fig. A. ($\times 30$)
Quartz Grain, from Quartz-Mica Diorite, Netherly, Rothies
Showing numerous (Rutile?) needles.

Fig. B. ($\times 50$)
Quartz Grain showing Cleavage Structure, from Glengrant Granite.

the impression of a single angular fragment, whereas what is really observed is an aggregate, made up of several rounded fragments.

Most of these exceptions, it will be seen, arise from the difficulty or impossibility of precise determination on the objective side, and it is reasonable to suppose that if greater precision were attainable in this respect, an even closer conformity to law than is observed would be easily demonstrated. At the same time it is worth noting that there are some minerals that, were we to decide the question on *a priori* grounds, would be naturally ranked as exceptions which are found to be in notorious conformity to law. Such a mineral is muscovite. The flat plates of this mineral are the first to show the rounding process, and appear in many sandstones, and notably in sea sand, as ovoid or nearly circular discs. From this result it would appear that they have a tendency to get on their edges and be trundled along by the current, somewhat after the manner of a boy's hoop. Biotite exhibits the same peculiarity, but though it comes out relatively to the muscovite invariably in the right position, still, in relation to the other minerals it is always too low. This is due to its higher susceptibility to chemical change and consequent greater friability, so that in the preliminary treatment of a sand it readily goes to pieces. It is frequently met with in crescentic fragments with one border rounded, the other frayed and irregular, due to the breaking up of originally rounded plates.

Several examples of sands and sandstones containing a proportion of heavy particles have been worked out in detail, and the relative amount of rounding of the several mineral groups noted. The list of minerals thus worked out—though it must be admitted by a highly artificial method—follow more or less closely the order derived from calculation. Such exceptions as occur may be readily explained by actual differences in the average size of the particles of the several mineral groups, by possible differences in the relative distances over which they have been carried, or by the operation of one or more of the subsidiary causes just described.

I. The first specimen of a sand examined in detail was one exceptionally rich in heavy particles, which I found in the collection of the late Dr Gordon, of Birnie, labelled "*Amethystine and Iron Sand, shore at Macduff, 1850.*" It was found to contain 41 per cent. of opaque particles, presumably in large part iron ores, 37 per cent. of garnet, 7 per cent. zircon, 6 per cent. staurolite, 1 per cent. quartz; and the remaining 8 per cent. was made up of kyanite, tourmaline, hornblende, hypersthene, and other undetermined fragments. In this specimen the characters

of the different particles as regards roundness or angularity were observed and noted, as shown in the following table:—

Mineral Particles.	No. Observed.	No. Rounded.	No. Subangular.	No. Angular.	Average Rounding.
Staurolite,	31	—	8	23	1.26
Garnet,	65	19	12	34	1.77
Opaque [Iron ores],	72	20	18	34	1.8
Zircon,	31	19	7	5	2.45

Average rounding represented by 1.82

The numbers in the last column are obtained by counting 3 for a rounded particle, 2 for a subangular, 1 for an angular, and dividing out the sum by the number of particles in each group—a highly artificial method, but one that facilitates comparison of the different groups one with another. Here the iron ores are out of place, and the zircons relatively too high—in all probability for reasons that have been already detailed.

II. A sample of river sand from the Spey at Craigellachie—*i.e.*, just before the river sand can possibly get contaminated by admixture with particles derived from the sandstones, was found, from its general angularity, to be incapable of numerical treatment, but its general characteristics were as here described. No appreciable rounding of the quartz grains was observed, though they certainly did not all of them present the marked angularity of the quartz grains in most boulder clays. The feldspars also were angular. Some of them being kaolinised in a high degree, did exhibit considerable blunting of their angles. The flakes of chlorite were generally rounded, but presented rather frayed, irregular edges. The muscovites showed some rounding of their prominent angles, but were generally of irregular outline. The same description applied to the biotites. Hornblende was present in minute ragged-edged fragments that showed no tendency whatever to rounding. Garnets were notoriously angular. The zircons occasionally showed ground glass surfaces, but were generally angular. These results fairly conform to law, but they are really so trivial as to prove nothing. Still they must represent, so far as the rounding process is concerned, in at least a proportion of the particles, the net result of not less than 70 miles of water carriage.

III. A very fine grained sandstone from the slab of *Holophtychius* found at Cuttishillock, now in the Elgin Museum, and presumably belonging to the U.O.R. series. This specimen was found to be very rich in heavy particles. These were roughly separated by the method of Carus Wilson,¹ and the micas in

¹ *Nature*, vol. xxxix. p. 591.

Fig. 1. ($\times 10$)
Matrix of L.O.R. Conglomerate.
Gollachy Mill, Port Gordon.

Fig. 2. ($\times 10$)
Slab of *Gordonia Duffiana*,
Elgin Museum.
From Reptiliferous Sandstone of
Cuttieshillock.

Fig. 3. ($\times 10$)
Culbin Sand.

Fig. 4. ($\times 10$)
Garnet Sand, from Shore near Macduff.

