(*Students' Paper, No. 300.*)

“Fly-Wheels and Governors.”¹

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ON a previous occasion the Author read a Paper² before the Students, giving an account of an instrument for the accurate measurement of the speed of rotating shafts, together with some examples of the results obtained by its aid. The present communication has been founded on certain experiments on gas- and steam-engines carried out during 1891 with this and other instruments.

The primary object of these trials was to obtain a knowledge of the cyclical velocity variations, (1) under normal working conditions, (2) with variable fly-wheel power, and (3) at different (mean) speeds. As it was impossible to ascertain the cyclical velocity variations without also obtaining a record of the influence of the governor on the general speed, a few remarks are added, as a secondary consideration, on this latter subject.

LIST AND ORDER OF TRIALS.

The trials will be considered in this Paper in the order in which they were carried out. The account of the “Cycle” gas-engine will be given far more in detail than those of the other cases, because it is novel in construction, and the difficulties attending its analytical examination are far greater than those met with in ordinary steam-engine mechanism, although the principles involved are in both cases identical.

¹ This communication was read and discussed at a meeting of the Students on the 12th of February, 1892, and was awarded the Miller Scholarship, Session 1891-92.

² Minutes of Proceedings Inst. C.E., vol. xeviii. p. 357.

The following is a list of the trials undertaken :—

Engine.	Cylinder Diameter.	Working Stroke.	Description of Fly-Wheels.	Number of Trials.
	Inches.	Inches.		
4 HP. "Cycle" . .	$8\frac{1}{4}$	$8\frac{3}{8}$	Two fly-wheels . .	19
" " " " . .	$8\frac{1}{4}$	$8\frac{3}{8}$	One fly-wheel . . .	7
2 HP. " " " " . .	$6\frac{1}{4}$	$8\frac{3}{16}$	Two small fly-wheels	8
" " " " " " . .	$6\frac{1}{4}$	$8\frac{3}{16}$	Two larger fly-wheels	7
E size Otto . . .	$8\frac{1}{2}$	10	Two fly-wheels . .	6
" " " " " " . .	$8\frac{1}{2}$	10	One fly-wheel . . .	4
4 HP. " " " " . .	7	15	" " " " " " . .	12
Steam-engine . . .	9	15	" " " " " " . .	35

THE "CYCLE" GAS-ENGINE.

As perhaps many are unacquainted with this new and ingenious mechanism for achieving the ends aimed at in gas-engine construction, it may be as well to describe the model of the link-work now on the table.

It is claimed by the inventor (Mr. Atkinson) that by this arrangement he is not only able to obtain one impulse in every revolution with only one cylinder, but that the working-charge is expanded to twice its original volume during the working-stroke, thereby effecting great economy. It should be noticed also that all four strokes of the piston are of unequal lengths, a point which, as will be seen afterwards, has considerable influence on the cyclical velocity variations.

Before discussing the results of the trials on this engine, it will be desirable to describe the drawings and calculations necessary for the construction of the theoretical cyclical velocity variation curve; as in this way only, can many of the curious results obtained by experiment be in any way explained.

To make these calculations, the following data were found necessary, and will, for the time being, be supposed obtainable. The methods of ascertaining some of these data will be described later on. During any given cycle,—(1) The mean velocity of the crank-pin must be accurately known during that particular cycle; (2) The indicator-diagram for the same cycle (and not a mean of the diagrams taken immediately before and after) must also be known; (3) The resistance to the motion of the engine must be constant during that cycle; (4) The weights and dimensions of all the moving parts of the engine must be known.

From (2) the impulse given to the piston is known from a

combination of (1) and (4) it may be found how far this impulse is modified by inertia of the reciprocating parts, before it reaches the crank-pin; and from (3) arises the certainty that the mean-height line of the modified twisting moment diagram may be taken as the base-line of the same curve, read as the crank-pin acceleration diagram. The crank-pin acceleration diagram having been obtained, there is no difficulty in constructing its velocity curve, the weights and dimensions of the fly-wheels being known. The curve so obtained may be then compared with the velocity-curve for that identical cycle, obtained experimentally with the aid of the cyclometer. Although this calculation, the outlines of which have just been given, is not one of great difficulty in the case of steam-engine mechanism, it becomes by no means simple or easy in the case of this gas-engine, where no assumptions can be made with safety regarding the proportion of the reciprocating masses, which may be assumed to move with the piston or crank-pin.

In these drawings, no dynamical assumptions whatever were attempted, and the only factor neglected was friction. Each link in the engine was carefully measured and weighed, while the centre of gravity was obtained both experimentally and analytically. In addition to this, the moment of inertia of each link was determined both by calculation and experiment. From the moment of inertia of the link, its radius of gyration about its centre of gravity could be calculated, and with this knowledge it became unnecessary to make any assumptions as to momentum. It may be here pointed out that it is fortunate that no assumptions were made, and examination of the drawings of twisting-moment and modified indicator diagrams show that the speed of the engine is affected nearly as much by the weight of the links as by the explosion in the cylinder. A description of the construction and proofs are given in Appendix A.

DETAILS OF TRIALS ON A 4-HP. ENGINE.

The speed of the crank-pin was ascertained experimentally by means of the cyclometer, by which its velocity of rotation could be recorded at every position for fifty consecutive revolutions. The resistance to the engine was applied by means of the ordinary form of rope-brake, and it was so arranged that it might be suddenly removed at any given instant. With regard to the indicator-diagrams, it was felt that, in a gas-engine, the explosions

varied to such a considerable extent that the ordinary form of indicator would not be applicable, when it was required to know accurately the precise impulse given during each cycle, for a period of fifty consecutive revolutions. Hence the Richardson continuous indicator, with some alterations, was adopted.

