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abandoning those projections nominally, in favour of close equivalents that have no clear geometrical significance? Shall we not conclude rather that the modern preference for the Conical Orthomorphic of Lambert, and the Gauss Conformal or transverse Mercator, is supported in the strongest way by Tissot's investigations, and that they may be employed with reinforced confidence under their habitual names? The idea that the Stereographic is the best projection for a map of equal extent in latitude and longitude is not, I think, supported by recent practice, but clearly requires examination.

The thanks of all interested in the subject will be given to Colonel Bellot for his very important explanation of the ideas underlying the "Tables de Projection (Système Lambert)," that were destined to play an essential part in the operations of the Allies, had not other causes precipitated the victory.

A Modified Rectangular Polyconic Projection

G. T. McCaw

THE common rectangular polyconic is designed to give the scale true on the equator. The scale errors on the meridian and parallel, to terms of the second order, are:

$$\begin{array}{llll} \text{Scale error in direction of meridian} & +\frac{1}{2}\omega^2 \cos^2 \lambda \\ \text{" " " " parallel} & -\frac{1}{4}\omega^2 \sin^2 \lambda \end{array}$$

where λ is the latitude and ω the longitude from the central meridian.

Now it is not difficult to reduce the scale error on the parallel. If the scale is made true on a parallel central to the map, instead of the equator, the scale error on the parallels generally will be reduced. But this is nothing but a partial improvement, since it leaves the scale error on the meridians unaffected.

It then becomes a question whether the scale error on the parallel could be brought up to the scale error on the meridian, thus making the projection orthomorphic; we could then reduce the scale error over the whole map by making use of a scale factor in the ordinary way. Now orthomorphism had been already obtained by Lagrange, whose projection had therefore to be investigated.

The Lagrange projection involves some elegant geometry, but it has peculiar properties. The projection, as regards scale error, is somewhat lop-sided, in that it introduces a scale error on the meridian and the line of zero error is a curve whose equation is not simple; this, however, is no fatal objection to a representation otherwise excellent. A point of even less importance in general is the fact that the northern and southern hemispheres are not symmetrical. Geometrically, the projection is not difficult to construct; but, as is well known, geometrical nicety of construction does not necessarily imply practical convenience for the map-maker; indeed,

experience proves rather the opposite. A good example is furnished by the rectangular polyconic itself: O'Farrell's construction is geometrically very neat, yet not of much practical value. The real objection, however, to the Lagrange is its lack of adaptability to the construction of general tables; each map demands a computation for itself. In fact, in this sense it is a better projection for the surveyor than the cartographer.

The writer was thus led back to the common rectangular polyconic. An orthomorphic form, indicated by J. I. Craig in another connection, was suggested to the writer by A. E. Young, and, suitably modified, proved fruitful. To explain this, some mathematical relations of the projection are unavoidable.

The equation of the meridians in the common rectangular polyconic is

$$\tan \frac{1}{2}\gamma = c \sin \lambda$$

where γ is the convergence or angle of the cone, and c is a constant *which must not involve the latitude*, though it may be any suitable function of the longitude. This c in the common form is $\frac{1}{2}\omega$ in radian measure.

Now supposing that we take

$$\tan \frac{1}{2}\gamma = \frac{1}{2}\omega \sin \lambda \{1 + \frac{1}{12}\omega^2(1 + \cos^2 \lambda)\}$$

we shall obviously have a projection which violates the condition above; we shall, in fact, have an orthomorphic projection which may be described as approximately polyconic.

Suppose, however, that in the term within the brackets we put $\lambda = \lambda_1$, where λ_1 is constant and equal to the latitude of the central parallel of the map, while λ outside the brackets remains unchanged; the polyconic condition will obviously not be vitiated. This, then, is the modification adopted. It leads to a rectangular polyconic which is approximately orthomorphic, and which is almost truly orthomorphic on the central parallel.

The scale errors on this modified polyconic are, to terms of the second order:

On the meridian	...	...	...	$+\frac{1}{2}\omega^2 \cos^2 \lambda$
,, parallel	...	...	...	$+\frac{1}{4}\omega^2(\cos^2 \lambda + \cos^2 \lambda_1)$

It remains to consider whether the modified projection can be set out from general tables. If $xy, x'_x y'$ be respectively the co-ordinates of the common rectangular and the modified rectangular projection, it is found that for the ordinates

$$y' = y \{1 + \frac{1}{12}\omega^2(1 + \cos^2 \lambda_1)\}, \text{ very approximately}$$

This contains λ_1 as a constant for the map, so that we may tabulate the fractional increase of the y of the common rectangular polyconic against the longitude as argument, and in the row representing the mean latitude of the map.

