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DIELECTRIC STRENGTH OF AIR.

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To investigate the dielectric strength of air, tests were made of the striking distances in air between parallel cylinders of different diameters, between sharp points, and between spheres of different sizes. Further tests were made on the effect of inserting a conductor in or near the path of the disruptive discharge and investigating its influence on the striking distance.

As source of power, an A-S smooth core alternator of the Thomson-Houston type was used, of about 30-k. w. capacity and a frequency of 125 cycles, giving practically a sine wave. In other tests a 60-k. w. high-frequency ironclad alternator of the Thomson-Houston type was used, giving the well known saw-tooth form of wave. The alternator was driven by a direct current motor of ample capacity.

With alternating E. M. F.'s of only 125 cycles, the luminous phenomena, as brush discharges from conductors, etc., were much less marked and brilliant than observed even at lower voltages with oscillating currents of very high frequency, as given by a Thomson transformer.

For the purpose of these tests four high-potential transformers were built, of the ratio of transformation 1 : 18 and of a rating of about 7.5 k. w. each. They are of standard air-blast type, but were used when immersed in large wooden tanks filled with oil.

Each of these transformers contains two primary and two secondary coils, the two high-potential secondary coils in the center, the low-potential primary coils at the outside.

The utmost care was taken in the insulation of these transformers and it proved very successful, since over and over again

the voltage has been run up to 150,000 volts and beyond and later even to 172,000 volts without ever breaking down the transformer.

The high-potential circuits of the four transformers were connected in series. To avoid the danger of a disruptive discharge taking place from the outside of the high-potential transformer at one end of the series through the low-potential coils to the outside of the high-potential transformer at the other end of the series, the low-potential primaries were not connected directly to the generator in parallel with each other, but were connected to the high-potential sides of four standard type F transformers of

FIG. 1.

General Electric make of 7.5 k. w. each, and these transformers were again fed by another set of four standard type F 7.5 k. w. transformers, the latter being connected in parallel to the generator.

Thus the diagram of connections was as follows (Fig 1): The generator G feeds in parallel the high-potential sides of four transformers F' which step down in the ratio 10 : 1 or 10 : .5. These transformers F' feed four transformers F which step up in the ratio 1 : 10 or .5 : 10 and feed with their high potential side the low-potential coils of the four transformers H_1, H_2, H_3, H_4 .

An internal disruptive discharge thus has to pierce two high-potential transformers H_1 and H_1 , and four standard F transformers.

Incidentally hereby the ratio from the generator to high-potential circuit could be changed between 1 to 360, 1 to 720, and 1 to 1440 by changing the ratio of the F transformers, that is connecting their low-potential coils in multiple or in series.

The generator was driven by a direct current motor and was separately excited. The voltage was varied by varying the generator excitation, by varying the ratio of transformation of the F transformers, and by cutting more or less of the high-potential transformers out of circuit. Thus in the lowest readings only one high-potential transformer was used, in the highest readings all four. When cutting out transformers always the first transformer H_4 , then H_1 , and last H_3 were cut out, leaving ultimately only H_2 in circuit.

The F transformers were generally for the higher voltage used with the ratio 1 : 10 in F' and .5 : 10 in F'' .

The high potential voltage was derived by the ratio of transformation, and for this purpose two Weston alternating voltmeters with 10 : 1 voltmeter transformers were inserted on the low-potential side of the transformers H_2 and H_3 , as shown by v_1 and v_2 in Fig. 1. They were calibrated before and after each test. A third voltmeter was inserted into the generator circuit with voltmeter transformer, and is shown as v_3 . Its readings were not used for deriving the high-potential voltage, since too many transformations were intermediate. Thus it was used merely as a check and for guidance in operating the generator field rheostat.

The tests were made in the usual manner by setting the electrodes for a certain distance, then closing the generator switch and gradually raising the generator excitation while following the rise of the voltmeter needle, until the discharge took place and the voltmeter dropped. Then the exciter switch was opened, the voltmeter reading recorded, and the electrodes reset for the next test.

For the tests between points, $2\frac{1}{2}$ " sewing needles were used and for every test a new pair of needles taken, since it was not considered sufficient to re-sharpen the needles after the discharge had taken place.

For the tests between cylinders and spheres it was not considered sufficient to have the electrodes polished, since no polish can give an absolutely perfect surface without any minute un-

evenness. To secure a perfect smoothness, a film of liquid quicksilver was used as electrode surface, that is polished brass cylinders and spheres were used, and were immersed in a solution of mercuronitrate and afterwards wiped off with a clean cloth. The fairly thick film of quicksilver deposited on the electrode surface forms a liquid and thus perfectly smooth surface. After every discharge the electrodes were reamalgamated.

The effect of this precaution was that even with very small distances, the discharge appeared constantly at the same voltage, while with merely polished electrodes with very small distances, a more or less erratic behavior takes place due to minute unevenness of the surface.

Before undertaking the final tests, a number of preliminary tests were made to determine,

1). The constants of the generator and the transformers and the total power thus available in the discharge circuit.

2). The equality of the ratio of transformation of all four sets of transformers, since only at two out of the four transformers was the voltage measured.

3). The constancy of the ratio of transformation, since it was relied upon for determining the high potential voltage.

4). The shape of the generator wave and its variation under the conditions of test.

1st.—The synchronous impedance of the generator was found as,

$$Z_o = r_o - j x_o = 3 - 12j,$$

that is, at an excitation corresponding to the voltage v at open circuit with a current C in the armature, the E. M. F. $C r_o$ is consumed in phase with the current, the E. M. F. $C x_o$ in quadrature with the current, thus giving the terminal voltage $V = C (r_o - j x_o)$.

Of the transformers F' and F'' , of rating $F-125-7.5-1040-104-52$, reduced to the high potential or 1000-volt coils, it is :

The total impedance,

$$Z^1 = r^1 - j x^1 = 2.1 - 20j.$$

The primary admittance, that is, admittance of the exciting circuit,

$$Y^1 = g^1 + j b^1 = (.13 + .24j) 10^{-8}.$$

The total number of primary turns is 280, the total number of secondary turns 28, of which, as before said, two could be connected in parallel. The cross-section of the magnetic circuit is 32.4 sq. in.

Of the transformers H_1 , H_2 , H_3 , H_4 of rating OC-125,-7.5,-27000,-1500 reduced to the low potential or 1500-volt coils, it is:

Total impedance,

$$Z_1 = r_1 - j x_1 = 2.5 - 8.0 j.$$

The primary admittance,

$$Y_1 = g_1 + j b_1 = (.32 + .5j) 10^{-3}.$$

The total number of high-potential turns is 3458 per transformer, of wire No. 24 B & S, subdivided in two coils.

The total number of low-potential (1500 volt) turns is 192, subdivided in two coils also. The cross-section of the magnetic circuit is 46.5 sq. in. in the center, 72.9 sq. in. in the outside shell.

Herefrom we get for the whole system in the connection used for the highest voltage, that is with F' connected 10 : 1, and F'' connected .5 to 10, reduced to the high-potential discharge circuit.

Total impedance inclusive generator,

$$\begin{aligned} Z &= 4 \times 18^2 Z_1 + 4 \times 18^2 Z^1 + 4 \times 36^2 Z^1 + 144^2 Z_0 \\ &= (80 - 390 j) 10^3 \text{ ohms,} \end{aligned}$$

or absolute:

$$z = 398 \times 10^3 \text{ ohms,}$$

or reduced to the generator circuit:

$$Z_2 = r_2 - j x_2 = 3.9 - 18.8 j \text{ ohms.}$$

Total admittance,

$$\begin{aligned} Y &= \frac{1}{4} \frac{1}{18^2} Y_1 + \frac{1}{4} \frac{1}{18^2} Y^1 + \frac{1}{4} \frac{1}{36^2} Y^1 \\ &= (.37 + .62 j) 10^{-6} \text{ mhos.} \end{aligned}$$

or reduced to the generator circuit,

$$Y_2 = (7.7 + 12.8 j) 10^{-3} \text{ mhos.}$$

Thus with 150,000 volts between the discharge terminals at the total ratio of transformation 144, the terminal voltage of the generator is about 1,100 volts and its field excitation corresponds to an induced voltage at open circuit of about $V = 1,280$ volts. It follows herefrom that the maximum current which can be produced in the discharge circuit is:

$$\frac{V}{144 \sqrt{r_2^2 + x_2^2}} = .465 \text{ amps.,}$$

and the maximum power available in the discharge circuit is :

$$\frac{V^2}{2(r_2 + \sqrt{r_2^2 + x_2^2})} = 35.5 \text{ k. w.}$$

At this voltage the maximum density in the core of the transformers H is 42.5 kilo lines per sq. in., or 6,600 lines per sq. cm. In the transformers F" it is 41.5 kilo lines per sq. in., or 6,400 lines per sq. cm., hence far below saturation.

2nd.—To determine the equality of the ratio of transformation, all four high-potential transformers were connected in parallel with their high-potential coils and in series with their low-potential coils, as shown in Fig. 2, and the voltages across the low-potential coils read. They were,

$$\frac{a}{419} \quad \frac{b}{405} \quad \frac{c}{401} \quad \frac{d}{401}$$

showing that the first transformer has a slightly higher ratio of transformation, and the last two a slightly lower ratio than the average.