The peculiarity of this instrument is that the diagrams are not superposed, but are described consecutively and distinct from one another on a long band of paper, which is regularly and automatically fed round the drum. After the engine had been running with the brake for some little time, the cyclometer and the indicator were simultaneously thrown into gear. Frequently, after about twenty-five revolutions from the start, the brake was suddenly removed, and at the same instant a mark was made, by means of a small scribe, across that particular line of waves which the tuning-fork was then writing on the cyclometer drum. After the trial it was easy to identify any particular revolution of the cyclometer with the corresponding diagram made by the indicator, owing to the peculiar combination of explosions and cut-outs which the gas-engine governor almost invariably produces. As was mentioned before, the tracings on the cyclometer were marked at the instant that the brake was removed, and therefore, if necessary, the corresponding point could be found on the indicator-diagrams. Hence, at the close of the trial, a record had been obtained for a period of fifty consecutive revolutions, of each particular impulse with its corresponding change in velocity, together with the mean resistance.

In some later experiments, the small changes in resistance which must occur with every brake, were also simultaneously and automatically recorded, but for the present these trifling changes may be neglected. The same methods were pursued whilst trials were separately made under the following conditions:—(1), With greater or less fly-wheel power; (2), while the engine was running at different speeds; (3), with and without the brake; and (4), when the engine was starting or stopping.

DRAWINGS.

It may be mentioned here that those drawings relating to the analytical examination of the engine mechanism have been, for want of space, relegated to the Appendix, together with a description of the methods employed. Even then, it is impossible to present more than the mere skeleton of the work involved in the

results which may be found there. The drawings chiefly considered in the text, are those derived from experiment. In Figs. 10, Plate 6, are given a number of velocity-diagrams, and the indicator-diagrams corresponding to Trial 3 (Fig. 10*b*, Plate 6) are shown as samples in Fig. 4, Plate 5.

Figs. 5, 6, 7, 8, Plate 5, represent four typical cyclical velocity-diagrams for the "Cycle" gas-engine under normal load conditions, but with varied fly-wheel power. The four curves in Fig. 9 show graphically the effect of speed on cyclical velocity variations. The results obtained by the continuous indicator are extremely interesting, and especially so in combination with the cyclical velocity variation diagrams. The indicator-diagrams so obtained appear to show up a number of points in the working of the engine, which would probably be missed if taken in the ordinary way. There is hardly space enough in this Paper to treat them as fully as they deserve, and therefore only a few comments can be made on them now; some further tables relating to them will be found in the Appendixes.

Perhaps the most striking feature is the marked difference between the intensity and character of the two explosions immediately after a cut-out, when the engine is light. Take in Trial 3, for instance, the two last explosions illustrated. The first of these two explosions gives an initial pressure of 195 lbs. per square-inch, and a mean pressure of 44.4 lbs. per square-inch; whereas in the second the initial pressure is only 150 lbs. per square-inch, and the mean pressure 42.3 lbs. per square-inch. When the engine is loaded, the explosion, after a miss, gives 195 lbs. per square-inch initial pressure, and 46.9 lbs. per square-inch mean pressure; and the succeeding explosion gives 176 lbs. per square-inch initial pressure, and 46.9 lbs. per square-inch mean pressure. The figures given are only typical of what generally occurs in this engine, which, it should be mentioned, has no timing-valve. The probable explanation of the low pressures succeeding an explosion, is that the cylinder has not been so thoroughly cleansed as would have been the case had a miss occurred. The clean state of the cylinder after a miss, would also account for the abnormally high explosions immediately succeeding it. The lower mean pressures occurring when the engine is light, probably indicate a cool cylinder after five or six consecutive misses, and also a weaker charge due to the alteration in speed of the engine.

A good example of the reduction in mean pressure, apparently due partly to the increase in speed, is given in Appendix C. The mean pressures of explosions occurring at about 140 revolu-

tions per minute, are more than 61 lbs. per square-inch; but when the speed reaches 156 revolutions, the explosions yield a mean pressure of less than 53 lbs. per square-inch. It may be that the air in the suction-pipe has an oscillatory motion along the length of the pipe, when the engine is working at a regular speed; and the natural period of oscillation, due to the length of pipe employed, is made to agree with the time of opening of the suction-valve. If this be the explanation, it is clear that, if the speed of the engine varied, less air would be drawn in, than if the time of opening of the suction-valve agreed with the independent and natural oscillatory motion of the air in the suction-pipe. In support of this theory may be mentioned the well-known fact, that the addition of a few feet to the length of the suction-pipe will materially alter the horse-power of an engine. Another point to notice in connection with these indicator-diagrams, is the variable time at which ignition occurs in those explosions immediately succeeding a miss. The charge, being drawn into a clean cylinder, is more explosive than when the cylinder is foul, and, consequently, often ignites slightly before the end of the compression stroke. From this it would appear that it would be practically impossible to use tube ignition without a timing-valve, unless the clearance at the end of the exhaust stroke be extremely small, as is the case in this engine.

Leaving the indicator-diagrams, it may be well to consider next the typical cyclical velocity variation curves of this engine (Figs. 5, 6, 7, 8). These are diagrams selected from many, because they appeared to approach most nearly, under the given conditions, to the mean of all those considered under the same conditions. The effect of increasing the fly-wheel power is obvious, but, as figures will shortly be referred to on this subject, it will not now be considered. It may be well to remark here that the 4-HP. engine is always sold with two fly-wheels, and that one fly-wheel was removed solely for the purposes of these experiments. It may be also noticed that all these curves bear a striking resemblance to the twisting-moment diagram, shown in Fig. 2, and they all possess two maxima, the second one occurring about the end of the suction-stroke.