The main portion of the x co-ordinate is given in the column under longitude 0° , and is therefore the length from the origin measured on the

central meridian to the parallel of the latitude of the points. The remaining part of the x co-ordinate is the arc-versine of the point, that is to say, the offset from the perpendicular to the central meridian at the latitude of the parallel of the point.

Now in the modified rectangular this offset is found to be increased in the ratio

$$\frac{1}{6}\omega^2(1 + \cos^2 \lambda_1), \text{ approximately}$$

that is, the offset is increased in just double the ratio of increase of the ordinate.

It is evident, therefore, that it is necessary to tabulate a single modifying factor only, applied directly to the ordinate y and doubled before being applied to the offset, which is that part of the abscissa x which represents the arc-versine.

The following table shows the reciprocal of the scale error on the meridian parallel of the modified projection when the central parallel is in latitude 45° .

Lat.	15°		30°		45°		60°		75°	
Long.	Mer.	Par.	Mer.	Par.	Mer.	Par.	Mer.	Par.	Mer.	Par.
15°	31	41	38	47	59	60	115	81	430	109
20°	17	23	21	27	32	33	64	47	241	65
25°	11	15	14	17	21	22	41	31	154	44
30°	8	10	9	12	16	16	28	23	107	33

The exaggeration of scale can never be reduced by a scale factor. We give the preference to that factor which makes the scale error (positive) at the right and left margins of the map equal to the scale error (negative) on the central meridian, since this reduces the maximum scale error to the least possible. If ω_2 be the longitude of the margin of the map, reckoned from the centre, the longitude of the standard meridians will then be ω_1 , where

$$\omega_1 = 0.707\omega_2$$

In the example above, for a longitude of $25^\circ = \omega_2$, we must then take $\omega_1 = 17^\circ 40'$. Accordingly, the reciprocal of the scale error now becomes—

Lat.	30°		45°		60°	
Long.	Mer.	Par.	Mer.	Par.	Mer.	Par.
0°	— 43	— 43	— 43	— 43	— 43	— 43
15°	+466	—404	—158	—147	— 68	— 90
$17^\circ 40'$	+ 83	—182	0	0	— 87	—150
20°	+ 45	+ 74	+145	+179	—123	—435
25°	+ 21	+ 30	+ 42	+ 48	+1612	+121

The means of the errors on the meridian and parallel are found to be very nearly the same as the Lagrange projection. Compare the errors

with those in the common rectangular polyconic; it is sufficient to give the results for longitude 25° .

Lat.	30°		45°		60°	
Long. 25°	Mer. +14	Par. -84	Mer. +21	Par. -42	Mer. +42	Par. -28

Thus, while the modification here indicated reduces the scale error by nearly one-half, it at the same time has the effect of rendering the representation nearly orthomorphic.

The modified projection is suitable for a meridional rather than a longitudinal configuration, but for areas whose total width does not exceed 40° of longitude it seems to be little, if any, inferior to the best forms of representation where the meridional extent is also considerable. The scale error is very much the same as that of the Transverse Mercator (Gauss Conformal). It would form an excellent projection for a general map of Egypt and the Anglo-Egyptian Sudan, or a country of similar proportions.

The Projection of the Sphere on the Circumscribed Cube

ARTHUR R. HINKS

THE projection of the sphere on a circumscribed cube was, according to D'Avezac, first used by Reichard in a work published at Weimar in 1803. The poles were at the centres of two opposite faces. A small sketch of a projection on the cube touching the sphere at 30° N. and 100° W. was given by G. A. Schott in the Reports of the United States Coast and Geodetic Survey, Washington, 1882, and is reproduced in the writer's book on 'Map Projections' published by the Cambridge University Press, though by a regrettable oversight there is no acknowledgment of the source. In this example the meridian through the above point is drawn parallel to the edge of the face. Similar projections of the celestial sphere have been used by the writer for studying the galactic distributions of star clusters and nebulae (Monthly Notices, R.A.S., vol. 71, p. 693). In all these examples the poles were placed in the median line of a face. It will be convenient to call this class the "inclined" projections on the cube, reserving the term "oblique" for the class in which the pole is not on the median line of a face.

The Washington projection referred to above cuts up the continents very awkwardly, and the same thing happens if one draws an inclined projection with London at the centre of a face. But by giving the cube a suitable obliquity it is possible to get a map of the world in which the principal land-masses are fairly well disposed on the faces of the cube,