FIG. 2.

Allowance therefor was made in reducing the readings. The correction in the extreme case did not exceed 1.33%.

3rd.—Since the high potential was derived indirectly by voltmeter readings on the low-potential side of the transformers, it was necessary to determine whether the voltage read in this manner was correct and whether the capacity of the discharge circuit has any effect on the ratio of transformation.

For this purpose the high-potential transformers were connected in pairs in parallel with their low-tension and in series with their high-tension sides, the one pair used as step-up transformers, the other pair as step-down transformers, and the voltage read in both low-tension circuits, as shown in Fig. 3. Then step-up and step-down transformers were interchanged, that is, the generator was connected first at *a*, then at *b*. This test was repeated after

inserting an oil condenser into the high-potential circuit at *c*. This condenser consisted of twelve iron plates of about $\frac{1}{2}$ sq. ft. each, separated by $\frac{3}{4}$ " of oil. Two such boxes of condensers were used in series, giving a total condenser surface of about $5\frac{1}{2}$ sq. ft. per electrode and $1\frac{1}{2}$ " oil distance, thus a capacity far in excess of that of any discharge circuit.

A number of tests were made, of which I give one set.

FIG. 3.

Generator at <i>a</i> : Ratio: 1 ÷ 360.				Generator at <i>b</i> :			
No Condenser.		Condenser.		No Condenser.		Condenser.	
<i>V</i> ₁	<i>V</i> ₂	<i>V</i> ₁	<i>V</i> ₂	<i>V</i> ₁	<i>V</i> ₂	<i>V</i> ₁	<i>V</i> ₂
11.5	11.0	11.0	11.0	10.1	11.8	10.6	11.9
25.2	23.5	20.3	19.0	24.2	25.1	19.8	20.6
44.9	42.8	30.0	28.5	43.2	45.0	29.5	30.5
66.8	64.2	39.9	38.3	65.3	67.0	39.4	40.4
93.0	89.2	45.6	43.7	91.0	92.7	44.5	45.8
102.1	98.5	56.0	53.5	100.4	102.0	54.7	56.0
117.6	113.0			115.8	117.3		
128.0	123.5			126.6	128.6		
134.4	129.6			132.2	133.6		

Average ratio:

1.0405 | 1.0455 | 1.0201 | 1.03

Average without condenser, 1.0303.

Average with condenser, 1.0396.

Hence 1.0303 is the change of ratio of transformation due to three times the exciting current. Thus the correction for the exciting current is 1.01%. The relatively very large condenser inserted into the high-potential circuit does not change the ratio of transformation by as much as 1%. Thus the very much smaller capacity of the discharge circuit is entirely negligible in its effect on the ratio of transformation.¹

To investigate the equality of the ratio of transformation of the type F transformers, the transformers were connected as in Fig. 4, that is the same way as during tests, but with the high-potential circuits disconnected, and the voltage at the low-potential

FIG. 4.

terminals of the high-potential transformers read at a , b , c , d and at the generator g . It was found :

g	a	b	c	d
50.3	47.5	48.1	47.2	47.8
74.6	70.0	70.7	70.6	71.0
100.6	97.0	97.2	97.4	97.3
127.3	122.8	123.2	123.0	123.0
352.8	337.3	339.2	338.2	339.1

Average $a, b, c, d = 338.45$,

1. This is due to the relatively very low self-induction of the step-up transformers compared with the self-induction of, for instance, a Ruhmkorff coil, etc.

Average ratio:

1.0425 | .9966 | 1.0022 | .9993 | 1.0019
that is, the four sets of intermediate transformers F' and F'' are

FIG. 5.

of identical ratio of transformation within a small fraction of one per cent., and the drop of voltage in them (which, however, is unessential for the test) amounts to 4.25%. Furthermore it follows that the voltmeter readings at the low-potential side of two of the step-up transformers give the high-potential voltage correct within one per cent., hence are perfectly reliable.

4th.—The shape of the generator wave was determined by instantaneous readings,

a. At open circuit. *b.* With the high-potential transformers and the type *r* transformers feeding them connected to the generator. *c.* With the above described condensers in the high-potential circuit.

The same connections were used as shown in Fig. 3, and the wave shape determined at the generator terminals *a* as well as at the terminals of the step-down transformer *b*, so as to investigate whether a change of wave shape takes place by the transformation. The following values were found:

Degrees.	At generator, open circuit.	At generator, no condenser.	At step down transformers, no condensers.	At generator, condenser.	At step-down transformer, condenser
10	13	7.25	5.8	5.3	3.7
20	28.5	18	14.4	26	25.8
30	44.1	35.5	29.7	43.3	42.3
40	64.4	52.6	48.2	56.2	53.9
50	13.6	61.6	56.5	64.8	60.7
60	82.5	66.0	60.1	72.6	69.3
70	89.6	71.9	66.5	79.9	77.5
80	95.5	77.3	72.8	85.0	81.9
90	97.7	80.1	74.2	86.1	82.3
100	95	77.5	72.5	83.6	79.8
110	91.4	74.25	70.8	79.2	75.1
120	86.5	70.6	66.6	74.6	70
130	76.8	61.6	58.6	69.4	65.8
140	63.2	49.9	48.1	59	56
150	50.5	41.25	40.2	42.1	38.7
160	36.6	28.6	27.1	23.6	21.8
170	19.9	13.3	13.4	14.75	13.4
180	2.6	.6	.9	7.6	8.3

Ratio $\frac{\text{max.}}{\text{eff.}}$	1.42	1.44*	1.43	1.42	1.42

* As seen in Fig. 5, this particular wave has a small peak at the maximum reading which causes a slightly higher ratio, but is probably merely an error of observation of this individual reading.

that is 1.008 times the value corresponding to the sine wave.

Hence, the generator wave has within less than 1% the same ratio of maximum to effective value as the sine wave, and this ratio is not changed by transformation nor by capacity in the high potential circuit.

The maximum value of voltage can therefore be calculated from the effective value by multiplying by $\sqrt{2}$. In all the tests the effective values of voltage are given.

To conclude, we get as results of preliminary tests:

Voltmeter readings on the low-potential side of the high-potential transformers give a striking voltage correct within 1%, irrespective of the capacity of the discharge circuit, within the limits of the tests.

The wave of the smooth core alternator is practically a sine wave, and is not changed by transformation or by capacity of the discharge circuit.

Since the accuracy of the striking distance determination cannot well exceed 1 to 2%, these preliminary tests show the method as correct within the errors of observation.

In the tests all distances are given in inches, since they were measured in inches, and the reduction to cm. is omitted.

The voltages are given in kilo-volts effective, and the maximum voltages thus are $\sqrt{2}$ times as large.

In general each of the readings given in the table is the average of from two to four tests. The tests are recorded in tables I. to IX. and plotted in Figs. 6 to 17.

It is:

Table I., striking distances between sharp points with smooth core and with ironclad alternator, the latter in air, fog and steam.

Table II., striking distance between $\frac{1}{4}$ " spheres.

Table III., striking distance between $\frac{1}{2}$ " spheres.

Table IV., striking distance between 1" spheres.

Table V., striking distance between 2" spheres.

Table VI., striking distance between parallel cylinders of .313" diameter with smooth core alternator only.

Table VII., striking distance between parallel cylinders of .111" diameter with smooth core alternator only.

Table VIII., striking distance between $\frac{1}{4}$ " spheres with brass disks inserted into the path of the discharge, with smooth core alternator.

Table IX., striking distance between $\frac{1}{4}$ " spheres with wet plank parallel to the path of the discharge.

Fig. 6, sharp points with smooth core alternator, observations marked by crosses.

Fig. 7, sharp points with smooth core alternator. Average of two sets of tests. Observations marked by crosses.

Fig. 8, $\frac{1}{4}$ " spheres. Two sets of tests. Smooth core alternator.

Fig. 9, $\frac{1}{4}$ " spheres. Average of two sets of tests. Smooth core alternator.

Fig. 10, $\frac{1}{2}$ " spheres. Smooth core alternator.

Fig. 11, 1" spheres. Smooth core alternator.

Fig. 12, 2" spheres. Smooth core alternator.

Fig. 13, sharp points and spheres. Comparison of results with smooth core alternator. Sharp points showing dotted line.

Fig. 14, .313" cylinders with smooth core alternator. Maximum electrostatic gradient marked by crosses.

Fig. 15, .111" cylinders with smooth core alternator. Maximum electrostatic gradient marked by crosses.

Fig. 16, sharp points with ironclad alternator. Two set of tests in air, one in fog and one in steam. Smooth core alternator, curve marked by dash-dotted line.

Fig. 17, sharp points and spheres. Comparison of results with ironclad alternator. Curve of sharp points in dotted lines. Sharp points with smooth core alternator marked with dash-dotted lines.