The four curves, giving the cyclical velocity variation in revolutions per minute for each of the four engines when running at different speeds (Fig. 9), are all deduced from the cyclometer records taken whilst the engines were starting from rest. The mean speed of the engine is plotted horizontally, and the cyclical velocity variation at that speed is plotted vertically. Of course these trials could

only be made when there was no load on the engine. It should be mentioned that the explosions which take place in a gas-engine at starting are often very irregular in every respect, and these irregularities produce corresponding differences in the cyclical velocity variation, quite independently of any change in speed. To minimise this difficulty in the case of the 4-HP. engine, continuous indicator-diagrams were taken simultaneously. The mean of the diagrams thus taken throughout start was then calculated, and the cyclical velocity variation in any one revolution was multiplied by the ratio of the mean of all the mean pressures to the particular mean pressure produced during that revolution.

From a comparison of the results which can be obtained from these diagrams, with the figures given in the table of results (Appendix B) referring to the loaded engine, it would appear that the loaded fluctuation of the 4-HP. engine with two fly-wheels is $13\frac{1}{2}$ per cent. less than when light; and when the fly-wheel power is halved, it becomes 14 per cent. less—the speed in each case being from 140 to 150 revolutions per minute. In the case of the 2-HP. engine, the cyclical velocity variation with the small fly-wheels is $16\frac{3}{4}$ per cent., and with large wheels 14 per cent., less. Plate 6 exhibits examples of velocity curves extending over a number of consecutive revolutions. Particulars are given on this sheet concerning fly-wheels and loading. It may be noticed that in several instances the load is suddenly removed, and on those occasions the governor is tried to its utmost limits, in all cases cutting out an explosion within 2 or 3 revolutions from the time of the removal of the load. Thus, expressed in time, the governor acts in between 0.86 second and 1.2 second from the moment of the removal of the load. As this is not to be regarded as an important practical test, the figures relating to governing do not include these abnormal cases.

REMARKS ON THE TABLE OF RESULTS (APPENDIX B).

As might be expected, the cyclical velocity variation is exceptionally low in this type of gas-engine, being in fact less than in many steam-engines. This is all the more remarkable when the low speed and extremely unequally distributed pressures (due to the expansion of the charge to twice its original volume) are considered. In the case of the 2-HP. engine Trial 1₂, it was only 3 per cent., or a variation of 4.7 revolutions, in each cycle;

and in none of the trials made with this engine was the load more than half of its maximum. The mean fluctuations over a number of trials of this type of engine are given in the Table, together with particulars as to speed and fly-wheel power. For the purpose of comparing the reduction in cyclical velocity variation with increase in fly-wheel power, it was found necessary to calculate the reduced inertia of each wheel. In comparing two trials of the same engine, it must be remembered that the difference in speed at which any two trials were made must have some effect on the cyclical velocity variation. In comparing engines of a similar type, but of different power, some correction should also be made for the proportionate differences in fly-wheel power.

The following Table has been arranged for the purpose of facilitating these comparisons, by allowing for the differences in speed at which the engines ran, and also by giving the calculated values of the cyclical velocity variations; on the assumption that each engine possessed fly-wheel power to the extent of 7·756 units of mass per square inch of piston-area situated at the crank pin; A denoting piston-area, and R reduced inertia of fly-wheels:—

Engine.	Actual Quantities.				Speed=150 Revolutions.	Speed=150 $\frac{R}{A}=7\cdot756$
	R	$\frac{R}{A}$	Revolutions per Minute.		Revolutions per Minute.	
			Speed.	Cyclical Velocity Variation.	Cyclical Velocity Variation.	Cyclical Velocity Variation.
2 HP. ₁	149·9	4·887	151·2	8·9975	9·1215	5·748
2 HP. ₂	237·9	7·756	153·25	5·0000	5·2200	5·220
4 HP.	184·0	3·442	143·8	7·8455	7·2110	3·201
4 HP.	92·0	1·721	141·2	16·4200	14·5520	3·2286

From the actual cyclical velocity variation column it may be seen that the cyclical velocity variation for the 4-HP. engine with one fly-wheel ($R = 92$) is 16·4 revolutions per minute. With this figure as a basis the cyclical velocity variation can be calculated for the conditions of speed = 150 revolutions per minute, and $\frac{R}{A} = 7\cdot756$; the result of this calculation is given in the

last column as 3·22 revolutions per minute. It will be noticed

that the fluctuation under standard conditions calculated for the 4-HP. engine from entirely different data, is almost exactly the same; and the calculated cyclical velocity variations for the 2-HP. engine agree closely. The calculations are based on the assumption that the cyclical velocity variation varies inversely as the square of the speed, and also as the value of $\frac{R}{A}$. These assumptions are only partially true, as is shown by the following Table:—

Engine.	a. Small Wheels. $\frac{R}{A}$	b. Large Wheels. $\frac{R}{A}$	$\frac{a}{b}$	c. Cyclical Velocity Variation at 150 Revolu- tions, for a.	d. Cyclical Velocity Variation at 150 Revolu- tions, for b.	$\frac{d}{c}$
2 HP.	4·887	7·756	0·6301	9·1215	5·22	0·5722
4 HP.	3·4425	1·721	2·00	7·211	14·552	2·018

Thus, in the case of the 2-HP. engine, the value of $\frac{a}{b}$ is 10 per cent. greater than the corresponding value of $\frac{d}{c}$; and, in the case of the 4-HP. engine, it is 0·9 per cent. less. It should be remembered that the heavy links also modify the cyclical velocity variations, their influence being more felt in the case of the 2-HP. than in the 4-HP. engine, and would tend to reduce the value of $\frac{a}{b}$.

GOVERNING.

It will be noticed that the last two columns in the Table of results (Appendix B) refer to governing. The first of these two columns, headed "Range," gives the difference between the maxima and minima speeds reached during any particular trial. In estimating the range, the cyclical velocity variation is not included, as the action of the governor must be entirely independent of cyclical velocity variation. The mean speeds during each consecutive revolution is known, and the difference between the greatest and least of these means is due to the governor.

