THE INFLUENCE OF MATHEMATICS ON THE PROGRESS OF PHYSICS.*

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In discussing the value of a given study, a lecturer is by common consent allowed—sometimes even in private duty bound—to exaggerate the importance of his subject, and to present it to his audience enlarged, as it were, through the magnifying power of a projecting lens, so that the details with which he has necessarily to deal may be brought into more prominent view. In an introductory lecture such as it is my duty to give to-day, the speaker need the less feel any scruples in following the usual custom, as different subjects are treated of in successive years, and the hearer may, after the lapse of a short cycle, strike a pretty fair balance between the various branches which have successively been brought before him. But although I might have felt tempted to-day to insist on the advantages of applied mathematics as a separate subject not only worthy of study, but second to none in interest and importance, and though I feel no doubt you would have accorded to me the indulgence which everybody requires who endeavors to lay an abnormal stress on the merits of a single branch of human knowledge, I prefer to found the claims of the subject which I have the honor to represent in this college, not so much on its intrinsic value as on the influence it has had on the progress of other sciences. For no subject can stand by itself, and the utility of each must be measured by the part it takes in the play of the acting and reacting forces which weave together all sciences into a common web.

The growing importance of mathematics as an aid to the study of all sciences is daily becoming more apparent, and it may indeed be questioned whether at the present time we can speak of physics as apart from applied mathematics. Riemann's opinion that a science of physics only exists since the invention of differential equations is intelligible; but however close the connection between physics and mathematics may be or may become, their growth in the earlier stages has been altogether independent. Galileo may be said to have been the founder of mathematical physics, and among his successors have been many who showed a greater inclination toward pure mathematics than toward physics proper. On the other hand, we can trace back the ancestry of our experimental physicist and that of our modern popular books on science to the Middle Ages, where we reach J. Baptista Porta and his books on natural magic. Even eighty years ago the fullest account of the state of experimental science was to be found in "Wiegler's Natürliche Magie," a book of twenty volumes, in which scientific experiments and conjurers' tricks are alternately described. But since the beginning of this century the importance of the mathematical treatment of purely physical subjects has steadily grown, and fifty years ago the two sciences were already sufficiently united to induce the founders of the British Association to join them together into one section. From that time until the present year, when the mass of work necessitated a temporary separation, the experimentalist and the pure mathematician could be seen at the annual meetings listening, or at least appearing to listen, to each other's investigations, and the influence which men of science on these occasions had on each other may be taken to represent roughly the mutual influence of the two sciences themselves; it was substantial, though in great part unconscious. I could not attempt to-day to give you a complete historical survey of the effect which the contact—one might often say the collision—of the two sciences had on the progress of each; even that part of the subject which I have chosen for special consideration is too vast to be successfully confined within the limits of a single lecture, and an incomplete sketch is all I can offer.

The influence of mathematical investigations on physical theories is not restricted to any single stage, but makes itself apparent throughout the whole course of their evolution. Before a theory is even started, the mathematician is often necessary to prepare its way. He has to classify complicated facts in a systematic manner, and working backward from the phenomena presented by nature, he endeavors to find out which of them are necessary consequences of others, and which of them require independent hypotheses for their explanation. It is in this way that the works of Poisson, Green, Gauss, and of all those who have followed in their footsteps, may be said to have laid the foundation of the theory of magnetism and electricity, although we do not as yet possess any physical notions as to the causes of these phenomena. The true power of mathematics, however, comes into play only when the physical inventor has done his work, and has formed distinct materialistic conceptions which allow themselves to be expressed by mathematical symbols. It is then that the consequences of the theory are to be worked out and tested by experiment. In order to be convinced of the truth of any hypothesis, the scientific world wants quantitative experiments. Numbers form the connecting link between theory and verification, and they always imply mathematical formulæ, however simple these may be. Often two rival theories are on their trial and the mathematician is supposed to find out where their conclusions differ and where crucial experiments are most likely to decide definitely between them. It is remarkable, however, how much more often physical or even metaphysical considerations have decided between two theories than arguments derived from mathematical reasoning. So called crucial experiments, as a rule, come either too early or too late. Sir Humphry Davy's experiment was absolutely conclusive against the corpuscular theory of heat, but scientific ideas were not ripe yet for the discovery, and his experiment had no marked effect on the progress of science. The crucial experiment here did not involve any mathematical deductions; it is otherwise with that which might have decided between the two theories of light. According to the corpuscular theory, light travels more quickly in water than in air; according to the undulatory theory, the passage through water is the slower, and this distinction is founded on the necessity to account mathematically for the laws of refraction. But when Foucault actually made the experiment, and gave a death-blow to the corpuscular theory, that theory was already dead. There was then only one scientific man of note left who still viewed the undulatory theory with suspicion, and his suspicions were not allayed by the crucial experiment. But if mathematical deductions have not decided as often as they might have done between two rival theories, they have constantly strengthened and confirmed our belief in physical hypotheses by inventing new cases which might test the theory, and which might, if experiment supported the mathematical deduction, establish it on a yet firmer basis.