The following has to be remarked regarding these tests:—

All distances given are net, that is, from the surface of sphere and cylinder to the surface of the other cylinder and sphere, etc. Connection of the high-potential transformers H_1, H_2, H_3, H_4 and the intermediary transformers F' and F'' with the generator G and the voltmeters v_1, v_2, v_3 , were in all the tests, as shown in Fig. 1. As stated before, the voltmeter readings of v_1 and v_2 were averaged to calculate the striking voltage, while the voltmeter reading v_3 was used merely as check. An ammeter was also inserted into the generator circuit at A as check to observe capacity effects.

Generally the intermediary transformers near the generator, F' , were connected for the ratio 1 to 10, the transformers F'' for the ratio 1 : 20.

For lower readings the high-potential transformers were gradually cut out in the order H_4 , H_1 , H_3 , leaving ultimately only H_2 in circuit.

Under the floor of the testing room were a number of steam pipes from which some steam was escaping, so that the air in the testing room was always moderately moist.

TABLE I.

POINTS.

$2\frac{1}{2}$ " needles. 125 cycles.

Smooth Core Alternator: A-10-30-1500.				Ironclad Alternator: A-10-60-1500.					
Distance, inches. <i>d</i> .	Kilovolts: effective.			Distance, inches. <i>d</i> .	Kilovolts: effective.				
	7-21-'96 * in air.	7-17-'96 † in air.	Average.		7-24-'96 ‡ in air.	Jan. '95 in air.	Average.	Jan. '95 in fog.	Jan. '95 in steam
.25	4.25			.25	4.13				
.5	10.0			.5	9.0	10.0	9.5	11.0	14.5
1.0	20.4			1.0	16.9	18.5	17.7	22.0	25.3
1.5	29.3			1.5	24.3	26.0	25.1	31.0	35.5
2.0	35.2			2.0	30.5	30.5	30.5	38.0	43.0
2.5	40.4			2.5	33.9	35.0	34.4	43.5	50.3
3.0	45.6			3.0	36.3	38.0	37.1	48.0	54.5
3.5	49.4			3.5	42.2	41.7	42.0	51.0	63.0
4.0	52.5			4.0	41.3	45.0	43.2	55.5	
4.5	59.6			4.5	45.5	48.0	46.7	61.0	
5.0		61.0		5.0	48.4	57.5	49.5		
5.5		65.7		5.5	53.0	55.0	54.0		
6.0	69.8	69.5	69.65	6.0	56.1	58.8	57.4		
6.5	73.4	74.7	74.05	6.5	59.8	62.0	60.9		
7.0	77.5	79.2	78.35	7.0	63.3	64.7	64.0		
7.5	83.8	83.0	83.4	7.5	67.5	69.0	68.3		
8.0	86.8	87.3	87.05	8.0	70.9	73.4	72.1		
8.5	90.5	90.2	90.35	8.5	75.8	76.0	75.9		
9.0	95.0	93.7	94.35	9.0	79.8	79.2	79.5		
9.5	97.7	96.3	97.0	9.5	84.8	82.5	83.6		
10.0	101.5	99.0	100.25	10.0	88.8	86.4	87.6		
10.5	107.0	103.0	105.0	10.5	93.5	89.5	91.5		
11.0	111.5	107.5	109.5	11.0	97.7	93.0	95.4		
11.5	114.0	110.5	112.5	11.5	102.0				
12.0	121.0	116.0	118.5	12.0	107.7				
12.5	125.5	120.0	122.75	12.5	111.0				
13.0	133.0	123.0	128.0	13.0	117.5				
13.5	135.0	127.0	131.0	13.5	122.5				
14.0	140.0	129.0	134.5	14.0	128.0				
14.5	144.0	136.0	140.0	14.5	134.4				
15.0	150.0			15.0	138.3				
15.5	155.0								
16.0	159.5 §								

* 85° F. Weather sultry.

† 75°-80° F. Weather clear and bright.

‡ 70° F. Weather cool and cloudy.

§ Internal discharges in intermediary transformers F' F".

In most of the tests the temperature and the weather are given from the record sheets. The meteorological record of the days during which tests were made is given in Table X. To check the influence of the weather, a few readings with 1" spheres and

ironclad generator were repeated on a sultry and rainy day, but found to agree with the readings taken on a clear and bright day.

With the E. M. F. of the smooth core alternator brush discharges at the electrodes were observed by the hissing noise produced by them when the potential approached the striking

FIG. 6. Points, Smooth Core Alternator, 125 Cycles.

voltage, and at greater distance even far below this, with sharp points with smaller spheres and with parallel cylinders of smaller diameter, less pronounced with larger spheres and parallel cylinders of large diameter.

When approaching 160,000 volts effective with a smooth core alternator, electrostatic discharges took place inside of the intermediary transformers F' and F'' , apparently between primary and secondary. It seems the transformers acted as a system of six condensers in series, the two outside high-potential transformers H_1 and H_4 and the two sets of intermediary transformers F' and F'' connected thereto with the insulation between primaries and secondaries as dielectric. At or near 160,000 volts the electro-

FIG. 7. Points, Smooth Core Alternator, 125 Cycles.

static stress exceeded the disruptive strength of the insulation between primary and secondary in F' and F'' . Since, however, the insulation between H_1 and H_2 resisted, no arc followed the discharge. Nevertheless, even at lower voltage, a number of intermediary transformers, F' and F'' , were pierced.

With the ironclad alternator with sharp points as well as with brass spheres, the breaking-down point was quite unstable.

Although the brass spheres were polished, amalgamated and rubbed smooth before each test with the same care as in the tests with the smooth core alternator, nevertheless the readings are far more erratic and in several instances the same disruptive voltage was found with a change of the striking distance of as much as 1". In the record sheets all questionable readings are omitted.

With the ironclad alternator, at voltages of 80,000 and upward, almost always several heavy electrostatic discharges took place between the electrodes before the disruptive voltage was reached,

TABLE II.
¼" SPHERES.
Amalgamated. 125 Cycles.

Smooth Core Alternator: A-10-30-1500.				Ironclad Alternator: A-10-60-1500.	
Distance, inches. <i>d</i> :	Kilovolts; effective.			Distance, inches. <i>d</i> :	Kilovolts; effective. 7-25-'96. *
	7-13-'96. 7-14-'96.*	7-25-'96.	Average.		
.125	8.18			.125	6.4
.25	12.9			.25	11.3
.50	18.65			.5	13.4
.685	22.4			1.0	18.2
1.47	24.8			1.523	23.3
2.0	34.7			2.0	28.2
2.97	43.6	44.8	44.2	3.0	37.8
3.51		50.0		4.0	47.7
4.01	56.5	56.4	56.45	4.97	53.8
5.03	67.0	66.0	66.5	5.97	58.0
5.5		71.0		7.0	65.8
5.99	77.0	76.4	76.7	8.0	75.1
7.0	86.8	83.6	85.2	8.97	86.2
8.0	96.0	90.6	93.3	10.0	98.4
9.03	103.5			11.0	108.5
10.02	108.7	100.6	104.62		
11.01	111.5				

* 80°-90° F. Weather clear and bright.

† 75°-80° F. Weather rainy and sultry

frequently as much as 1,000 to 1,500 and even 2,000 volts below the actual striking voltage. These discharges were never observed with the smooth core alternator, but were most marked with ½" and 1" spheres and with sharp points. To investigate the frequency, a Thomson transformer, that is a transformer consisting of a small number of primary and of secondary turns wound in air (without iron core) was inserted in series in the discharge circuit with sharp points as electrodes, and the secondary terminals of this Thomson transformer separated by a gap of $\frac{3}{8}$ ". When approaching the striking voltage, but before the final disruptive

discharge took place, a continuous discharge passed across the $\frac{3}{8}$ " air-gap of the secondary circuit of the Thomson transformer, showing thus the existence of an oscillatory or very high frequency effect.

Photographic investigations of the phenomena taking place during the disruptive discharge have shown that it always consists of an oscillating electrostatic discharge with at least two or three well marked oscillations followed afterwards by an arc, as I have discussed more fully in a previous article on "Photographic Study of a 150,000-volt Power Discharge."

FIG. 8. $\frac{1}{4}$ " Spheres. Smooth Core Alternator, 125 Cycles.

Condenser effects were observed only once when two insulated brass disks were inserted in the path of the discharge between $\frac{1}{4}$ " spheres separated by $9\frac{1}{8}$ ", and these brass disks brought very close, within $\frac{3}{4}$ " to the discharge terminals. In this case the primary current in the generator circuit rose. As above discussed, capacity within a reasonable limit has no effect on the ratio of transformation, but would show a rise of the current in the generator circuit, and no such rise was observed, except in the above mentioned case.

Regarding the effect of frequency on the striking distance, within the range of frequency easily accessible, that is from 40 to 125 cycles, no noticeable change was observed.

Of more general interest are the tests of the striking distance made with the smooth core alternator in which the wave is nearly a sine wave and the ratio of maximum to effective E. M. F. thus constant and known. The striking distance with the ironclad alternator are of interest only in so far as they show which devia-

FIG. 9. $\frac{1}{4}$ " Spheres. Smooth Core Alternator, 125 Cycles.

tions from the sine wave curve can be expected with this type of machine most commonly in use, of sawtooth wave shape. Since, however, during the tests the voltage was varied by varying the alternator excitation and the wave shape of the ironclad changes more or less with the excitation, these curves are of comparative interest only.

