$= 2 \sin^{-1} \left(\frac{\sin b}{\cos a} \right)$; whence we see that A, B are poles with respect to the sphere of the cyclic arcs of the conic.

Also, since $AQ = \frac{1}{2}AP$, $AQ' = \frac{1}{2}AP'$, we see that—If through the pole of a cyclic arc an arc be drawn cutting a conic, the product of the trigonometrical tangents of the arcs is constant.

If now we cause a point P to describe a small circle on a sphere by means of a link having the same curvature as a great circle, we only require a linkage to enable us to bisect any arc AP.

Such a linkage is the following:—

$QB'P$, $QB'P'$ are two curved links $= 180^\circ$; $B'P$, PB' , $B'A'$, $A'B''$ are four equal links, $= \alpha$, a constant. It is evident that

Q, P, A', P' are four points on the same great circle, and $QP' = 180^\circ$.*

If, then, A be a point on the sphere, and A' its antipode,

$$AA' = 180^\circ = QP',$$

$$\text{or} \quad \begin{aligned} AQ &= A'P' \\ &= QP, \end{aligned}$$

or AP is bisected in Q.

The application of this to the description of a sphero-conic is evident.

A Parallel Motion. By Mr. HART.

[Read May 13th, 1875.]

I. ABCD is a skew parallelogram, and the links in the figure satisfy the following conditions:—

$$AB = CD = 2CM = 2r,$$

$$EB = BG = GC = \frac{1}{2}AD = p,$$

$$EF = FG = a,$$

$$NG = NK = NL = \frac{qr}{2a},$$

(where $q = r + p$),

$$LM = r - p;$$

then, if $rq^2 = 4a^2p$,

K always lies in LM produced,

G „ LG perpendicular to LM.

* By employing a linkage ABCD analogous to that given in the *Messenger*, Oct. 1874, it is easily seen that only five links are requisite for the description of a sphero-conic.

In the same way I deduced Mr. Kempe's motion, not knowing of its existence, from the other three-bar motion, which, owing to its symmetry, only gives one result; the general condition I gave (*Messenger*, Dec. 1874)

$$\{a^2m + b^2c\}^2 = b^2(a^2 - b^2)(a^2 - c^2);$$

putting $c + 2m = \pm a$, $2b^2 = \pm a \{\pm a - c\}$.

June 10th, 1875.

Prof. H. J. S. SMITH, F.R.S., President, in the Chair.

Messrs. Hunter, Rhodes, and Wright were elected Members; Mr. W. Tanner, B.A., Jesus College, Oxford, was proposed for election; and Mr. J. H. Röhrs was admitted into the Society.

Communications were made by Prof. Cayley, "On some Figures of Curves in Three-bar Motion; Dr. Sylvester, "On James Watt's Parallel Motion"; and Mr. T. Cotterill, "On the Correspondence of Points Collinear with a Fixed Origin." The Chairman and Dr. Hirst spoke on the subject of the last communication.

The following presents were received:—

"Bulletin de la Société Mathématique de France," tome iii., Avril, No. 1.

"Jahrbuch über die Fortschritte der Mathematik" (Ohrtmann, Müller), Fünfter Band, Jahrgang 1873, Heft i.

"Bulletin des Sciences Mathématiques et Physiques," Fev. 1875.

"Mémoires de la Société des Sciences de Bordeaux," tome x., 2^e cahier. Paris, 1875.

C. G. Reuschle, "Tafeln complexer Primzahlen welche aus Wurzeln der Einheit gebildet sind." Berlin, 1875.

"Sur la résolution des équations du 2^{me}, 3^{me} et 4^{me} degré par la fonction $r^n(X)$," par Dr. Axel S. Guldberg (20 Dec. 1872).

"Beretning om der internationale Meter-Kommissions Møde i Paris, 24 Sep.—12 Octbr. 1872," af Dr. O. J. Broch. Christiania, 1874.

"Partielle Differential-Gleichungen 1, 0, in denen die unbekannte Funktion explicite vorkommt von Sophus Lie" (besonderer Abdruck nach den Verhandlungen der Gesellschaft d. W. zu Christiania, 1873).

"Neue Integrations-Methode eines 2n-gliedrigen Pfaffschen Problems," von Sophus Lie (vorgelegt im October 1873).

"Ueber partielle Differential-Gleichungen 1, 0," von Sophus Lie.

"Zur analytischen Theorie der Berührungs-Transformationen," von Sophus Lie.

"Bidrag til Theorien for Dissociationen," af C. M. Guldberg (10 Mai 1872).

"Bemærkninger om Formelen for Højdemaaling med Barometer," af C. M. Guldberg.

The last eight pamphlets presented by the Royal University of Norway, Christiania.

On the Integration of Discontinuous Functions.

By HENRY J. STEPHEN SMITH, F.R.S.

1. Riemann, in his Memoir "Ueber die Darstellbarkeit einer Function durch eine Trigonometrische Reihe" (Abhandlungen der k. Gesellschaft der Wissenschaften zu Göttingen, vol. xiii., p. 87), has given an important theorem which serves to determine whether a function $f(x)$ which is discontinuous, but not infinite, between the finite limits a and b , does or does not admit of integration between those limits, the variable x , as well as the limits a and b , being supposed real. Some further discussion of this theorem would seem to be desirable, partly because, in one particular at least, Riemann's demonstration is wanting in formal accuracy, and partly because the theorem itself appears to have been misunderstood, and to have been made the basis of erroneous inferences.

2. Let d be any given positive quantity, and let the interval $b-a$ be divided into any segments whatever, $\delta_1 = x_1 - a$, $\delta_2 = x_2 - x_1$,, $\delta_n = b - x_{n-1}$, subject only to the condition that none of these segments surpasses d . We may term d the *norm* of the division; it is evident that there is an infinite number of different divisions having a given norm; and that a division appertaining to any given norm, appertains also to every greater norm. Let $\epsilon_1, \epsilon_2, \dots, \epsilon_n$ be positive proper fractions; if, when the norm d is diminished indefinitely, the sum

$$S = \delta_1 f(a + \epsilon_1 \delta_1) + \delta_2 f(x_1 + \epsilon_2 \delta_2) + \dots + \delta_n f(x_{n-1} + \epsilon_n \delta_n)$$

converges to a definite limit, whatever be the mode of division, and whatever be the fractions $\epsilon_1, \epsilon_2, \dots, \epsilon_n$, that limit is represented by the symbol $\int_a^b f(x) dx$, and the function $f(x)$ is said to admit of integration between the limits a and b . We shall call the values of $f(x)$ corresponding to the points of any segment the *ordinates* of that segment; by the *ordinate difference* of a segment we shall understand the difference between the greatest and least ordinates of the segment.