The most important of all the functions of mathematical physics, however, and perhaps the only one through which mathematics has had unmitigated beneficial influence on the progress of physics, is derived from its power to work out to their last consequences the assumptions and hypotheses of the experimentalist. All our theories are necessarily incomplete, for they must be general in order to avoid insurmountable difficulties. It is for the mathematician to find out how far experimental confirmation can be pushed, and where the new hypothesis is necessary. Facts apparently unconnected are found to have their origin in a common source, and often only a mathematician can trace their connection. It is here that the pure experimentalist most often fails. A new experiment gives results to him unexpected, and he is tempted to invent a new theory to account for a fact which may only be a remote consequence of a long-established truth. Many examples might be given to show how mathematics often finds a connection unsuspected by the pure experimentalist, but one may be sufficient. A ray of light passing through heavy glass placed in a magnetic field, in the direction of the lines of force, is doubly refracted as it comes out. To none but a mathematician is it clear that this is only a direct consequence of Faraday's discovery that the magnet turns the plane of polarization of the ray on its passage through the glass. Happily this fact was first worked out theoretically; had it been otherwise, we should have heard much of the power of the magnet to produce double refraction.

In addition to the many services actually rendered by mathematical treatment, the mere attempt to put physical theories into a form fit for such a treatment has often been invaluable in clearing the theory of all unnecessary appendages, and presenting it in the simple purity which may bring its hidden failings to light, or may suggest valuable generalizations. Instead of dealing, however, in a general manner with the various ways in which mathematics have been useful in the prosecution of physical investigations, it will be better to give a short account of the growth of some of our physical theories, and to illustrate the subject of this discourse by a few digressions suggested by the historical development.

As a first example I chose the progress of the undulatory theory of light. There is no other branch of physics in which the power of mathematics has been more successfully shown, nor is there one which shows the relations of experimental to mathematical physics in a truer light. At first we had experimental facts ahead of theoretical explanations; then we had the undulatory theory, which placed theory in advance of experiment; and now again a reversal has taken place, and unexplained experiments will remain unexplained until we shall be able to form more definite ideas of the relations between matter and the luminiferous ether.

Huyghens first worked out scientifically the hypothesis that light consisted of the undulations of an all-pervading medium. But as those who adopted the rival theory professed to explain equally well all phenomena which were then generally known, the scientific world preferred to walk in Newton's footsteps, and to reject what they believed to be the complicated and unnecessary assumption of a universal medium. The corpuscular theory could easily explain the ordinary laws of reflection and refraction. Its attempts to explain the colors of thin plates and the fringes of shadows were less successful, but experimental investigations of these phenomena were not sufficiently advanced to bring these facts prominently into view, nor had their true explanation as yet been given. It was only when mathematical analysis was applied to the undulatory theory that its enormous advantages were discovered. Neither of the men to whom we owe the greatest advance which has yet been made in the science of light was a professed mathematician. Young was a medical man; Fresnel was an engineer; nor was the subject, when these men took it up, in a state which would have attracted a mathematician. Conceptions distinctly physical had to be formed, and assumptions not quite satisfactory had to be made. Their chief claim to our gratitude rests, not so much on the mathematical treatment they have given, as on the fact that they left the subject in a state sufficiently advanced to allow mathematicians, even without special physical proclivities, to take it up, extend it, and establish its foundations more firmly than otherwise they could have done.