Restricting thus at first our attention to the tests made with the smooth core alternator, we find the striking distance between

parallel cylinders in Tables VI and VII and Fig. 14 and 15 represented by curved lines, the striking distance increasing faster than the voltage. The curve of parallel cylinders of larger diameter resembles the lower part of the curve of cylinders of smaller diameter in increased scale, as is to be expected.

Very interesting is the curve of striking distance between sharp points, Table I and Figs. 6 and 7. It starts in a straight line, the striking distance being proportional to the voltage at the rate of 20,000 volts per inch. At about $1\frac{1}{4}$ " distance the curve bends upward and passes into a second straight line which it fol-

TABLE III.

 $\frac{1}{2}$ " SPHERES.

Amalgamated. 125 Cycles.

Smooth Core Alternator: A-10-30-1500.		Ironclad Alternator: A-10-60-1500.	
Distance, inches. <i>d</i> :	Kilovolts; effective. 7-23-'96. 7-14-'96.*	Distance, inches. <i>d</i> :	Kilovolts; effective. 7-20-'96.†
.125	8.2	.125	6.86
.25	14.6	.25	11.9
.485	22.0	.5	17.5
.94	31.3	1.0	27.0
1.5	36.1	1.5	26.5
2.0	39.4	2.0	35.4
2.5	44.4	2.97	44.0
2.90	47.2	4.0	51.2
4.0	59.4	4.95	50.1
5.0	68.0	5.94	60.0
6.06	79.0	6.97	68.8
6.97	84.0	7.97	77.5
7.97	95.0	8.97	83.0
9.0	98.3	9.97	97.3
9.93	105.5	11.03	108.8
11.0	112.0		

* 80°-90° F. Weather clean and bright.

† 75°-80° F. Weather rainy and sultry.

lows from 3" distance upward, the distance increasing at the rate of 1" per 8,000 volts effective. This second straight line, however, does not pass through the origin, but through a point of the voltage of 22,000 effective.

At very high voltages, about 100,000, the two sets of readings taken diverge from each other, the one falling below, the other passing above the straight line, as shown in Fig. 6.

The average, however, follows very closely the straight line as shown in Fig. 7, where the average values of the two sets of readings are marked by crosses.

The four sets of curves representing the striking distance between spheres of various diameters have a considerable similarity with each other, the same curve repeating itself in increased scale with the increased diameter of the spheres. They differ, however, greatly from the curve of sharp points.

All the four curves start at the same slope for very small distances, of about $\frac{1}{8}$ " per 10,000 volts. Then they bend upwards and pass into a second straight line, and the middle of the bend

FIG. 10. $\frac{1}{4}$ " Spheres. Smooth Core Alternator, 125 Cycles.

lies at about twice the diameter of the spheres. After the bend they follow a straight line for a certain range, but bend upwards once more for high voltages and then approach rapidly the curve of sharp points. The second upward bend is marked with $\frac{1}{4}$ " and $\frac{1}{2}$ " spheres, while with 1" spheres the curve extends only to what is probably the end and with 2" spheres the beginning of the straight lined part. At very high voltages the $\frac{1}{2}$ " and $\frac{1}{4}$ "

TABLE IV.
1" SPHERES.
Amalgamated. 125 Cycles.

Smooth Core Alternator : A-10-30-1500.		Ironclad Alternator : A-10-60 1500.	
Distance, inches. <i>d</i> :	Kilovolts ; effective. 7-13-'96. 7-14-'96.*	Distance, inches. <i>d</i> :	Kilovolts ; effective. 7-17-'96.†
.125	8.95	.125	6.75
.25	15.9	.25	11.6
.5	26.1	.485	19.6
1.0	43.6	1.00	33.3
1.47	52.8	1.188	37.2
1.94	59.2	1.97	46.0
2.97	67.7	3.0	52.5
3.99	76.0	4.0	66.7
5.0	81.2	4.99	69.2
6.03	86.5	6.0	74.7
6.61	89.0	6.99	79.5
7.28	100.0	8.2	86.0
7.99	99.0	8.99	92.0
8.99	104.5	10.0	97.5
9.99	108.7	10.97	107.0
10.94	122.0		

* 80°-90°. Weather clear and bright.

† 75°-80°. Weather clear.

FIG. 11. 1" Spheres. Smooth Core Alternator, 125 Cycles.

spheres have practically approached each other, the 1'' spheres are coming close to the same value, while the 2'' spheres show a bend, which as seen in Fig. 8, would bring the curve beyond the

TABLE V.

2" SPHERES.

Amalgamated. 125 Cycles.

Smooth Core Alternator: A-10-30-1500.		Ironclad Alternator: A-10-60-1500.	
Distance, inches. <i>d</i> :	Kilovolts; effective. 7-13-'96. 7-14-'96.*	Distance, inches. <i>d</i> :	Kilovolts; effective. 7-20-'96.†
.125	8.95	.125	6.67
.25	15.9	.25	12.5
.493	26.7	.50	22.6
1.08	51.0	1.03	39.3
1.454	65.2	1.50	50.8
1.688	70.8	1.95	57.8
2.25	83.8	2.51	66.7
3.0	94.0	2.99	75.0
3.438	102.0	3.5	80.3
2.952	101.5	4.0	82.7
5.08	108.0	4.49	92.0
5.60	114.5	4.97	94.0
		5.47	96.5

* 80°-90° F. Weather clear and bright.

† 75°-80° F. Weather rainy and sultry.

FIG. 12. 2" Spheres. Smooth Core Alternator, 125 Cycles.

range of test up to the same value. At high voltages, however, the two sets of tests with $\frac{1}{4}$ " spheres deviate from each other, the one leaving the straight line sooner than the other. Thus their average has been plotted in Figs. 2, 9 and 13.

While the striking distances between spheres of different diameters are very different at intermediary voltages, they seem to coincide at very low and very high voltages, but differ essentially from the curve taken between sharp points, except at extremely high voltages.

FIG. 13. Comparison of Points and Spheres. Smooth Core Alternator, 125 Cycles.

Comparing the striking distance between small spheres, and those between needle points as shown in Fig. 13, the remarkable feature is observed that the $\frac{1}{4}$ " spheres approach sharp points at

comparatively low voltages, and even intersect the curve of sharp points, so that within a certain range of voltage, from 28 to 49 kilovolts, the striking distance between $\frac{1}{4}$ " spheres is greater than between sharp points, but beyond this range these curves separate again widely, far beyond the possible error of observation, and at very high voltages approach once more. The intersection of the curve of sharp points and of $\frac{1}{4}$ " spheres is very

TABLE VI.

PARALLEL CYLINDERS.

.318" diameter. Radius of Curvature : 87". Length : 24"

Amalgamated brass. 125 cycles.

Smooth Core Alternator : A-10-30-1500. 4-14-96.					
Distance, inches. <i>d</i> :	Kilovolts, effective :	Mean Gradient. <i>g</i> :	$\sigma\sigma_o$ *	Maximum gradient : <i>g_o</i> :	Δ :
.065	4.78	73.5	.934	78.6	+ 1.5
.080	5.7	71.2	.921	77.2	+ .1
.174	10.85	62.2	.849	73.4	- 3.7
.347	18.55	53.4	.742	72.0	- 5.1
.439	23.7	53.9	.698	77.2	+ .1
.47	22.9	48.8	.685	71.2	- 5.9
.565	23.0	40.7	.643	63.3	- 13.8
.69	30.5	44.2	.600	73.7	- 3.4
.84	34.3	40.9	.556	73.5	- 3.6
.93	33.1	35.6	.533	66.8	- 10.3
1.18	41.5	35.1	.478	73.4	- 3.7
1.44	47.3	32.8	.434	75.5	- 1.6
1.72	51.4	29.9	.396	75.7	- 1.4
2.12	53.8	25.3	.357	71.0	- 6.1
2.62	65.0	24.5	.312	78.5	+ 1.4
2.82	67.8	24.1	.298	80.5	+ 3.4
3.16	71.0	22.5	.276	81.6	+ 4.5
3.18	69.8	21.0	.275	79.6	+ 2.5
3.69	72.5	19.65	.251	78.4	+ 1.3
8.72	73.3	19.7	.250	78.8	+ 1.7
4.06	75.6	18.6	.236	79.0	+ 1.9
4.09	77.4	18.95	.234	81.4	+ 3.3
4.34	77.3	17.8	.226	79.0	+ 1.9
4.53	77.7	17.15	.220	77.8	+ .7
4.84	80.0	16.5	.211	78.3	+ 1.2
5.28	83.5	15.8	.199	79.3	+ 2.2
5.46	83.7	15.35	.195	75.6	+ 1.5
5.84	93.6	16.0	.186	86.0	+ 8.9

Average : 76.0
Second average : 77.1

* $\sigma\sigma_o$ = ratio $\frac{\text{mean}}{\text{max.}}$ gradient, calculated.

interesting and quite marked, and cannot well be due to errors of observation, since either curve represents two independent sets of tests taken at different times which coincide, and the same intersection at a somewhat lower range is strongly marked also in the tests with the ironclad alternator, Fig. 17.