It is interesting to note how the efficiency of the governor varies with the fly-wheel power. On comparing the reduced inertia and

the range columns, it appears that, in each case, increase in fly-wheel power corresponds to a higher governor efficiency.

The following Table, perhaps, will make this clearer :—

—	Engine.	$\frac{R}{A}$	Range per Cent.
B	2 HP. ₁	4·887	3·6
C	2 HP. ₂	7·756	3·12
D	4 HP.	3·442	4·735
E	4 HP.	1·721	9·35

The ratio of $\frac{R}{A}$ for lines B and C = 0·630

whereas the inverse ratio of ranges per cent. = 0·867

The ratio of $\frac{R}{A}$ for lines D and E = 2·000

and the inverse ratio of ranges per cent. = 1·975

The difference between the two values of $\frac{R}{A}$ for the 2-HP. engine is large, whereas the corresponding change in governing efficiency is comparatively small; whereas in the case of the 4-HP. engine the change in fly-wheel power is almost exactly proportional to the increase in governing efficiency. In the first case, the inertia of the smaller wheels was probably sufficient to prevent changes in speed from occurring too rapidly for the governor to rise; and consequently considerable increase in fly-wheel power might include no corresponding increase in governing efficiency. In the case of the 4-HP. engine, it is fairly clear that, with one fly-wheel, the changes in the fly-wheel revolutions did occur too rapidly for the governor; but it by no means follows that, were twelve larger fly-wheels substituted, the governing efficiency would increase in the same ratio.

Before concluding this account of his experiment on the “Cycle” gas-engine, the Author wishes to acknowledge his great indebtedness to Messrs. Manlove, Alliott & Co., Limited, for the facilities which they gave him to carry out the trials at their works. His thanks are also due to Professor Heaton for permitting him to test the 2-HP. engine in connection with the Physical Laboratory at University College, Nottingham. Some help in working out the “cyclometer” diagrams was also most kindly given by Mr. Jackson.

TRIALS ON THE "OTTO" GAS-ENGINE.

Two engines of this type were tested, one of them being specially constructed for electric-lighting purposes, the other being an ordinary commercial 4-HP. engine. The 4-HP. engine was tested in a precisely similar way to that already described in connection with the 4-HP. "Cycle" gas-engine. The Richardson indicator, rope-brake, and cyclometer were applied to this engine simultaneously as before, and the moving parts, including the fly-wheel, were carefully measured and weighed, &c. From these data drawings, corresponding to those already described for the 4-HP. "Cycle" engine, were prepared.

As the constructions necessary for the complete solution of dynamical problems relating to the ordinary slider-crank mechanism are well known, and a more elaborate example involving similar principles has been already described in this Paper, it is not essential to enter into the details of the present case. It is sufficient to say that no assumptions were made with regard to the inertia of the connecting-rod or of any other moving part, and that friction only was again neglected.

The 4-HP. engine tested had slide ignition, and was unfortunately not quite a new sample. Some of the cyclical velocity variation curves show certain irregularities, which can only be accounted for by the presence of a very loose main-bearing, which was discovered after the experiments had been concluded. The other type of Otto engine tested was what is commercially known as the E-size electric-lighting engine. No indicator diagrams were taken from this engine, and the load was applied by means of a dynamo supplying current to 52 16-candle-power incandescent lamps, and one 300-candle-power lamp.

The nominal maximum working load is equivalent to 72 16-candle-power lamps. It is fitted with tube ignition and timing-valve. The two fly-wheels are very heavy, and the speed is 250 revolutions per minute. The governor fitted to this type of engine is arranged to admit a weaker or stronger charge, according as the speed of the engine increases or diminishes. The strongest charge, only admitted when the engine is fully loaded, produces a mean pressure of about 71 lbs., and the weakest only 42.6 lbs. per square-inch. There being five stages in all, this constantly varying intensity of explosion produces inequalities in the amount of cyclical velocity variations. Therefore no less

than five different values of the cyclical velocity variations may be expected, each one corresponding to a certain intensity of explosion.

INDICATOR-DIAGRAMS OF 4-HP. OTTO ENGINES.

When the engine is working steadily, the diagrams present a very uniform appearance. The initial pressures are lower than those of the Cycle engine, while the exhaust pressures are higher; from which facts it is found, as might be expected, that the pressures throughout the working stroke are more uniform, and therefore tend to minimise the cyclical velocity variation. The effect of a miss on the succeeding explosion is overcome by some special arrangement. The mean pressure of the explosion following a miss is generally 3 or 4 per cent. less than the average of the mean pressures, although this difference often becomes more marked when the engine is unloaded. These explosions also occur more slowly than when the engine is unloaded, or is taking gas in every cycle. The effect of speed on the explosions is not very perceptible, and perhaps this may be partly due to the absence of any suction-pipe. A diagram showing the mean pressures and speed of an engine starting is given in Fig. 11, Plate 5, from which it may be seen that there is a certain speed at which the explosions are most powerful, although the differences are not very great. In Fig. 10, Plate 6, a considerable number of cyclical velocity variation diagrams are shown. With regard to the 4-HP. engine, it should be stated that the normal speed is 200 revolutions per minute, and the trial at 155 revolutions per minute was only made for the purposes of these experiments. Also the E-size engine is always sent out with two fly-wheels, and one wheel was removed solely for experimental purposes.

The typical cyclical velocity variation diagrams are given in Figs. 12, 13, 14, Plate 5. The influence of the reciprocating parts is clearly shown in all three cases, but mostly so in the high-speed engine. The resemblance between the cyclical velocity variation diagram, during the compression stroke, and the corresponding portion of the indicator and twisting moment diagrams, is clearly marked.

TABLE OF RESULTS (APPENDIX D).