LAWS THAT GOVERN THE ROUNDING OF PARTICLES OF SAND. 307

another sample by levigation, and examined with the following result:—

Mineral Particles.	No. Observed.	Rounded.	Sub- angular.	Angular.	Average Rounding.
Tourmaline,	17	4	9	4	2
Biotite,	36	9	16	11	2
Opaque [Iron ores],	34	17	7	10	2.2
Zircon,	72	40	16	16	2.3
Sphene,	21	10	9	2	2.4
Muscovite,	71	48	18	5	2.5
Felspar, }	56	23	16	17	2.1
Quartz, }	58	14	14	30	1.7

Average rounding represented by 2.15

The quartz and felspar particles were at least twice the diameter of the other particles, and some allowance must be made for this. The opaque particles are again too low to be exclusively iron ores. Biotite is relatively to the muscovite in its right position, but is relatively too low as compared with the other minerals. The probable reason of this has been already given.

IV. A specimen from the Culbin Sands. This particular sample was fairly rich in heavy particles. Great differences in this respect have been observed in these sands, as also in regard to the general degree of rounding of the particles presented by different specimens, though it has, no doubt, been marked in all. At present I can only offer tentative reasons for these differences. The following results were obtained in the specimen examined:—

Mineral Particles.	No. Observed.	Rounded.	Sub- angular.	Angular.	Average Rounding.
Staurolite,	2	—	—	2	1
Kyanite,	2	—	1	1	1.5
Garnet,	45	13	14	18	1.9
Hornblende,	5	3	1	1	2.4
Tourmaline,	13	8	4	1	2.5
Opaque [Iron ores],	10	7	3	—	2.7
Sphene,	9	8	—	1	2.8
Zircon,	7	7	—	—	3
Felspar, }	16	10	5	1	2.6
Quartz, }	16	6	9	1	2.3

Average rounding represented by 2.27

Here the order is very much in accordance with law, but the number of particles in some of the groups is too small to insure

a fair average. The quartz and feldspars were here also much larger than the other fragments.

V. A specimen of sand from the sea-shore, in the sandy reach midway between Lossiemouth and Covesea, was found to be very poor in heavy particles. This sand gave the following results:—

Mineral Particles.	No. Observed.	Rounded.	Sub- angular.	Angular.	Average Rounding.
Hornblende,	10	—	5	5	1.5
Kyanite,	2	—	1	1	1.5
Biotite,	16	3	7	6	1.8
Opaque (Iron ore),	3	1	1	1	2
Chlorite,	4	1	2	1	2
Muscovite,	20	7	9	4	2.2
Quartz, }	38	6	13	19	1.7
Feldspar, }	26				
		8	13	5	2.1

Average rounding represented by 1.8

VI. A specimen from the Reptiliferous of Cuttishillock was found to be notoriously poor in heavy particles. So was a second specimen from the slab of *Gordonia Duffiana* in the Elgin Museum. [It is interesting to compare specimen No. III. with this. The difference as regards the number of heavy particles present is certainly remarkable.]

Mineral Particles.	No. Observed.	Rounded.	Subangular.	Angular.	Average Rounding.
Tourmaline,	1	—	1	—	2
Opaque (Iron ores),	6	4	2	—	2.7
Zircon,	5	5	—	—	3
Rutile,	1	1	—	—	3
Quartz, }	43	33	11	1	2.7
Feldspar, }	25	20	2	3	2.7

Average rounding represented by 2.7

The quartz and feldspar here come out equal. This confirms a previous impression that the feldspars were more frequently found angular relatively to the quartz particles than in any other formation. The general percentage of feldspar in this series being low, the reason probably is that most of the feldspars, having been ground to dust, only those that have been carried a relatively short distance remain, and these in consequence are in large proportion angular.

VII. A sample from the matrix of the L.O.R. conglomerate, Gollachy Mill, was examined. The particles of this specimen

LAWS THAT GOVERN THE ROUNDING OF PARTICLES OF SAND. 309

were generally so angular that results were not represented numerically. It showed a small number of heavy particles, among those being zircon, usually in idiomorphic, but occasionally in spindle-shaped forms. The iron ores were mostly sub-angular. The garnets were in angular fragments, but some of them showed idiomorphic outlines. Staurolite was present in markedly angular fragments, frequently showing chonchoidal fracture. The muscovites were many of them rounded, while some of the biotites showed crescentic edges. The feldspars were generally angular, though an occasional fairly rounded specimen was seen. Of the quartz grains some approached subangularity, The degree of rounding of these latter, though not much, was certainly higher than in the specimen of river sand above described, and the same remark applies to the average degree of rounding of the particles. In this specimen a large number of very angular fragments of a dark indigo-blue schist were separated among the heavy particles. Similar fragments, but of much less intensity of colour, have been observed in the sand of the river Fiddich—in fact the whole facies of the particles of this specimen have a remarkable resemblance to the sand of that river. The blue fragments have been provisionally referred to ottrelite, and their angularity is ascribed to their breaking up during the preliminary treatment of the sand, some regular crystals possibly to be referred to barytes, and evidently of secondary origin, were also observed in this specimen.