From all these tests it should be expected that a law on the disruptive strength of air could be derived.

The disruptive electrical discharge bears a striking analogy to mechanical rupture. Thus, as the mechanical strength of a beam remains practically unimpaired up to a certain load, and at this load suddenly falls to zero by mechanical rupture, so the resistance of an air space is practically infinite up to a certain potential, at which, under disruptive discharge, it suddenly falls down to practically nothing. In looking for a physical law represent-

FIG. 14. Parallel Cylinder .313" Diameter. Smooth Core Alternator, 125 Cycles.

ing the electric disruptive discharge, it thus was to be investigated how far the laws of mechanical strength would represent the observed effects.

Under mechanical stress, rupture takes place as soon as the stress exceeds the breaking strain anywhere in the material, and this breaking strain is a constant of the material. Thus I investigated whether the disruptive discharge of air takes place as

TABLE VII.
PARALLEL CYLINDERS.

1.11" diameter.

Radius of curvature: 140"

Length: 20.

Amalgamated brass.

125 cycles.

Smooth Core Alternator : A-10-30-1500. 4-6-96.					
Distance, inches, <i>d</i> :	Kilovolts, effective.	Mean Gradient. <i>g</i> :	$\sigma\sigma_0$ *	Maximum gradient. <i>g_0</i> :	Δ :
.094	5.9	62.7	.973	64.5	+ 4.5
.109	7.02	54.4	.969	66.5	+ 6.5
.172	9.94	57.8	.951	60.8	+ 8
.188	11.6	61.7	.947	65.2	+ 5.2
.320	19.2	60.0	.914	65.6	+ 5.6
.407	21.6	53.0	.892	59.4	+ 6
.492	27.0	55.0	.874	63.0	+ 3.0
.891	41.0	46.0	.797	57.8	+ 2.2
1.31	52.0	39.7	.729	54.5	+ 5.5
1.49	60.5	40.6	.704	57.6	+ 2.4
1.78	66.2	37.2	.667	55.8	+ 4.2
1.79	68.4	38.3	.666	57.6	+ 2.4
2.66	74.0	36.0	.637	56.5	+ 3.5
2.19	79.0	36.0	.623	57.7	+ 2.3
2.37	84.6	35.7	.605	59.1	+ .9
2.46	86.7	35.3	.597	59.2	+ .8
2.54	93.0	36.5	.589	62.0	+ 2.0
2.90	98.5	34.0	.559	60.8	+ .8
3.19	103.7	32.5	.536	60.7	+ .7
3.72	118.4	31.8	.502	63.5	+ 3.5

Average: 60.4

Second average: 59.6

Mean: 60.0

* $\sigma\sigma_0$ = ratio $\frac{\text{mean}}{\text{max.}}$ gradient, calculated.

FIG. 15. Parallel Cylinder, 1.11" Diameter. Smooth Core Alternator, 125 Cycles.

TABLE VIII.
Brass Disks of $\frac{5}{32}$ " Thickness, in Path of Discharge of $\frac{1}{4}$ " Spheres, at Right Angles Thereto.
Spheres Amalgamated, Disks suspended by three silk threads, 125 cycles.

Smooth Core Alternator: A-10-30-1500. 7-22-'96.										
Number of disks	Diameter of disks, inches.	Distance between spheres and disks.		Total distance between spheres. <i>a</i> ;	Net air distance between spheres. <i>b</i> ;	Air distance between spheres around edge of disk. <i>c</i> ;	Kilovolts, effective.	Δ_a	Δ_b	Δ_c^*
		Inches.	Per cent.							
1	15			$8\frac{1}{8}$		.....	98			
1	10			$8\frac{1}{8}$		.....	99			
1	10			$8\frac{1}{8}$		.....	100			
1	10			$9\frac{1}{8}$		.....	101			
1	10			$9\frac{1}{8}$		.....	101.3			
2	10			9 185		.....	110			
2	10			10.75		.....	123.5			
2	10	$4\frac{1}{8} + 4\frac{3}{8}$	.50 + .50	$8\frac{1}{8}$	$8\frac{1}{8}$	.....	126	23.5	24.5	
1	10	$4\frac{1}{8} + 4\frac{3}{8}$	.50 + .50	$8\frac{1}{8}$	$8\frac{1}{8}$	.....	126	26	27	
1	10	$3 + 5\frac{1}{8}$	.34 + .66	$8\frac{1}{8}$	$8\frac{1}{8}$	.....	124	24	25	
1	10	$1\frac{3}{8} + 7\frac{3}{8}$	.19 + .81	$8\frac{1}{8}$	$8\frac{1}{8}$	.....	116	16	16	
2	10	$2\frac{1}{8} + 2\frac{1}{8} + 2\frac{1}{8}$	.33 + .33 + .33	$9\frac{1}{8}$	$8\frac{1}{8}$	.....	127	26	28	
2	10	$\frac{3}{4} + 7\frac{1}{8} + \frac{3}{4}$	.085 + .83 + .085	$9\frac{1}{8}$	$8\frac{1}{8}$	.....	94.3†	— 6.7	— 4.7	
3	10	$\frac{3}{4} + 3\frac{3}{8} + 3\frac{3}{8} + \frac{3}{4}$	.087 + .413 + .413 + .087	$9\frac{1}{8}$	$8\frac{1}{8}$	.....	94.7†	— 6.3	— 3.3	
1	6	$4\frac{1}{8} + 4\frac{1}{8} + 4\frac{1}{8}$	.50 + .50	$8\frac{1}{8}$	$8\frac{1}{8}$	10.75	115.5	15.5	10.5	5.5
1	2	$4\frac{1}{8} + 4\frac{1}{8}$	.50 + .50	$8\frac{1}{8}$	$8\frac{1}{8}$	9.185	108.5	8.5	9.5	7.2

* Δ_a = kilovolts observed minus kilovolts required for distance *a*. Δ_b = kilovolts observed minus kilovolts required for distance *b*. Δ_c = kilovolts observed minus kilovolts required for distance *c*.

† Decided capacity effect, with increase of primary current, in generator circuit, to 8 amperes.

soon as the dielectric stress, that is the electrostatic gradient anywhere in the path of the discharge exceeds a certain constant value, which would be called the "dielectric strength of air" in analogy to the "mechanical strength" of a material. The mechanical strength of a material is tested by subjecting it to a uniform mechanical stress, the usual test of tensile strength by

FIG. 16. Points in Air, Fog and Steam at Atmospheric Pressure. Ironclad Armature, 125 Cycles.

pulling a test piece of known cross-section to pieces. The analogous test of the dielectric strength of air would require a uniform electrostatic field, that is, parallel plates of perfect polish as electrodes. This is obviously not feasible. Thus I chose a shape of electrodes giving a relatively simple, although not uniform electrostatic field.

Between spheres, the lines of electrostatic force and equipotential surfaces are curves and surfaces of higher order. Between parallel cylinders, however, lines of force and equipotential lines are two linear systems or pencils of circles or rather cylinders, that is, all lines of electrostatic force are circles intersecting each other in two points, the fundamental points of the pencil, and all

TABLE IX.

Wet Plank Parallel to Discharge of $\frac{1}{4}$ " Spheres.

Grain parallel to discharge. Mohawk river water. Spheres amalgamated.
125 cycles.

Smooth Core Alternator : A-10-30-1500.					7-22-96.					
Distance, inches, between :		Kilovolts, effective :	Struck to :	Average Kilovolts,* effective. 9	$\Delta_a :$	$\Delta_b :$	$\Delta_c :^\dagger$			
Spheres :	Sphere and plank.									
6	∞	76.5	—	—	}	From Table 2.				
8	"	93	—	—						
10	"	105.6	—	—						
12	"	118	—	—						
$4\frac{1}{4}$	"	58	—	—						
$6\frac{1}{8}$	"	77.7	—	—	}	7.0	11.5	18.5		
9	"	99.5	—	—						
$10\frac{7}{8}$	"	110.5	—	—						
6	2	67.7	plank.	{ 69.5						
"	$2\frac{1}{4}$	71.3	spheres.							
"	$2\frac{1}{2}$	71.3	"	{ 82.1	10.9	4.4	15.3			
"	3	71.3	"							
8	$2\frac{3}{4}$	78	plank.	{ 82.1						
"	3	80.7	"							
"	$3\frac{1}{8}$	83.5	spheres.	{ 82.1	10.9	4.4	15.3			
"	$3\frac{1}{2}$	86.5	"							
10	$3\frac{3}{4}$	88	plank.	{ 103				2.6	3.5	6.1
"	$4\frac{1}{2}$	102	"							
"	$4\frac{1}{2}$	104	spheres.	{ 103						
"	$4\frac{5}{8}$	105	"							
12	$5\frac{1}{4}$	116.5	plank.	{ 117.3	.7	6.8	7.5			
"	$5\frac{3}{8}$	119.5	"							
"	$5\frac{1}{2}$	115	spheres.	{ 117.3						
"	$5\frac{5}{8}$	121	"							

* That is, voltage where the direct path between the spheres, and the path over the wet plank have the same dielectric strength.

$^\dagger \Delta_a$ = kilovolts of direct discharge between spheres, minus 9.