It is unfortunate that in testing the E-size engine no indicator-diagrams were taken. For this reason, the figures given for the cyclical velocity variation diagrams, with one and two fly-wheels, cannot be compared, as it is impossible to say for certain how much gas the governor was allowing in each case. It is probable, however, that the total gas-supply in each trial was not very different, as the load was in each case nearly similar. The maximum cyclical velocity variation recorded, amounted to 5 revolutions per minute, or 2·17 per cent., and the minimum was as low as 1·9 revolution, or 0·8 per cent. When the engine is fully loaded, the cyclical velocity variation would probably approach most nearly to the higher value, and the smaller cyclical velocity variation occurs when there is little or no resistance. The high speed of this engine, together with the regulating governor, also rendered the maximum possible range of (governor) speed difficult to obtain over such short trials.

In Trial 7, with one fly-wheel, the governor allowed the speed to rise from 249 revolutions to 266 revolutions per minute, without a cut-out occurring. In Trial 6, with two fly-wheels, the speed rose 8·75 revolutions per minute, without a cut-out. The Author was informed, whilst testing this engine, that the governor was not in perfect working order, which perhaps accounts for the somewhat large range of speed permitted. In the case of the 4-HP. engine, it must be remembered that the fly-wheel power and speed are considerably less than with the E-size engine.

The following figures give the relative fly-wheel power of the two engines:—

Type of Engine.	Reduced Inertia of Fly-Wheels.	Reduced Inertia per Square-inch of Piston Area.	Cyclical Velocity Variation. Revolutions per Minute.
E size	2,300·6	42·02	4 (?)
4 HP.	353·4	9·18	10·5

The ratio $\frac{42\cdot02}{9\cdot18} = 4\cdot57$; that is to say, the E-size engine has four-and-a-half times the fly-wheel power of the 4-HP. engine; and, in addition to this, its speed is 25 per cent. greater.

Taking 10·5 revolutions per minute as a basis, and allowing

for the differences between the two engines, the calculated cyclical velocity variation for the E-size engine is 1.5 revolution per minute. A greater reduction might certainly have been expected in similar engines, whose main constructive details differ only in the lengths of their connecting-rods and strokes. The ratio of the connecting-rod to the stroke, in the case of the E-size engine, is 2.275, and for the 4-HP. engine it is 2.68.

In Fig. 10, Plate 6, Trial 10, a cyclical velocity variation diagram for the 4-HP. Otto engine, running at 156 revolutions per minute, is given. Here the cyclical velocity variation reaches 8.6 per cent. when loaded, and 9.2 per cent. when light. The mean cyclical velocity variation, at 199.5 revolutions per minute, amounts to 5.285 per cent., under nearly similar conditions as to load. The cyclical velocity variation, at 156 revolutions per minute, is 13.42 revolutions per minute; and $13.42 \left(\frac{156.0}{199.5} \right)^2 = 8.2$ revolutions per minute. Thus, if only the cyclical velocity variations at 156 revolutions per minute were known, the calculated value at 199.5 revolutions per minute would be 8.2 revolutions per minute, instead of 10.54 revolutions per minute, as is actually the case. This difference has been usually found throughout these trials, and appears to be due to the momentum of the reciprocating parts being increased. In one case, the cyclical velocity variation became actually higher by increasing the speed beyond a certain limit.

In reference to the experiments on the Otto gas-engine, the Author wishes to express his best thanks to Messrs. Crossley Bros., Limited, for the great assistance which they rendered him whilst at their works. Without their entire and generous concurrence, the trials which have been described would have been practically impossible.

9 BY 15 INCHES STEAM-ENGINE.

The last series of trials which space permits to be described, are those on a steam-engine 9 inches diameter, 15 inches stroke. The engine was of a very old type, with an extremely large and heavy fly-wheel, fitted with an ordinary slide-valve giving a cut-off at about three-quarters of the stroke. Although the speed was low (about 85 revolutions per minute), it might also be expected that the cyclical velocity variation would be small, and this proved to be the case, and probably very few modern engines with single cylinders give much better results. In obtaining the

theoretical cyclical velocity variation curve, the same constructions and methods were used as for the Otto gas-engine. In this particular case, the load on the engine was only about one-third of what the engine could bear; and consequently the cyclical velocity variation is very much smaller than in other cases which will shortly be referred to.

A typical cyclical velocity variation diagram is shown in Fig. 15, Plate 5, and this should be taken as the representative curve for the engine. It should be mentioned here that, in the case of a steam-engine, it is impossible to identify a particular semi-revolution on the cyclometer diagram with the corresponding diagram on the indicator record, by the same means as those adopted in the case of the gas-engines. To get over this difficulty, an electrical time-marker was placed on each of the instruments, in such a way that the operator could place at will a mark on both drums simultaneously by merely pressing a key. The two time-markers were placed in parallel in one circuit; and, by an arrangement attached to the cross-head of the engine, this circuit was momentarily closed only when the piston reached the forward end of its stroke. In addition to this automatic interruption of the circuit, a key, or switch, was also placed in the circuit, by means of which it could be permanently broken at the will of the operator at the cyclometer.

From these observations it may be gathered that after both instruments have been working for, say, a dozen revolutions, the operator may, by merely pressing the key, make a mark on the drums of both instruments at the close of the thirteenth revolution. If the key be raised immediately after one or two revolutions have been marked, it may again be pressed later on during the trial to form a check. The whole of the fifty diagrams on each drum may be identified, one with the other, by merely counting backwards and forwards from the marked diagrams. The Author was unable to make trials on this engine with different fly-wheels, owing to the great expense attending the necessary charges, without proper appliances, on a permanently bedded engine. The results derived from trials made when the engine was fully loaded, are given in Appendix E. These figures show that the cyclical velocity variation is slightly greater during the forward stroke than during the back stroke. This is in all probability due to the fact that the mean steam-pressure on the piston during the forward stroke is slightly greater than during the return stroke.

