somewhat tedious but the same cannot be said of the graphical method. It is theoretically exact.

By it one may obtain results in a few hours that by the analytical or any other graphical method it might take weeks to accomplish and the author has found that the error need not exceed one or two per cent. if only ordinary care is used. The process throughout is as nearly mechanical as possible, thus not only preventing the danger of gross error but also getting rid of the burden of mental labor resulting from the algebraic numerical computations necessary in other methods.

SOME FACTS RELATING TO THE ARTICLE ON THE FORCE OF IMPACT OF WAVES AND THE STA- BILITY OF THE SUPERSTRUCTURE OF BREAKWATERS.

BY L. D'AURIA.

In the *Journal of the Franklin Institute*, for November, 1890, the writer, in his article "On the Force of Impact of Waves and the Stability of the Superstructure of Breakwaters," for the first time enunciated the following general principle: "The amount of energy which is necessary to cause the propagation of a wave is equal to the energy which such wave would require if it had to be moved *en masse* with its velocity of propagation upon a frictionless level surface" (p. 373.) This energy, when computed per linear foot of wave crest, is approximately expressed by

$$E = \frac{s k \gamma v^2}{4 g}$$

in which s is the wave length; k its height measured from crest to trough; γ the weight of one cubic foot of water; and g the acceleration of gravity.

When a wave comes in contact with a breakwater the energy E is discharged against such structure in the interval of time which the wave requires to propagate its own length, so that, denoting by f the mean force of impact of

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the wave against every foot of breakwater during such interval of time, we find

$$f = \frac{k \gamma v^2}{4 g}$$

According to observations the force of impact of waves against breakwaters is manifested only on that portion of the superstructure extending from crest to trough, so that, such force, at the time when the trough comes in contact with the breakwater, it must be zero and must gradually increase toward a maximum. Hence, the maximum force of impact per linear foot of superstructure can be expressed approximately by twice the mean, or by

$$F = \gamma \frac{k v^2}{2 g}$$

(See equation (1), *Journal Franklin Institute*, p. 374, November, 1890.)

The mean force of impact per square foot, deduced from this equation, is

$$p = \gamma \frac{v^2}{2 g}$$

and since the force of impact per square foot is found to vary from zero to a maximum, this maximum can be expressed approximately by twice the mean, or by

$$P = \gamma \frac{v^2}{g} \quad (1)$$

This ought to accord with the maximum force of impact which can be observed by means of marine dynamometers placed against the superstructure of breakwaters. This has been proved to be a fact by the editor of *Engineering News*, in his editorial article on "Wave Pressure," published September 6, 1890, two months before the writer's article on the force of impact of waves, etc., appeared in the *Journal of the Franklin Institute*. This article, however, is a report the writer made to Col. Henry M. Robert, Corps of Engineers, U.S.A., while in the U. S. Engineer Office of Philadelphia, and was filed January 24, 1890. It was begun sometime in 1889 when the project to close the gap of the

Delaware Breakwater was being studied in that office. At that time the Assistant Engineer, Mr. L. Y. Schermerhorn, consulted the best works on the subject, but was unable to find any formula by means of which the maximum force of impact of a given wave against every linear foot of the superstructure of a breakwater could be ascertained, and the proper dimensions to be given to such superstructure determined. The want for such formulæ has always been felt by marine engineers, and it is no mean achievement for any one who succeeds in making such contribution to the dynamics of waves. The question now is how the editor of *Engineering News* reached the conclusion given in formula (1) of this paper, without the least knowledge of the energy of waves, as is shown by the manner in which he has handled the subject. Had there been no proposal for building the Diamond Shoal Lighthouse perhaps no one would have discovered that the editor of *Engineering News* possessed such valuable information regarding the wave pressure, as he calls it, for it was on this occasion, and immediately after the contract for that lighthouse had been awarded, that he presented himself as an authority in matters relating to wave pressure. This occurred on September 6, 1890, in his editorial already mentioned, in which, speaking of the bidders and the views they took in regard to wave action against the proposed lighthouse, he says: "There was evidently a decided contrast of views in regard to this question among the bidders on the structure, all of whom were engineers or contractors of high reputation."

Then, after reviewing the project of one of the bidders, he asks the question, "Are those allowances sufficient for a case of this kind?" To determine this he proceeds, saying, "Let us first run over certain general facts and theories," which he does, and accordingly reaches the conclusion, by Rankine's formula, that a wave 600 feet long, forty-three feet high, and moving at a rate of fifty-four feet per second, would probably at sea exert a pressure per square foot not exceeding 300 pounds. But he says: "The preceding laws hold only when the water is about as deep as the length of a wave or several hundred feet. Any person accustomed to surf-bath-

ing does not need to be told that the whole upper surface of the water moves forward with great speed on the crests and with hardly any in the troughs; while there is a constant undertow setting out from shore."