Δ_b = 9 minus disruptive voltage between spheres at twice the distance of wet plank.

$\Delta_c = \Delta_a + \Delta_b$

equipotential lines are circles intersecting the first set at right angles, and containing the cross section of the cylinders as one pair.

This being the relatively simplest case, it was chosen for the investigation. Obviously the length of the parallel cylinders

could not be infinite, and therefore, to avoid the influence of the ends of the cylinders, they were given a slight backward curvature of a very large radius compared with the diameter of the cylinders and their distance, as shown in Fig. 18, which represents the relative proportions of the larger cylinders at the highest observed striking distance. The effect of this curvature was investigated and correction made therefor.

FIG. 17. Comparison of Points and Spheres. Ironclad Armature, 125 Cycles.

From the distance between the cylinders d and the striking voltage V , the mean electrostatic gradient $g = \frac{V}{d}$ was calculated and is given in the third column of Tables VI. and VII. in kilovolts per inch. From diameter and distance of cylinders, the ratio of mean to maximum gradient σ was calculated, and by

TABLE X.
METEOROLOGICAL RECORD AT DATE OF TESTS.

Date.	TEMPERATURE ° C:					BAROMETER, ":		SKY:		Tests made.
	Preceding nightly minimum.	8 h morning.	Daily maximum.	9 h evening.	Following nightly minimum.	8 h morning.	9 h evening.	Forenoon.	Noon. Afternoon.	
April 6	-2	+1.5	+11.5	+2	+5	29.9 "	29.8 "	cloudless	almost cloudless	1.11" cylinders, smooth core.
" 14	—	17	25	17	—	29.55 "	29.7 "	cloudless	cloudy	.313 "
July 13	26	—	30	24.5	23	29.7 "	29.6 "	almost cloudless	almost cloudless	" "
" 14	23	25	29	23	18.5	29.65 "	29.65 "	cloudless	cloudless	1/4", 1/2", 1", 2" spheres.
" 17	13	17.5	26	20	15.5	29.8 "	30.0 "	almost cloudless	almost cloudless	Smooth core.
" 20	20	21.5	25.5	23.5	21	29.9 "	29.7 "	rain	rain	Needle points, smooth core.
" 21	21	25	31	25	22.5	29.77 "	29.8 "	almost cloudless	almost cloudless	1/4", 1/2", 2" spheres, ironclad.
" 22	22.5	23.5	31.5	23	19	29.8 "	29.5 "	almost cloudless	rain and thunderstorm	Needle points, smooth core.
" 24	15.5	18	20.5	16	15.5	29.75 "	29.5 "	cloudy	rain	Brass disks and wet plank.
" 25	15.5	18	26.5	18.5	15	29.62 "	29.8 "	almost cloudless	almost cloudless	Needle points, ironclad.
										1/4" spheres, smooth core.

dividing the observed mean gradient g by the factor σ , the maximum gradient $g_o = \frac{g}{\sigma}$ would be derived for parallel cylinders.

To correct for the backward curvature of the cylinders, of radius r_o , the ratio of mean to maximum radiant was calculated for the distance d but a diameter of cylinders of $2r_o$. Since σ_o differs very little from one, being at the highest distance observed with large cylinders $\sigma_o = .994$ and with small cylinders $\sigma_o = 0.893$,

FIG. 18.

by dividing with σ_o the correction was far within the errors of observation.

Thus the maximum gradient g_o was derived from the observed mean gradient $g = \frac{V}{d}$ as: $g_o = \frac{g}{\sigma \sigma_o}$.

The calculation of σ and σ_o is as follows:

Let in Fig. 19 be represented the cross-section of the two parallel cylinders of equal diameter.

Let x = abscissæ with center 0 between the parallel cylinders as origin.

y = ordinates of one line or circle of electrostatic force of radius R .

d = distance inside between parallel cylinders.

r = radius of parallel cylinders.

p = distance of fundamental points A and A^1 of cylindrical pencil (common point of intersection of all lines of force) from origin.

FIG. 19.

At the surface of the cylinders, it is :

$$y_0 = \sqrt{R^2 - x_0^2} - \sqrt{R^2 - p^2};$$

in general it is :

$$y = \sqrt{R^2 - x^2} - \sqrt{R^2 - p^2};$$

it is :

$$p^2 = x_0 (x_0 + 2r).$$

Let g_0 = electrostatic stress at the cylinder surface in the central line, that is at x_0 ; then it is :

$$g_x = \frac{y_0}{y} = \text{electrostatic stress at point } x \text{ of central line,}$$

if the line of force 1 is infinitely near the central line, that is, if:

$$r = \infty.$$

Thus the ratio of average electrostatic stress g to maximum stress g_0 is:

$$\sigma = \frac{g}{g_0} = \frac{1}{x_0} \int_0^{x_0} \frac{y_0}{y} dx.$$

Substituting, it is:

$$\sigma = \frac{1}{x_0} \int_0^{x_0} \frac{\sqrt{R^2 - x^2} - \sqrt{R^2 - p^2}}{\sqrt{R^2 - x^2} - \sqrt{R^2 - p^2}} dx.$$

Introducing the transformation:

$$\sqrt{R^2 - x^2} = R - ux,$$

$$x = \frac{2uR}{1+u^2},$$

$$d = \frac{2R(1-u^2)}{(1+u^2)^2},$$

$$\sqrt{R^2 - x^2} = \frac{R(1-u^2)}{(1+u^2)},$$

$$u = \frac{R}{x} \left\{ 1 - \sqrt{1 - \frac{x^2}{R^2}} \right\},$$

$$\sqrt{R^2 - x_0^2} = a,$$

$$\sqrt{R^2 - p^2} = b,$$

it is:

$$\sigma = \frac{2R(a-b)}{x_0} \int_0^{x_0} \frac{(1-u^2) dx}{(1+u^2) \{(R-b) - (R+b)u^2\}}.$$

Substituting:

$$R - b = c^2,$$

$$R + b = g^2,$$

$$\delta = \frac{2R(a-b)}{x_0(g^2+c^2)} \int_0^x \left\{ \frac{1+u^2}{2} + \frac{g^2-c^2}{2c} \frac{g^2-c^2}{gu+c} - \frac{g^2-c^2}{2c} \frac{g^2-c^2}{gu+c} \right\} dx.$$

$$\frac{2R(a-b)}{x_0(g^2+c^2)} \arctan u + \frac{g^2-c^2}{2gc} \ln \frac{gu+c}{gu-c} \bigg|_0^{x_0}$$

or substituting :

$$\sigma = \frac{R}{x_0} \left\{ \sqrt{1 - \frac{x_0^2}{R^2}} - \sqrt{1 - \frac{p^2}{R^2}} \right\} / 2 \operatorname{arc} \tan \frac{R}{x} \left\{ 1 - \sqrt{\frac{x^2}{R^2}} + \frac{R}{p} \sqrt{1 - \frac{p^2}{R^2}} \lg \frac{p \left\{ 1 - \sqrt{1 - \frac{x^2}{R^2}} \right\} + x \left\{ 1 - \sqrt{1 - \frac{x^2}{R^2}} \right\}}{p \left\{ 1 - \sqrt{1 - \frac{x^2}{R^2}} \right\} - x \left\{ 1 - \sqrt{1 - \frac{p^2}{R^2}} \right\}} \right\} \Bigg|_{x_0} = \frac{R}{x} \left\{ \sqrt{1 - \frac{x^2}{R^2}} - \sqrt{1 - \frac{p^2}{R^2}} \right\} 2 \operatorname{arc} \tan \frac{x^2}{R} \left\{ 1 - \sqrt{1 - \frac{x^2}{R^2}} \right\} + \frac{R}{p} \sqrt{1 - \frac{p^2}{R^2}} \lg \frac{x_0 \left\{ 1 - \sqrt{1 - \frac{p^2}{R^2}} \right\} + p \left\{ 1 - \sqrt{1 - \frac{x^2}{R^2}} \right\}}{x_0 \left\{ 1 - \sqrt{1 - \frac{p^2}{R^2}} \right\} - p \left\{ 1 - \sqrt{1 - \frac{x^2}{R^2}} \right\}} \Bigg\}^0.$$

In the central line between the parallel cylinder it is :

$$R = \infty.$$

Substituting this, it is :

$$\sigma = \sqrt{1 - \frac{x^2}{R^2}} = 1 - \frac{1}{2} \frac{x^2}{R^2} - \frac{1}{8} \frac{x_0^4}{R^4}.$$

Thus :

$$\sigma = \frac{p^2 - x^2}{2 p x_0} \lg \frac{p + x_0}{p - x_0},$$

the ratio of average to maximum strain in the central line, where :

$$x_0 = \frac{d}{2},$$

$$p^2 = x_0 (x_0 + 2 r).$$

Substituting this, it is :

$$\sigma = \frac{r}{p} \lg \frac{p + x}{p - x},$$

and if the distance x_0 is very small compared with the radius r , it is :

$$\begin{aligned} \sigma &= \frac{r}{p} \lg \frac{p + x_0}{p - x_0} \\ &= \frac{2r}{p} \left(\frac{x_0}{p} + \frac{x_0^3}{3p^3} + \frac{x_0^5}{5p^5} \right), \end{aligned}$$

or, since :

$$p^2 = x_o (x_o + 2r,$$

approximately :

$$\sigma = 1 - \frac{x_o}{2r}.$$

If now

r = diameter of cylinders,

r_o = ratio of curvature of cylinders (r_o being very large compared with r and x_o), it is :

$$\sigma = \frac{r}{p} \lg \frac{p + x_o}{p - x_o},$$

$$\sigma_o = 1 - \frac{x_o}{2r_o},$$

and

$$\sigma \sigma_o = \frac{r}{p} \left(1 - \frac{x_o}{2r_o} \lg \frac{p + x_o}{p - x_o} \right),$$

the ratio of the maximum to the mean electrostatic gradient within the path of the discharge.