The speed diagrams of this engine are also shown with those of

the other engines in Fig. 10e, Plate 6. During this trial, the full load was on the engine, and suddenly removed after half the trial had been run. In this trial (C_{31}) the speed rose 17 revolutions per minute during the three revolutions following the removal of the load, and would probably settle down to about 93 revolutions per minute instead of 86 revolutions, the speed at which it was running before the removal of the load. This was equivalent to a range of 19 per cent., and was due entirely to the governor.

If, on the other hand, the engine be running steadily at 86 revolutions per minute with only about a third of its maximum load, and that is suddenly removed, then the speed during the following five revolutions only rises to 94 revolutions, and finally settles down at 93 revolutions (Trial C_9). In this case the range is only eight revolutions per minute, or 9 per cent.

Again, if, with the smaller load on, only half of it is removed (Trial C_{15}), the change in speed amounts to only $4\frac{1}{2}$ revolutions per minute, or 5 per cent. Yet, when only one-third of the same load is removed (Trial A_{13}), the rise in speed amounts to 7 revolutions per minute, or 7.75 per cent.

These typical examples are apparently incongruous, but on further examination they can be satisfactorily explained. In Trial C_{31} , the stop-valve of the engine was fully open, and consequently the high-pressure steam in the valve-chest would be sufficient to raise the speed of the engine considerably, even after the governor had acted.

In Trial C_9 , where only one-third of the maximum load was on the engine running at the same speed, the stop-valve was admitting only one-third of the amount of steam into the steam chest; and consequently the same volume of steam would be inclosed in the valve-chest after the governor had acted; but it would be at a lower pressure, and therefore could not increase the speed of the engine to so great an extent.

The apparent discrepancy between Trials C_{15} and A_{13} is due principally to the fact that, in the first case, the governor-sleeve was running $1\frac{3}{4}$ inch above its lower stop (engine speed = 88.6 revolutions per minute) before the load was removed, and in the second case it was barely $\frac{3}{8}$ inch (engine speed = 85.2 revolutions per minute) above the same point. If the sleeve had been at the same height in both cases, namely, $1\frac{3}{4}$ inch, the ranges would have been $4\frac{1}{2}$ revolutions per minute, and $3\frac{1}{2}$ revolutions per minute, respectively. Therefore, in this case also, the greater range corresponds to the greater removal of load, although the reverse is apparently true.

From these observations on an ordinary Watt governor, it is tolerably clear that the fluctuation in speed, due to governing, may vary considerably under exactly similar conditions as to loading, but with different conditions as to initial speeds (*i.e.* speed before removal of load); and, conversely, it may also vary considerably under constant conditions as to initial speeds, but with variable conditions as to load removal.

These remarks have been confirmed by a large number of experiments, which it is impossible now to enter upon in detail, and the results are in keeping with what might have been expected from *à priori* reasoning.

Before concluding this Paper, the Author would again draw attention to the combination of time-marker and cyclometer, used in conjunction on the steam-engine with the automatic interruption on the cross-head. It is clear that the period during which the armature of the time-marker is depressed, is identical with that during which the engine cross-head closes the circuit—say during the last $\frac{1}{16}$ inch of its stroke. When the armature is depressed, it makes a mark on the cyclometer-drum, parallel to the rows of waves, and, consequently, the number of wave-lengths which go to make up the length of the mark, give the actual time of depression in $\frac{1}{812}$ second. In this way the actual time taken for the piston to traverse the last $\frac{1}{16}$ inch, or any portion of its stroke, could be directly measured. By this means it would be easy to measure the exact time during which a Worthington pump hangs at the end of its stroke, and, indeed, the system might be applied to many machines where it is useful to know the speed at any given point. In these experiments, it is not even necessary to place the cyclometer in gear with any portion of the machine which it is testing, but it can be, if required, placed in another room and turned by hand; and, in cases where periods of cessation of motion are to be measured, it would be essential to adopt such a course.

The Author regrets that many of the conclusions which might be deduced from the results obtained during these experiments, have been omitted from want of space, but he trusts that sufficient evidence has been given to establish the fact that there is a relation between governor and fly-wheel; which connection is, however, rendered so indeterminate by the unknown friction in the governor, and depends so largely on the manner in which the governor controls the steam- or gas-supply, that it could only be expressed in very vague terms. The figures given are, unfortunately, incomplete, but the difficulties attending the change of

a fly-wheel are frequently so great as to prevent its being often done. No doubt, were a fly-wheel designed specially for such experiments as these, such that its moment of inertia could be readily altered, more complete information could be easily obtained. Nevertheless, the figures and drawings described in this Paper have already proved of practical importance to the makers.

The Paper is illustrated by four sheets of tracings, from which Plates 5 and 6, and the figures in Appendix A, have been prepared.

APPENDIXES.

APPENDIX A.

CONSTRUCTION EMPLOYED TO DRAW THE THEORETICAL CRANK-PIN VELOCITY CURVE FOR THE "CYCLE" GAS-ENGINE.

The first step was to plot the positions of the linkwork for a large number of points during the rotation of the crank-pin. The path traced by the centroid of each link during the cycle was also marked. To find the retarding or impelling influence of the links on the motion of the crank-pin, it is necessary to know the linear velocity of each link, its weight, and the radius of gyration about its centroid.

The mean speed of the crank-pin is known, and the velocities of all other points in the mechanism were found in the following way:—

In *Fig. 1* let C be the crank-pin, OC the crank, V the vibrating link-head, VR the vibrating link, CVR the connecting-rod, LP the piston-link, L the piston-pin.

The point C is moving at right-angles to OC, and the point V is moving as at right-angles to VR.

Hence the virtual (or instantaneous) centre of rotation of the connecting-rod must be at the intersection of OC and VR.

Let them meet at O_{cv} .

Let the known velocity of C (linear velocity in direction of motion) be v_c , and let the velocity of V be v_v .