The different manners in which Young and Fresnel set to work to prove to the scientific world the truth of their favorite hypothesis, and the corresponding difference in their success, is especially interesting for the purpose which we had in view. Both men had considerable mathematical ability, and of the two, Young perhaps had the greater inclination toward pure mathematics, yet he avoided wherever he could the use of mathematical symbols, and disdained to bring forward experimental verification for what he considered sufficiently clear without.* It is to Young that we owe most of the physical conceptions which have secured a final success for the undulatory theory of light. He was the first to explain the principle of interference both of sound and of light, and he was the first to bring forward the idea of transverse vibrations of the undulations of light. The most diverse phenomena were explained by him, but their easy explanation was a sufficient proof to him of the theory he was defending, and he did not trouble to verify his conclusions by extensive numerical calculations. It thus happened that, although Young was first in the field in furnishing the true explanation of complicated phenomena, Fresnel, applying mathematical analysis to a much greater extent, had a much more potent influence in turning the scale of public opinion in favor of their common theory.

Though Fresnel's first memoir was published fourteen years after Young had established the principle of interference, Young's writings had remained unnoticed by him as well as by the scientific world in general, and Fresnel was surprised and irritated to hear that another had been in the field before him. But every one must agree that the chief share in securing the final triumph of the wave theory belongs to Fresnel, nor can there be any doubt that this is due to the mathematical calculations which he applied to cases easily verified by experiment. For there is a great fascination in a table with one column headed "calculated," another headed "observed," and a third giving the differences with the decimal point as much to the left as possible. And it is right that such tables should play an important part in the history of science, for whatever the ultimate fate of a partially accepted theory, the one solid legacy which it will leave behind after its death is the array of numbers for which in its successful stage it has given a sufficiently correct account.

Fresnel invented different pieces of apparatus to test Young's simple supposition, independently made by him,

that waves may be made mutually to destroy one another by addition, the crest of one wave being superposed on the hollow of another. It is necessary that the waves should originally be derived from a single source of light, yet they must seem to diverge from two different points. The necessary experimental conditions were fulfilled by the ingenious device of reflecting the light from two mirrors slightly inclined to each other. The light diverging from the two images of one source was allowed to cross, and bands alternately luminous and dark were measured at the places where the waves overlapped. A rough micrometer of his own construction served to measure the intervals between the bands at various distances from the mirror, and Fresnel succeeded in obtaining sufficient data to test his theory. It cannot be my purpose to follow Fresnel and to describe all the various devices which he invented to confirm his views, and to establish the true theory of diffraction. Though he succeeded in making a convert of Arago, the greatest authorities then living, and the most influential men in scientific matters, both Laplace and Poisson, disdained to consider the theory. The mathematical basis on which the theory rested seemed to them to be weak and insufficient. No doubt they were right; for many assumptions made by Fresnel were daring, and only justified by the results of further more careful investigations; some of his assumptions even were inaccurate. It was only when the phenomena of polarization and double refraction were explained that Laplace acknowledged the great power of the undulatory theory, and with a remarkable inconsistency publicly stated his admiration for Fresnel's work, after a paper which is more unsatisfactory from a mathematical point of view than anything else written by Fresnel. The opposition to the undulatory theory offered by the strictly mathematical school, no doubt prevented its rapid acceptance by the general body of scientific men, but it is doubtful whether its final success was delayed. On the contrary, Fresnel was spurred on to greater exertions, and the excitement caused by the violent views taken by the opposed parties, rendered the question a burning one, which it was necessary to settle definitely. The impartial observers had, at the time of which we are speaking, one strong argument for suspending their judgment. One great class of phenomena, now known under the title of phenomena of polarization, were unexplained as yet, and it seemed doubtful to them whether the undulatory theory could successfully overcome the difficulty. Then, as before, it was Young who first gave the physical explanation, while it was reserved for Fresnel again to show how the explanation was sufficient to account numerically for all the observed facts.