Collating the averages of these results we get:—

For	II. River sand of Spey, represented by a number	
	not much over,	1
„	VII. L.O.R., Gollachy certainly higher, but not,	1.5
„	I. Garnet Sand, Macduff,	1.8
„	V. Sea Sand, Lossiemouth,	1.8
„	III. Upper Old Red; Holoptychius Slab,	2.15
„	IV. Culbin Sands,	2.27
„	VI. Reptiliferous, Cuttieshillock,	2.7 ¹

It will thus be seen that the two highest results have been obtained, in the one case where the rounding of the particles is known, in large part, to be due,—in the other, where it is believed on hypothetical grounds to be due to the agency of winds.

This leads us to investigate why wind should be a more potent agent in the rounding process than water. In air the particles roll with their full weight. In water they lose weight, by the weight an equal volume of water, so that in water they

¹ These numbers are not in any way proportional to the degree of rounding of the several specimens. If we subtract 1 from each number the results should then be approximately proportional.

roll with only a fraction of their weight. This is one reason why much larger particles are moved by water than by wind. It should be remembered, however, that in the rounding of the larger particles or pebbles, it is only the exceptional force of the current that is concerned, whereas in the case of wind, a relatively large proportion of the work of rounding is probably done at comparatively low velocities.

To compare the relative efficiency of wind and water as rounding agents, we have—

$$\text{For wind, } R \propto \frac{x^3 spg d}{mh}, \quad \text{for water, } R = \frac{x^3 (spg - 1) d}{mh},$$

$$\therefore \frac{\text{wind agency}}{\text{water agency}} = \frac{spg}{spg - 1} \quad \text{or, for quartz} = \frac{2.65}{1.65},$$

or, wind = 1.6 times water.

But it is very evident, I think, that the relation is not of the simple kind now stated. The momentum of the moving particles at the time of impact must be taken into account—in other words, the amount of rounding will be in proportion to their capacity for work that is as $\frac{1}{2}mv^2$. Now $m = x^3 spg$ for air, and $x^3 (spg - 1)$ for water. So that $\frac{\text{wind agency}}{\text{water agency}} = \frac{\frac{1}{2}x^3 spg v^2}{\frac{1}{2}x^3 (spg - 1) v_1^2}$, where v and v_1 are the velocities of wind and water respectively.

This cancels down to $\frac{\text{wind agency}}{\text{water agency}} = \frac{spg v^2}{(spg - 1) v_1^2}$. Now as spg is always greater than $spg - 1$, and v than v_1 , the effect of wind over water must be marked.

Take the following hypothetical case for a particle of quartz of spg 2.65, the velocity of water being two miles an hour and wind eight miles an hour. What is the relative efficiency of the two agents?

$$\frac{\text{wind agency}}{\text{water agency}} = \frac{2.65 \times 64}{1.65 \times 4} = 29 \text{ nearly,}$$

or wind = 29 times water (nearly).

From the law of squares already worked out, this will mean that under the conditions stated, particles less than $\frac{1}{29}$ th the diameter of those rounded by water will be rounded to an equal degree by wind.

These are no doubt somewhat surprising results, but I am convinced that the difference between the two agencies in relation to the rounding of particles of sand is much greater than is generally supposed.

Note.—The relation of the size of a particle to the force required to move it is expressed by $x^3 spg \propto x^2 v^2$, x being the

LAWS THAT GOVERN THE ROUNDING OF PARTICLES OF SAND. 311

side of a hypothetical cube. Supposing the substance to be constant this becomes—

$$\left\{ \begin{array}{l} x^3 \propto x^2 v^2 \\ = x \propto v^2 \\ = \sqrt{x} \propto v \end{array} \right\} \text{ or } x^3 \propto v^6.$$

That is to say, the carrying power of wind or water varies as the 6th power of the velocity—a result, I believe, already arrived at for water (Prestwich, *Geology*, vol. i. page 83), but I have not been able to consult the reference there given.—Quart. Journal Geo. Society, vol. iv. pp. 92-93.

Starting with $\sqrt{x} \propto v$ (a much easier formula to use), we have an index of the velocity of the wind necessary to move a particle. Thus the Culbin sand is mostly about $\frac{1}{160}$ inch in diameter, but exceptional samples are found containing grains up to $\frac{1}{80}$. These, however, are less rounded than the $\frac{1}{160}$, showing that winds of sufficient velocity to move the latter are exceptional at the present day. In the Reptiliferous of Elgin, we have grains up to $\frac{1}{80}$, and these are more rounded than the $\frac{1}{160}$ of the present Culbins. If we take $\frac{1}{160}$ as representing the particle moved by the mean average velocity of the wind, say 10 miles per hour, then we have the diameters, being respectively as 1, 4, 8.

For Average Culbin Sand, $\sqrt{1} = 1 \times 10 = 10$

„ Largest Culbin Sand, $\sqrt{4} = 2 \times 10 = 20$

„ Largest Reptiliferous, $\sqrt{8} = 2.83 \times 10 = 28.3$ miles per hour, as the velocities required to convey them.

These results are only to be taken as approximate, but observations are in progress that make for establishing a more correct result. It may be concluded, however, that winds over 25 miles an hour in velocity were a much more frequent occurrence in Reptiliferous times than winds over 20 miles an hour are at the present day.