Then, if the velocity of any point on a rotating body be known, the velocity of any other point in that body can be deduced from it, as their velocities are in proportion to their radii.

In the present case the link is for the moment rotating about O_{cv} .

$$\therefore v_v = v_c \frac{O_{cv} V}{O_{cv} C}.$$

Clearly, so far as the vibrating link is concerned, the velocity of V determines the velocity of any point in the link, and therefore need not be further considered. The velocity of any point in the piston-link is not so readily determined.

But it is known that P is moving at right-angles to $O_{cv} P$, because P may be considered for these purposes as a point either in the connecting-rod or piston-link. The virtual centre of the piston-link must lie therefore on $O_{cv} P$.

It is also known that L must move horizontally, and therefore the virtual radius of the link must also lie on the vertical through L.

Hence, the virtual centre of the piston-link must be at the point of intersection of $O_{cv} P$ and the vertical through L.

Let this point be called O_{pl} .

The virtual centres of all the links in the mechanism being known for every position which the links can take up, the velocities of the essential points can be determined for each position as shown in *Fig. 1*.

Produce OC and VR to meet in O_{cv} .

Join O_{cv} and P.

Set off, along $O_{cv} C$, a length $C C_1$, to represent the velocity of C. Through C_1 draw $C_1 V_1$ parallel to CV, and $C_1 P_1$ parallel to CP. Then $V_1 V$ is the velocity

varying with every position of the link, but the radius of gyration is of constant magnitude for a given link.

The methods just described for finding the accelerating effects of any link are precisely similar to those commonly adopted for a similar purpose in considering the compound pendulum. The analogy only differs in so far as, in the engine, the point about which any link is revolving is constantly changing its position, whereas, in the pendulum, the centre is fixed in position.

In the pendulum it becomes necessary to treat the whole mass as swinging, and concentrated at a point at the radius of inertia, in order to find its kinetic energy at any moment. This is precisely what has to be done with each of the links in succession, and for every fresh position which they may take up.

Their masses are known constant quantities; their radii of gyration vary infinitely from instant to instant, but are nevertheless calculable.

The forces which the acceleration of each link produces on its neighbour are called, in the drawing, acceleration-pressures, and are plotted as curves for each link to a distance base. The acceleration-pressures due to each link are then resolved separately back to the piston, and combined there with the indicator-diagram in the usual manner (Fig. 1, Plate 5). In this way, the individual action of each link can be compared with that of its neighbour, and also their combined effect in modifying the forces due to the explosion is evident on inspection of the corrected indicator-diagram.

It is almost needless to say that the importance of the changes produced by the inertia of the linkwork fully justified the trouble which had been taken.

It now became only necessary to resolve the corrected indicator-diagram through the linkwork to obtain the net effort tending to turn the crank-pin. *Fig. 2* gives one means of reaching this end, and, although not the shortest possible, it was adopted because it gave a considerable amount of information concerning the peculiar stresses to which parts of the mechanism are subjected.

Let aL = net force acting on the piston in pounds per square-inch of piston-area.

Resolve aL along LP , and vertically; then bL = tension per square-inch of piston-area in the piston-link, and ab = pressure on cylinder per square-inch of piston-area.

A force equal to bL acts on the connecting-rod at P . It is balanced by resistances applied at V and C , and therefore bH must be resolved in the direction VP and PC .

Set off, from P , $dP = bL$.

Draw cd parallel to CP , and produce PV to C . Then cP = tension per square inch of piston-area between V and P , and dc is the force per square inch of piston-area transmitted direct to the crank-pin.

cP acts on V , and this is balanced by the resistance along VR and that portion of the connecting-rod between V and C .

Hence cP must be resolved along CV and VR .

Produce Pc , and let $eV = Pc$.

Draw, through e , a line ef parallel to VC .

Produce VR .

There are now two forces acting through C ,—namely, dc along PC , and ef along VC .

Let their combined effect be represented by Cg in magnitude and direction.

Resolve Cg along CO , and tangentially to the circle at C .

Then Lg is the force per square-inch of piston-area tending to turn the crank and acting at the crank-pin C .

The twisting-moment diagram so obtained (Fig. 2, Plate 5) was also checked

by another shorter and more accurate method for the resolution of forces depending on the principle of virtual centres.

From the twisting moment diagram it is easy to construct the crank-pin velocity diagram, if the weights and dimensions of the fly-wheels and crank-shaft are accurately known, but it is not necessary to enter upon the details here.

In Fig. 3, Plate 5, the theoretical cyclical velocity variation curve is given on a large scale, and the experimental curve for the identical revolution of the engine is plotted with it for comparison.

The difference between the two curves is not considerable, and can easily be accounted for, when it is remembered that friction has been neglected throughout the calculations.

A peculiar feature in the curves is the sudden rise in velocity during the latter part of the suction stroke.

A glance at the corrected indicator-diagram (Fig. 1, Plate 5) at once explains the reason. During this period, the vibrating link is reaching its lowest position,

Fig. 2.

and both the other links are accompanying it. It is suddenly arrested at the end of the suction stroke, and its motion is reversed (corresponding to the reversal of motion of the cross-head in the ordinary steam-engine). Whilst its motion is being suppressed, it is clear that it will exercise a considerable influence on the speed of the engine, and the amount is shown on all the cyclical velocity variation curves.

The actual work done during the suction stroke by these links falling, is shown by the second area on the twisting moment diagram above the mean-height level.

This area represents 380 foot-lbs. of work, and the whole of it is applied solely to increase the speed of the engine.

During the explosion, 2,114 foot-lbs. are applied to the same purpose, and given out again by the fly-wheels during other portions of the cycle.

An examination of the typical cyclical velocity variation curves (Figs. 5, 6, 7, 8, Plate 5) will show that this calculation is corroborated in every case by experiment.

APPENDIX B.