Those who first started the idea of luminous undulations founded their belief in great part on the analogy between the phenomena of light and those of sound. In a wave of sound each particle moves in the direction in which the waves are propagated, and it was natural to make the same supposition for the waves of light. Yet the mass of unexplained facts forced Young to consider the alternative case of waves in which the motion is in a plane at right angles to the direction of propagation. The waves of water in which such a motion partly takes place may have given to Young the first idea of a supposition which, as he showed, could account for many apparently singular phenomena. But his want of taste for calculations, as well as for experimental verification, prevented him from reaping the full fruits of his fertile ideas. Fresnel tells us that when he first conceived independently the idea of transverse vibrations, he considered the supposition so contrary to received ideas on the nature of vibrations of elastic fluids, that he hesitated to adopt it, and he adds: "Mr. Young, more bold in his conjectures, and less confiding in the views of geometers, published it before me, though he perhaps thought it after me." But when once the question was raised, Fresnel applied to it the patient skill which, either by strict mathematical deductions, or by happy guesses and assumptions, surmounted all difficulties. The phenomena of double refraction, and their connection with polarization, were now explained, and all the varied phenomena of light seemed naturally to follow from the simple supposition of waves of transverse vibrations. Such a successful application of mathematical calculations to the investigation of physical phenomena had not been heard of since the time of Newton, and could not fail in the end to produce its due effect. The supporters of Young and Fresnel became more numerous and confident, and the scientific societies duly acknowledged the services rendered by both. Young was elected one of the eight foreign members of the French Academy, and Fresnel received the Rumford Medal of the Royal Society, which, however, only reached him on his deathbed.

The undulatory theory now entered on a stage in which it could be taken up by the mathematician pure and simple. Its foundations had to be rendered more secure, and its consequences had to be worked out to a greater extent than even Fresnel had done.

The scruples which hindered most of the French mathematicians from accepting Fresnel's views were shared by Poisson, who deduced from his equations a result apparently paradoxical. According to Fresnel's formulæ, the center of the shadow of a small circular disk formed by a luminous point should be as bright as if the disk were absent. But, however curious this result might be, it had been observed just 100 years before Fresnel's time, and as that experiment had been completely forgotten, Poisson's theoretical conclusion had again to be subjected to the test of experiment, when it was found to be completely in accordance with fact.

But the most remarkable discovery made solely by calculation was the so called conical refraction, theoretically deduced from Fresnel's wave surface by Sir Wm. Hamilton. That great mathematician had found that a point, when looked at through a crystalline plate cut in a certain direction, should appear not as a point, but as a ring, and the fact was verified experimentally by Prof. Lloyd. This discovery has always been considered one of the greatest triumphs of mathematical physics, and justly ranks on equal terms with the discovery of the planet Neptune by Adams and Leverrier. It is necessary to remark, however, that strange and unexpected conclusions, especially when they have been arrived at after complicated mathematical transformations, tempt us sometimes to exaggerate the additional support which their verification gives to the theory by means of which those conclusions have been reached. It is extremely unlikely that any theory should account for all the facts explained by Fresnel, and not also for all those discovered by his successors. As a matter of fact, Fresnel's wave surface is not the only one which has been suggested, but as they all contain the singular points at which the conical refraction is produced, this phenomenon is no proof that Fresnel's equations are strictly correct. It often happens in mathematical explanations of physical phenomena that the equations originally deduced contain a series of con-

* A lecture introductory to the Session, 1881-82, of Owens College, Manchester, by Arthur Schuster, Ph.D., F.R.S., Professor of Applied Mathematics.