It seems that neither the bidders for the Diamond Shoal Lighthouse (all of whom are engineers or contractors of high reputation) nor any of the investigators or able marine engineers who have studied the subject in question, ever enjoyed the pleasure of a surf-bath, for otherwise they would have thought themselves warranted in assuming, as the editor of *Engineering News* assumes, without any other support, that "the surface of the sea in a harbor or on a bank moves forward with a velocity equal to that of the wave," and would have arrived at the "interesting results" found by him on such assumption. But let us see how he reaches these results. He quotes Rankine's *Civ. Eng.*, p. 754, in which this celebrated author says: "In water that is very shallow, compared with the length of the wave, the velocity is nearly independent of the length, and is nearly equal to that acquired by a heavy body in falling through half the depth of the water added to three-fourths of the height of a wave." The editor of *Engineering News*, however, instead of "very shallow compared with the length," substitutes, on his own authority, "when the depth of water is only a few times the height," and then proceeds to find the velocity of the waves observed at Wick, Scotland, which waves were forty feet high, and moved in water a trifle over sixty feet deep. Hence, he finds for such waves a velocity of sixty-two feet per second. Armed with this velocity, he considers the wave itself as a jet impinging upon the breakwater with such velocity, applies the formula for the impact of a jet of water against an unlimited plane surface, and comes out triumphantly with a wave pressure of 3.7 tons per square foot. Here he remembers for an instant that according to Rankine's formula, he had found that in the open sea the maximum pressure per square foot exerted by the greatest-observed wave would not exceed 300 pounds, is staggered for a moment, and then remarks: "This result seems absurd as compared with that just deduced for the pres-

sure of waves in mid-ocean, but the absurdity is only apparent."

The reason given for this is that a maximum pressure observed by marine dynamometers is very nearly the one he has found by * * * experience in surf-bathing, in the absence of any theory. He, therefore, concludes: "Hence it will be seen that the assumption of a velocity of the particles of water equal to that of the wave, although not in accordance with theory, still leads to results agreeing with experiments." On the contrary, we have shown at the head of this paper that our theory leads exactly to the same result which are obtained by experiments. The principle upon which this theory is based has been proved to be correct in the article, "A New Theory of the Propagation of Waves in Liquids," which the writer has published in the *Journal of the Franklin Institute* for December, 1890. In this article, by the application of the same principle above-mentioned, a general formula for the velocity of propagation of any wave has been obtained, which, when applied to various cases, has shown the most remarkable agreement between practice and theory. Take, for example, the case of the tidal wave in which it is known that the horizontal displacement of the liquid particles from surface to bottom is constant, we have found that the velocity of propagation of such wave would be equal to that which a heavy body would acquire in falling through half the depth, a formula well known to those familiar with the subject, and which the editor of *Engineering News* evidently ignored. In fact, had he been acquainted with it, he would have understood Rankine differently and would have noticed that the generalization which this illustrious scientist had in view when he assumed the depth to be very shallow compared with the length of the wave, was referred to tidal waves, not to waves forty feet high and probably 500 feet long, moving in water sixty feet deep. In a wave of the class which was intended by Rankine and to which his generalization could be applied, the liquid particles are supposed to possess the same velocity of translation at the same instant and at all depths in the same cross-section, and the vertical motion must be supposed inconsider-

able in comparison with the horizontal motion. The tidal waves which are observed in tidal rivers are nearly waves of this kind, and the pressure which such waves exert upon a plane surface at right angle to their motion is almost insignificant compared with that due to their velocity of propagation.

It seems strange that the editor of *Engineering News*, while in a state of such ignorance about the whole subject of waves (with the exception of surf-bathing), should undertake to catechise "engineers and contractors of high reputation" on this very subject.

The most remarkable thing, however, about the incident, is that the editor of *Engineering News*, in the issue of November 15th, while engaged in the discussion of wave pressure with a correspondent, did not notice the writer's article on the force of impact of waves, etc., published in the *Journal of the Franklin Institute*, for November, 1890, though he noticed and reprinted from this *Journal* an article by Theo. N. Ely, on "Steam Heating of Passenger Trains," which ends on the same page where the article "On the Force of Impact of Waves," etc., begins. (See p. 373, *Journal of the Franklin Institute*, November, 1890.)

That the editor of *Engineering News* could fail to notice this article which, at that time, ought to have concerned him most, is impossible, and he did not even protest against the writer's arrogation of originality. Was it magnanimity, or was the editor of *Engineering News* aware of the fact that the writer's article had been filed in the United States Engineer Office of Philadelphia, January 24, 1890?

Indeed, this last alternative is not improbable, since the editor of *Engineering News* is not a stranger at that office. In fact, he is an old friend and schoolmate of the assistant engineer, with whom he often corresponds on engineering matters, and it may be that accidentally the conclusions reached by the writer in the above paper were propagated, so to speak, from Philadelphia to New York, which would solve the enigma without adding another word in the way of explanation.