TABLE OF RESULTS FOR "CYCLE" GAS-ENGINES.

Engine.	Trial.	Conditions.					Cyclical Velocity Variations in Revolutions per Minute.				Speed Variation due to Governor.	
		Brake HP.	Number of Fly-Wheels.	Moment of Inertia of Fly-Wheels.	Reduced Inertia of Fly-Wheels.	Mean Speed in Revolu- tions per Minute.	Mean Maximum Velocity.	Mean Minimum Velocity.	Difference.	Difference per Cent.	Range.	Range per Cent.
2 HP.	8 ₁	Dynamo.	2	91.5	149.9	150.8	156.65	147.55	9.075	6.02	5.66	3.77
"	3 ₁		2	91.5	149.9	151.6	156.98	148.10	8.880	5.85	5.20	3.43
"	1 ₂		2	145.2	237.9	152.8	155.56	150.84	4.720	3.09	4.60	3.01
"	7 ₂		2	145.2	237.9	153.7	155.96	150.68	5.280	3.44	4.97	3.23
"	3 ₂		2	145.2	237.9	178.8	180.65	175.42	5.230	2.92	6.47	3.62
"	5 ₂		2	145.2	237.9	106.1	108.12	102.38	5.740	5.41	6.81	6.41
4 HP.	1	6.2	2	140.9	184	142.5	145.97	138.16	7.811	5.48	6.70	4.70
"	3	7.0	2	140.9	184	145.1	149.76	141.86	7.880	5.43	6.93	4.77
"	16	6.9	1	70.4	92	142.4	149.45	133.40	16.05	11.27	12.70	8.92
"	22	6.7	1	70.4	92	140.0	147.05	130.26	16.79	11.99	13.70	9.78

APPENDIX C.

TABLE OF SPEEDS AND MEAN PRESSURES FOR 4-HP. "CYCLE" ENGINE
AT STARTING. TRIAL 13. TWO FLY-WHEELS.

Revolutions after Start.	Mean Speed in Revolutions per Minute.	Mean Pressures in Lbs. per Square-inch.	Revolutions after Start.	Mean Speed in Revolutions per Minute.	Mean Pressures in Lbs. per Square-inch.
3	54.32	28.70	15	116.25	58.44
4	Miss	Miss	16	124.40	49.35
5	60.35	43.4	17	129.6	40.95
6	70.61	38.15	18	134.42	31.5
7	79.43	31.15	19	Miss	Miss
8	Miss	Miss	20	138.1	61.6
9	85.55	47.95	21	Miss	Miss
10	95.6	46.2	22	142.8	61.25
11	108.36	44.45			
12	Miss	Miss			
13	109.18	57.75	39	Miss	Miss
14	Miss	Miss	40	155.9	52.85

APPENDIX D.

TABLE OF RESULTS FOR "OTTO" GAS-ENGINES.

Engine.	Trial.	Conditions.					Cyclical Velocity Variations in Revolutions per Minute.				Speed Variation due to Governor.	
		Brake H.P.	Number of Fly-Wheels.	Moment of Inertia of Fly-Wheels.	Reduced Inertia of Fly-Wheels.	Mean Speed in Revolutions per Minute.	Mean Maximum Velocity.	Mean Minimum Velocity.	Difference.	Difference per Cent.	Range.	Range per Cent.
4 HP.	9	6.6	1	137.5	353.4	202.8	207.40	197.54	9.86	4.86	16.1	7.94
"	8	5.5	1	137.5	353.4	196.2	201.45	190.25	11.20	5.71	15.7	7.99
Esize	6	Dynamo	2	398.2	2300.6	238.1	240.47	236.64	3.83	1.61	No cut-outs occurred during these trials.	
"	3		2	398.2	2300.6	251.4	252.91	249.46	3.45	1.37		
"	7		1	199.1	1150.3	250.2	253.78	247.80	5.92	2.37		

N.B.—In Trial 7 (E-size) the speed varied from 249.15 revolutions per minute to 266.2 revolutions per minute, being 6.85 per cent.

In Trial 6 (E-size) the speed varied from 237.4 revolutions per minute to 246.15 revolutions per minute, being 3.67 per cent.

In Trial 3 (E-size) the load was not altered, and the speed remained nearly uniform.

In Trial 6 (E-size) the cyclical velocity variations varied from 5.16 revolutions per minute to 1.901 revolutions per minute.

In Trial 3 (E-size) the cyclical velocity variations varied from 4.074 revolutions per minute to 2.173 revolutions per minute.

In Trial 7 (E-size) the cyclical velocity variations varied from 6.682 revolutions per minute to 2.824 revolutions per minute.

In the Table, no allowance has been made for the variable explosions (due to governor) in the figures given for the cyclical velocity variations of the E-size engine. The figures give the general average of the cyclical velocity variations taken consecutively during any trial.

APPENDIX E.

TABLE OF RESULTS FOR STEAM-ENGINE.

Trial.	Conditions.		Cyclical Velocity Variations in Revolutions per Minute.							
	Reduced Inertia of Fly-Wheel.	Mean Speed.	Forward Stroke.				Return Stroke.			
			Mean Maximum Velocity.	Mean Minimum Velocity.	Difference.	Difference per Cent.	Mean Maximum Velocity.	Mean Minimum Velocity.	Difference.	Difference per Cent.
C 29	631	87.1	88.34	86.11	2.22	2.55	87.82	86.12	1.70	1.96
C 30	631	86.9	88.14	85.9	2.24	2.57	87.65	85.79	1.85	2.13

Fig. 1.

Fig. 2.

Fig. 3.

FLY-WHEELS AND GOVERNORS.

Fig. 5.

Fig. 6.

Fig. 7.

Fig. 8.

Fig. 9.

Fig. 11.

Fig. 12.

Fig. 13.

Fig. 14.

Fig. 15.