* "For my part it is my pride and pleasure, so far as I am able, to supersede the necessity of experiments."—Peacock's "Life of Young," p. 477. Abstract of letter by Young.

stants which are then determined to fit the experiments. This process, which is perfectly legitimate, does, however, often prove only that the theory is successful in giving us a useful formula of interpolation, and need not be conclusive in favor of the ideas which have led to the formula. In a considerable number of cases, such as the reflection of light from metals, and even the theory of double refraction, we have different formulæ which all give, as far as we can test them, a sufficiently correct account of the facts, and none of them therefore prove anything in favor of the views which the different authors of the equations have put forward.

Before leaving our consideration of the services rendered by mathematics to the undulatory theory, we must not forget to notice the mathematical investigations by means of which its foundations have been placed on a safe dynamical basis. The investigations of Cauchy, those of Green, which followed, but especially those of Stokes, have secured for this theory such a firm support that even Laplace might have accepted it without further scruples. As a matter of history these investigations have done little toward the final victory of the theory. They came too late to affect the course of events, but they have increased the confidence of mathematicians in physical theories, and have prepared the way for further investigations.

As I have already remarked, it is one of the great objects of mathematical physics to investigate how far we can safely push certain assumptions and where a new hypothesis must be brought into play. And, indeed, when we have carried our calculations as far as we can, when we have experimented and measured as much as we can, we find that the undulatory theory as it stands at present, though following up to a certain point with marvelous accuracy the true course of nature, shares the common fate of all theories, and leaves a vast quantity of facts unexplained and waiting for more complete investigations. Nor is this to be wondered at; our assumptions as regards material media may in many cases give correct results, and no doubt answer very well as a first approximation, but we arrive at a point where such a material medium can no longer be considered homogeneous, and here our conclusions must break down; but it is to mathematics that we must look for the next great step. The progress of the science of optics during this century has shown us how much mathematical calculation can help to establish a great and important fact such as the existence of that all-pervading medium, the vibrations of which constitute light, and I may review more quickly the recent progress of other branches of science.

In the science of heat we do not require mathematical calculations to show the superiority of the mechanical over the corpuscular theory. Sir Humphry Davy's experiment shows conclusively that heat cannot be a substance, and Joule's experiments served further to illustrate the great advantages of the mechanical theory. The mathematical treatment of thermic problems was not required to establish a theory, but was suggested by practical considerations. The important question, how much work we can get out of a steam-engine, first attracted mathematicians, and out of this question the present science of thermodynamics may be said to have arisen.* Carnot, who gave the initial impulse to these mathematical investigations, assumed in his papers that heat was indestructible, though he seemed personally inclined to prefer the mechanical theory, which denied that indestructibility. Carnot's investigations were only gradually appreciated, and it was only when Clausius and Thomson corrected his theory so as to bring it into accordance with modern ideas, that general attention was directed to the subject. It was found that so many important consequences of physical interest (as the lowering of the freezing point of water by pressure) followed out of Carnot's corrected reasoning that the mechanical theory now rapidly made its way, and though, as already mentioned, the proof of its truth rests on a perfectly simple experiment, mathematics must be considered to have had an important share in the final establishment of that theory.

It seems impossible to speak of the services rendered by mathematics to the progress of our knowledge of heat without mentioning the great law of the dissipation of energy. No two sciences seem further apart than mathematics and metaphysics, yet mathematical propositions have often furnished material for metaphysical speculations on the workings of nature. Thus the many dynamical propositions involving minimum or maximum properties, such as the principle of least action, have been taken to show that nature always works with the least expenditure of force, and thus the important law of dissipation of energy, which asserts that the world must have a slow and gradual end, could not fail to be used in the discussion of its sudden and abrupt beginning. These metaphysical speculations react again on the progress of physics, but it seems doubtful how far this indirect influence of mathematics has been beneficial; at any rate mathematicians cannot be held responsible for such an extension of their power.