Thus, if g = mean gradient,

$$g_o = \frac{g}{\sigma \sigma_o} \text{ is the maximum gradient.}$$

With the cylinders used it was :

$$r = .1565 \text{ and } .555 \text{ respectively.}$$

Herefrom the numerical values of correction factor $\sigma \sigma_o$ and maximum gradient g_o are calculated and given in Tables VI and VII.

With the large cylinders, which were investigated first, the values of the maximum dielectric gradient g_o were found over the whole range from the lowest to the highest voltage to average 60. The difference of individual values of g_o from the average is not greater, but rather less than found under similar conditions in tests of tensile strength in mechanics. They are, however, greater than can be accounted for by the errors of observation, thus showing the existence of some secondary effect disturbing the results. The values of g_o show no noticeable tendency of deviation from constancy for very high or very low voltages, and appear rather low in the middle, high at either end of the curve.

The tests with the smaller cylinders gave an equally good agreement between the values of the maximum gradient g_o .

However, comparing both sets of tests we found that the "dielectric strength of air" averages 60 with the 1.1" cylinders but averages 77 with the .313" cylinders. Thus, while the max-

imum gradient is fairly constant with the same size of cylinders and different distances, it differs considerably with different sizes of cylinders, or in other words the dielectric strength of air as found in this way is not a constant, as the tensile strength of mechanics, but varies with the conditions.

Some further results on the question of the constant dielectric strength of air can be derived from the tests with points and with spheres as terminals.

Between mathematical points the dielectric field of force is similar regardless of the distance. Thus with the needle points used, for all distances great compared with the radius of curvature of the needle points, that is already the smallest distance investigated, the striking distance should be proportional to the voltage if it depends upon the maximum gradient. This, however, is not the case.

Between mathematical points even with the lowest voltage the gradient at the points is infinite. Thus with needle points very small voltages should strike over great distances. This is not the case either, but the curve of needle points does not differ much from that of $\frac{1}{4}$ " spheres. This leads us to one of the phenomena obscuring the true electrostatic gradient at the moment of disruption, the brush discharge.

Long before the disruptive voltage is reached between needle points and smaller spheres or cylinders, a brush discharge issues from the terminals into the surrounding air. The space surrounding the electrodes is filled with an infinite number of violet streamers issuing from the electrodes under a hissing noise and being constantly in motion, and this space is thus more or less broken down dielectrically, so that, when the final disruptive discharge takes place, the terminals of the discharge are no longer the original metal terminals, but the whole space covered by the brush discharge is more or less to be considered as discharge terminals.

An attempt was made to investigate this phenomenon, thus: Instead of the amalgamated metal cylinders of diameter D and distance d , two cylinders were substituted of the diameter $D + x$ and distance $d - 2x$ and for the latter cylinders the mean gradient $d = \frac{V}{d-2x}$, the ratio of mean to maximum gradient $\sigma\sigma_0$, and this the maximum gradient g_0 calculated for various values of x .

Representing then the length of the brush discharge by x , with constant D , d , and V , g_0 decreases with increasing x , reaches a minimum and then increases again; or inversely, at a constant maximum gradient g_0 , with increasing length of brush discharge, the voltage required to give this gradient g_0 will increase, reach a maximum and then decrease again. Thus if the terminal voltage is gradually increased, as soon as the maximum gradient at the terminals exceeds the breaking strain, a brush discharge will issue and thereby enlarge the size of the terminals by the space covered by the brush discharge, but at the same time decrease the maximum gradient at the surface of the effective terminals, that is, the space covered by the brush discharge, until the critical point is reached beyond which a further expansion of the brush discharge increases the gradient at its surface again. At this point a disruptive discharge will take place.

No satisfactory results were found, however, by this method, probably for the reason that:

1st. The brush discharge at high voltage does not cover a well defined distance, but consists of an infinite number of streamers issuing from the electrodes to a greater or lesser distance and being in rapid and violent agitation.

2nd. The space covered by the brush discharge is only partially broken down, that is while not having the dielectric strength of air before the break-down, its dielectric strength has not fallen to zero yet, as after the discharge. It thus represents electrostatically a similar condition as mechanically a medium under strain exceeding the elastic limit of the material. Under mechanical stress, rupture does not always take place as soon as the strain anywhere in the cross-section exceeds the breaking strain.

Comparing, for instance, a beam of the cross-section A in Fig. 20 with that in B of the same figure, which latter differs only by the addition of the two small ribs α on top and bottom, it is obvious that B has no lesser strength than A . But at the extremities of the ribs α the breaking strain is exceeded at a load far below that which would break down beam A . The result is that in the two thin ribs α , cracks are formed or the material strained beyond the elastic limit, and only at much greater load the break-down of the beam occurs.

The tests made with spheres allows some further conclusions. The electrostatic field of force between two spheres of diameter D and distance d is similar to the field between spheres of diame-

ter n D and distance n d . Thus the ratio between mean and maximum gradient is the same. Hence, if disruptive discharge takes place at the same maximum gradient, in the latter case n times the striking voltage would be required.

Since tests with four different diameters of spheres were made, this feature is easy to investigate. In the following a number of sets of readings are given abstracted from Tables II to V: Values are chosen in that range of voltage where the four curves differ most. The first column gives the diameter of the sphere

FIG. 20.

D , the second column the striking distance d chosen proportional to the sphere diameter, the third column the striking voltage, and the fourth column the ratio α of mean gradients of subsequent lines. As seen, while constant dielectric strength of air would require a ratio $\alpha = 1$, the ratio differs herefrom. This difference could not well be accounted for by a brush discharge depending upon the electrostatic gradient, and thus some further secondary effects are probable.

Diameter of spheres, inches. <i>D</i> :	Distance of spheres, inches. <i>d</i> :	Striking voltage, kilovolts. <i>V</i> :	Ratio of gradients. <i>a</i> :
1	8	98.7	1.194
$\frac{1}{2}$	4	59	1.152
$\frac{1}{4}$	2	34	
1	6	87	1.134
$\frac{1}{2}$	3	49.3	1.180
$\frac{1}{4}$	$1\frac{1}{2}$	29	
1	4	75.4	1.060
$\frac{1}{2}$	2	40	1.165
$\frac{1}{4}$	1	23.6	
2	6	115	1.194
1	3	68.5	1.035
$\frac{1}{2}$	$1\frac{1}{2}$	35.4	1.186
$\frac{1}{4}$	$\frac{3}{4}$	21	
2	4	103	1.155
1	2	59.5	1.010
$\frac{1}{2}$	1	30	1.154
$\frac{1}{4}$	$1\frac{1}{2}$	17.3	
Average.....			1.155

At half the distance, the gradient is 1.135 times as great. Thus, if g_1 = gradient at distance d_1 , it is, at distance :

$$d = \frac{1}{2_n} d_1,$$

the gradient :

$$g = 1.135^n g_1,$$

thus :

$$\log d = \log d_1 - n \log 2,$$

$$\log g = \log g_1 + n \log 1.135,$$

hence, eliminating n :

$$.183 \log d + \log g = .183 \log d_1 + \log g_1,$$

$$g d^{.183} = g_1 d_1^{.183}$$

or :

$$g = \frac{a}{d^{.183}},$$

$$V = a d^{.817},$$

where :

$$a = g_1 d_1^{.183} = \frac{V_1}{d_1^{.817}},$$

or :

$$d = b V^{1.22},$$

where :

$$b = \frac{d_1}{V_1^{1.22}}.$$

This gives a parabolic law of striking distances in uniform field.

According hereto the curve of striking distances should approach the zero point horizontally, that is for very small voltages the electrostatic gradient should approach infinity and the striking distance zero. This, however, is not the case, since even for very small voltages the striking distance and the gradient are finite. On the contrary, in the tests with spheres the gradient for very small distances approaches 75 to 80, about the same value as the dielectric strength of air as found with small cylinders. Hence, if a constant dielectric strength of air exists it would be somewhere between 60 and 80 kilovolts per inch. Some results are in favor but some in opposition to such a hypothesis.

The curve of striking distance between sharp points has a very interesting shape. Up to $1\frac{1}{4}''$ striking distance it is proportional to the voltage, and represented by the equation,

$$d = .05 V$$

or

$$V = 20 d,$$

then it curves upwards and becomes a straight line again from $3''$ upward to the limits of test, at $14\frac{1}{2}''$.

This straight line does not pass through the origin but through a point at the voltage 22 or the distance $-2\frac{3}{4}''$, and can thus be represented by the equation,

$$d = .125 V - 2.75,$$

or,

$$V = 8 d + 22.$$

This feature points to something like a spurious "counter E. M. F. of the spark." That is, beyond $3''$ distance, by assuming a potential of 22 as required for the discharge passing from the terminals to the air, the remaining voltage is proportional to the distance. For low voltages the curve leaves the straight line and slopes towards the origin, becoming straight again for still lower voltages.

This shape of curve bears a striking similarity to the volt-ampere curve of the electric arc or to that of electro-chemical polarization.

In the electric arc under constant conditions the voltage is approximately proportional to the current plus a constant value, the spurious counter E. M. F. of the arc.

In an electrolyte the current is proportional to the voltage for small voltages below the counter E. M. F. of polarization. Approaching the latter the current rises beyond proportionality, that is, the volt-ampere curve bends upwards, and ultimately turns into a second straight line, when the dissociation voltage V_o is passed, which latter straight line does not pass through the origin, however, but through the point $C = 0$, $V = V_o$, and thus can be represented by

$$C = a V + V_o.$$

Thus the theoretical volt-ampere curve of the electrolyte is of the same shape as the distance-volt curve between points.

Since with alternating E. M. F. the current in the electrolyte below the dissociation potential is essentially in quadrature with the E. M. F. the current over the whole range can be represented by the theoretical equation,

$$C = \sqrt{a^2 V^2 + b^2 (V - V_o)^2}$$

where the latter term appears only beyond $V = V_o$. It is of interest to note that the curve of striking distance between sharp points over the whole range from the origin to the limits of observation can be represented by

$$d = \sqrt{.05^2 V^2 + .115^2 (V - V_o)^2}.$$

This equation represents the curve absolutely as shown in the following comparison, where the first column gives the voltages, the second the striking distance taken from test, and the third the striking distances as calculated from this formula:

$$d = \sqrt{d_1^2 + d_2^2}$$

$$d_1 = .05 V$$

$$d_2 = .115 (V - 27.5.)$$

V :	d calc:	d obs:
10	.5	.5
20	1.0	1.0
27.5	1.88	1.38
30	1.53	1.53
60	4.8	4.8
90	8.5	8.5
120	12.25	12.25
150	16.0	16.0

The magnitude of this spurious counter E. M. F. or transition resistance of the spark is the same as that at which the brush discharge is formed. Not the electrostatic glow of bluish color which under favorable conditions can be observed already below 1,000 volts, but the real brush discharge consisting of violet streamers issuing from the terminals into the surrounding air.

The tests with spheres show also more or less marked the appearance of straight lined parts in the curve, passing through a point differing from the origin.

The most interesting feature in connection therewith is, however, if an electric conductor as a brass disk is inserted into the path of the discharge.

A considerable number of tests were made and are recorded in table 8, by inserting brass disks of different diameters and in different positions and different numbers into the path of the discharge between $\frac{1}{4}$ " spheres. The first column of table 8 gives the number of disks used, the second the diameter, the third and fourth the position of the disks between the spheres, the fifth and sixth the total distance and the net distance between the spheres and the seventh the striking voltage. The last three columns give:

As Δ_a the difference in the observed voltage and the striking voltage between $\frac{1}{4}$ " spheres at the same total distance without disks between them.

As Δ_b the difference in the observed voltage and the striking voltage between $\frac{1}{4}$ " spheres at a distance equal to the net air distance between spheres and disks.

As Δ_c the difference in the observed voltage and the striking voltage between $\frac{1}{4}$ " spheres at the same distance as in the test, but in a line over the edge of the disk.

It is of interest to note that the insertion of an electric conductor, as a brass disk, increases the voltage required to strike between the spheres, although the net air distance is reduced by the thickness of the disk.

Thus with one 10" disk midway between $\frac{1}{4}$ " spheres at about 9" distance, 126 kilovolts are required to strike across, while after withdrawing the disk 100 kilovolts will strike; or in other words, at voltages between 100 on 126 kilovolts between $\frac{1}{4}$ " spheres and 9" distance in air, a disruptive discharge will take place, but will not take place any more if an electric conductor as a brass disk is inserted between the spheres.

The disks were suspended and insulated by three silk threads. Between 10'' and 15'' disks, no difference seems to exist, but 6'' disks increase the striking voltage less, and still less 2'' disks. In either of the two last cases, however, the striking voltage is higher than would be required for a stroke from sphere to sphere around the edge of the disk.

This phenomenon could be explained by the fact that two sparks of half the length require a higher voltage than one spark of full length. This explanation, however, does not explain the observations on the 6'' and 2'' disks, where the voltage was higher than necessary for a single stroke through air around the edge of the disk, by 5.5 and 7.2 kilovolts respectively. If the disk is not midway between the spheres, but nearer to one side, the increase of voltage is less.

Two disks equidistant from each other and the spheres, increase the voltage by the same amount as one disk in the center, and less if they are nearer to the spheres, and disks very close to the spheres even reduce the striking voltage, under marked capacity effects. The same applies to three disks.

That means, if the disks are very close to the electrodes they are probably brought in perfect electric connection therewith by the brush, and thus form a new set of electrodes.

The increase of voltages due to the insertion of an electric conductor is from 23.5 to 27 kilovolts, that is about the same as the spurious counter E. M. F. of the spark, and would thus well agree with the assumption of such a phenomenon, since the insertion of a conductor increases the number of transitions from metal to air by two, and thus should require an increase of the striking voltage by the counter E. M. F. of the spark.

The next table, No. 9, gives tests of striking distances between $\frac{1}{4}$ '' spheres when an electric conductor, as a plank wetted with river water, is approached parallel to the discharge line. The effect thereof is a change of the striking voltage below that required for twice the distance between the spheres, but beyond that required for twice the distance between spheres and plank. That is, when the plank is approached so far that the single discharge between the spheres and the double discharge from spheres to plank have the same dielectric strength, the striking voltage is intermediate between that corresponding to the length of the direct discharge and a distance equal to the sum of the two discharges from spheres to plank. The difference in voltage

which would be required for direct discharge between spheres and for a discharge equal to twice the distance of the plank is 18.5 and 15.3 kilovolts respectively in the first two cases, less in the last two, where probably the resistance of the plank as an imperfect conductor was noticeable. Making allowance for the latter, we find again a voltage of the magnitude of the spurious counter E. M. F. of the spark, as discussed under sharp point.

Tests with the ironclad alternator between sharp points in air, in fog and in steam, point also to such a counter E. M. F. of the spark, or lesser magnitude with the ironclad alternator, due to the greater ratio of maximum to effective E. M. F.

These sets of tests with brass disks and wet plank were suggested by some observations made three years ago with an iron-clad alternator and recorded on table I.

To investigate whether the dielectric strength of air was reduced in fog, as occasionally suggested at that time, a steam pipe was arranged below the discharge circuit and steam made to escape very slowly, so as to fill the whole space between the needle points with dense fog. Against expectation it was found that a much higher voltage was required to strike across the same distance in fog as in dry air. This is seen in Fig. 16. Hence fog increases the dielectric strength of air and reduces the striking distance very markedly. This may be explained by the assumption that the discharge is by the fog broken up in an infinite number of minute successive discharges, between the conducting fog particles, requiring a higher total voltage than the direct discharge over the same total distance.

By turning full steam on, issuing from a boiler at 80 pounds pressure into the space between the needle points, the striking distance was still reduced further below that in fog, that is, live steam at atmospheric pressure has a much greater dielectric strength than dry air.

From these tests, the following results are derived:

1st. At constant voltage and constant wave shape, that is constant ratio between maximum and effective E. M. F., the striking distance is a constant, especially between sharp points, where the tests have been repeated over and over again, and independent of the atmospheric condition, the frequency, etc., to such an extent that the striking distance between needle points offers the most reliable means to determine very high voltages. For this reason, it is used in this manner as final check in all high potential insulation tests of the General Electric Company.

2nd. No physical law has been found to represent satisfactorily all the observations. Some point to the existence of a constant dielectric strength of air, analogous to the tensile strength of mechanics. Others point to the existence of a spurious counter E. M. F. of the spark or transition resistance from electrode to air.

3rd. Constant dielectric strength. Cylinders of 1.11" diameter give an average disruptive strength of air of 60 kilovolts per inch. Cylinders of .315" diameter, an average dielectric strength of 77. Spheres at very small distance point toward the latter value. As a disturbing factor in this case, enters the electrostatic brush discharge, which by a partial breakdown of the air surrounding the electrodes changes and increases the size and decreases the distance of the effective terminals.

4th. Counter E. M. F. of the sparks. The tests with sharp points give 22 kilovolts, or 11 kilovolts for a single transition from terminal to air. Spheres give curves pointing to a similar phenomenon. Electric conductors inserted at right angles into or parallel with the discharge, point to the existence of a counter E. M. F. of the same magnitude. The beginning of the electrostatic brush discharge is at a potential of this magnitude also.

5th. Potentials of 160,000 volts effective and even up to 170,000 volts have been experimented with. They are probably the highest voltages ever reached by man at ordinary frequencies with alternating currents of considerable power.