An offshoot of the mechanical theory of heat is the molecular theory of gases. The idea on which that theory is based is not new, but it remained a speculation merely until, chiefly through the labors of Joule, the mechanical theory of heat was experimentally established, and its laws investigated. There is, perhaps, no branch of science in which mathematics has had such unexpected results in forming and confirming our faith in purely physical conceptions. That matter is made up of atoms and molecules is a hypothesis which simplifies many physical and chemical problems. It may, on chemical grounds especially, be considered a highly probable hypothesis, but we could hardly have obtained the confirmation amounting to proof which the idea has received of late years, without the mathematical treatment which it has received at the hands of Clerk Maxwell and those who have followed in his footsteps. One of the most astonishing results obtained by Maxwell is the one subsequently verified by experiment, that so long as Boyle's law is true, the coefficient of viscosity, as well as that of the thermal conductivity in a gas, is independent of the pressure. This fact alone, which could never have been obtained without the aid of mathematics, is a sufficiently strong foundation on which we may rest our belief in molecules. It would be extremely interesting to follow out the more recent developments of the mechanical theory of gases, and to show how both mathematics and the absence of mathematics have advanced its progress, but if it is a good rule to say nothing but good of the dead, it is a better one to say nothing at all of the living.

I have already alluded to the mathematical treatment of electricity and magnetism. The aid of mathematics here was not required to confirm a theory, but rather to prepare the way for one. The complicated laws, regulating the attractions of electric and magnetic bodies, and of bodies

carrying electric currents, have by the aid of mathematics been reduced to their simplest form, and electrical units have been connected with the ordinary mechanical units. This interesting branch of physics will furnish us with an example of the services which mathematics has rendered in directing the efforts of experimenters into the proper groove. We need only compare the magnetic measurements which were made during the last century with those made in our own time. While the early investigations gave us only a series of numbers impossible to interpret without a large quantity of accessory data, which are generally omitted, modern measurements, even when made by non-mathematicians, have generally been suggested by mathematical calculations and very often serve a useful purpose.

I have hardly alluded, as yet, to the science of dynamics, which is the foundation of all applications of mathematics. Its progress has been steady since the time of Galileo, but all the marvelous results arrived at by Newton and his followers—results which first showed the great fertility of applied mathematics—are too familiar to need any enumeration from me. The modern researches in hydrodynamics may perhaps not as yet have led to any definite result of physical interest, but they are rapidly progressing toward that end, and we may look forward to an increasing number of physical discoveries made by the aid of mathematics.

In tracing the history of some of our modern theories, I have followed the usual plan of presenting the history of science as illustrated by the discoveries of our great scientific men. It is necessary, however, to draw attention to the fact, and I have tried to keep this point in view throughout this discourse, that it is not always the most conclusive arguments which carry the day, and that second-hand thinkers have often had a more potent influence in shaping the course of scientific history than those to whom we now justly ascribe the greater merit of discovery. In our historical studies, therefore, we ought to direct our attention not less to that which has influenced public opinion, than to the actual soundness and originality of each discoverer.

If we ransack old books of science we often come across passages of long-forgotten writings, in which, when they are properly construed, when new meanings are given to old words and obscure expressions are freely translated, we may trace a faint prophetic glimmering of a modern theory. Such passages have a peculiar charm for the student of scientific history; they are often the only reward for much patient and otherwise useless reading, and are interesting as showing the almost boundless ingenuity both of him who made the statement and of him who interpreted its meaning. But those who are fond of this process of exhumation ought not to forget that two parties are necessary to every advance in science—the one that makes it and the one that believes in it, and the course of history is as much affected by the second class as by the first.

"A jest's prosperity lies in the ear
Of him that hears it, never in the tongue
Of him that makes it."

A scientific man, in so far as he influences the progress of science, cannot be far ahead of his time, and though his writings may be read and admired centuries after his death, he will have written in vain if he has not been appreciated by his contemporaries or by those who immediately followed them. For our present purpose, then, we must consider not so much those mathematical arguments which appear now to us the most conclusive ones, but such as did appear conclusive to those whose opinion they were meant to affect. But if we try to discover what arguments have had the greatest power in removing old prejudices and in causing a solid advance in science, we find that they have often been of the most flimsy nature. Analogies, sometimes not even good ones, have succeeded where solid reasoning has failed, prejudices have been overcome only by other prejudices, and a rough illustration of a point of secondary importance may have made a previously obscure theory look more familiar, though not more clear, to the popular mind. What, for instance, has the existence of Jupiter's four satellites to do with the question whether the earth turns round the sun or the sun round the earth? Yet the discovery of these satellites has produced a greater revolution in favor of the Copernican theory than anything else that Galileo wrote on the subject.

If we look at the history of science from the point of view suggested by these considerations, we find that in addition to the legitimate influence of mathematics which we have traced, its practical effects, through less reasonable causes, have often been as powerful. The statement that in science authority is of no avail against argument, is one the proof of which must be looked for in the future, rather than in the past. There can be little doubt that authority has had a great effect in all scientific revolutions, and the authority of mathematicians was always greater than that of other men of science. Men are thoroughly convinced in one of two ways only; either by a train of reasoning which they can fully appreciate, or by one which is entirely above their comprehension. To those who are particularly amenable to the second kind of proof, mathematics has always been a magic power. Many results first obtained by the help of advanced mathematics have since been deduced by more elementary reasoning, but it seems questionable whether the original author would have been as successful in overcoming the inertia of his contemporaries, if he had confined himself to language intelligible to the greater number of his readers. It is, no doubt, due to this cause that mathematical papers have brought with them more widespread convincing power than we should now feel inclined to accord to them. The papers of Young, in which he avoided mathematical symbols, may appear to us sufficient to establish the undulatory theory of light; the arguments of Sir Humphry Davy, the experiments of Joule, may seem absolutely conclusive in favor of the mechanical theory of heat; but although the mathematical investigations of Fresnel, Clausius, and Thomson could be appreciated only by a much smaller number of readers, they had a more powerful influence in turning the scale of public opinion in favor of the modern ideas. It seems sometimes almost as if it required an experimentalist to convince a mathematician, and a mathematician to convince the general world. It is impossible to enter into greater detail or to exemplify more amply the assertions which I have made without touching on delicate and controversial matters, but on the present occasion it seemed to me to be specially fitting to point out that the course of science is as much affected by the appreciative faculty of receptive minds as by the creative faculty of the discoverer.

It is given to few only to take an active and successful part in the production of scientific work. The young man who begins life with the idea of making a name as a scientific discoverer is like the little girl in *Punch* who intended to become a professional beauty. They may both be successful, but if so, it will depend as much on the ready appreciation of their contemporaries as on themselves. The advance

of science takes place through many channels, and each generation has its own part to play. Particular ideas, particular faculties are wanted at particular times, and no one can foretell where success will be. Men who are now quoted as shining lights would have passed away unnoticed had they lived at other times, and many a life has been one of patient but unsuccessful work, because its energies were devoted to a subject which was barren, or at least lay fallow for a time. No one, for instance, who has attempted to read through J. B. Morinus's work (and I doubt whether any one has ever got beyond the attempt) can fail to notice in him qualities which might have made a successful discoverer. In his method of determining longitudes by lunar distances Morinus has left us a lasting legacy. During the greater part of his life, however, his energies were devoted to the study and application of astrology, and all the labor spent on that subject was thrown away, although he did his best to make his own prophecies come true, and, having predicted the end of the world for a certain year, went through with his share of the proceedings, and died a natural death at the appointed time. *A priori*, there was no reason why astrology when married to mathematics should not have produced a healthy progeny, and looking especially to the state of science at the time, we can have little fault to find with the old astrologers.

History then does not teach us any royal road to success. But more important for the ultimate progress of truth than a solitary success is the training of the faculty which enables the scientific man to judge correctly, and to appreciate the results of those who strike out new roads and extend the boundaries of knowledge. It seems to me to be one of the chief objects of an institution like this to bring up men, who, by conscientious consideration of scientific speculations, may help to give that solidity and elasticity to public opinion which is necessary for the rapid advance of science.

PHOTOGRAPHIC NOTES.

In the *Mittheilungen* there is an article, by Herr A. Stosch, upon the sensitizing of gelatine plates with caustic potash. He says:

When gelatine emulsion plates are rendered extremely sensitive by being dipped into an ammoniacal and alcoholic silver solution, such as I described in the *Photo. Mittheilungen* (No. 207, p. 70), the silver salt does not work in every case as a sensitizer only through its power of seizing bromine, but it also effects the extremely powerful alkaline reaction of the oxide of silver and ammonia. The omission of the ammonia from the formula, as has been recommended, I consider no improvement, any more than the addition of citric acid, with which I have not succeeded in making permanent plates with simultaneous shortening of the exposure. Jastrzebsky's observation that placing the emulsion in a bath of moncarbonate of soda has a favorable action upon the sensitiveness of the emulsion plates, led me to substitute caustic potash in alcoholic solution for oxide of silver and ammonia, and the result was very good. I have dissolved one and a-half, two and a-half, and even three grammes of potassium causticum siccum in 100 c. c. alcohol (of about 87° to 90°), placed gelatine plates in it for three or four minutes, and then dried them in a rather warm room (which, when the room is not too cold, or the atmosphere too much loaded with moisture, should not take much longer than four or five minutes, but the drying must be thorough, else the alcohol will have a harmful effect), after which the plates were exposed. The results exceeded my expectations. Besides an extraordinary shortening (to one-fourth) of the exposure the plates were very clear. The caustic alcoholic alkali saponified cold every trace of grease which was upon the gelatine film, and I consider grease the cause of most of the spots in gelatine plates. During the development the quantity of the alkali upon the gelatine plates is disagreeable. In the first place it leads to fog, and in the second it has a considerable tendency to crepiness. I therefore place the plates once more in a bath of alcohol before development and after exposure, and overcome the crepiness by a bath of common salt, in which the plates are placed after the development, and when taken out of it they are at once, without being rinsed, fixed as recommended by Dr. Vogel for such cases. I developed with a liberal quantity of ferrous oxalate and covered the plate well with it, so that the aqueous fluid was not repelled. This process has the advantage over the alcoholic ammonia and silver bath of the unpleasant reaction of the silver upon the gelatine being avoided; and it seems to me that if the alkali be very carefully washed off before development (I have not tried weak acid) the quantity of caustic potash may be increased with advantage. In order to accelerate the drying of the plates after the alcoholic bath it is also permissible to add a little ether, freshly distilled over calcium, to the alcohol. I do not recommend the addition of ether which has already stood some time, particularly if it has stood in bottles not perfectly full and exposed to the light. When trying the foregoing procedure attention must be given to seeing that the plate is thoroughly dry, otherwise it may happen that at those places where alcohol still remains there will either be no image at all or that it may be eaten away.

Herr Prumm gives the following process for reducing a gelatine negative: Lay the plate alternately three or four seconds—first in a fifteen per cent. solution of iron chloride and then in hyposulphite of soda. By alternating these two baths the picture may at last be made to disappear, only a strong relief remaining. It has been recommended to mix the two solutions, but Dr. Vogel points out that the mixture is hardly advisable, as sulphur is thereby precipitated.

Herr Ladewig gives the following as Herr Angerer's formulæ for developer for gelatine plates:

I.
Potassic oxalate..... 200 grammes.
Distilled water..... 1,000 "

II.
Ferrous sulphate..... 100 grammes.
Sulphate of iron and ammonia.. 100 "
Distilled water..... 600 "

III.
{ Gelatine ... 3 grammes.
{ Distilled water 400 "
{ Glacial acetic acid..... 30 "
{ Bromide of potassium..... 5 "
{ Distilled water..... 75 "
{ Alcohol, 40 per cent..... 75 "
(Mix equal parts of these two solutions.)

IV.
Hypsulphite of soda..... 1 : 200

To use, mix four parts of solution I. and one part of solution II.; and to every 100 grammes of the mixture add six to eight drops of solution III. and three to four drops of solution IV.

* Foucault's investigations, though of enormous mathematical importance, cannot be said to have had a direct influence on the progress of physics.