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ON THE RELATION OF STIMULUS TO SENSATION IN VISUAL IMPRESSIONS.

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The object of this investigation is briefly as follows: First, to produce what appears to the eye to be a smooth and even gradation from white into black through deepening shades of gray; secondly, to determine the proportional amounts of stimulus or physical excitation and to express them in a curve on the graphic method; and finally to ascertain the principles on which the amounts of stimulus and of sensation are related to each other. Colors have also been employed in the investigation with the primary object of obtaining a more extended series of data on which to found general principles. Secondary relations have however been brought out by their use.

The method adopted is based on that employed by Dr. Kirschmann.¹ Cardboard discs are prepared of a suitable size for use with a rotating table. Such a disc, say 374 mm. in diameter, is covered with black 'surface paper.' If a white sector, say of 30°, be affixed, and the disc be rapidly rotated, the black and white blend in visual sensation so as to give a uniform gray, the depth of which depends on the relative percentages of white and black. The blending is more perfect, however, if the same amount of white be added in three evenly-spaced sectors each of one-third the angular value, say 10°. It is clear that each sector increases in width from the center outwards; but the angle made by its boundaries is throughout the same. To increase or decrease the percentage of white we must increase or

¹ *American Journal Psychology*, Vol. VII., p. 386 (1897).

decrease the angle. And in all cases, at any distance from the center, the percentage is proportional to the angle subtended, or in the case of several sectors to the sum of the angles. This gives the method of introducing on to a disc any required percentage of white at any given distance from the center.

Let us suppose that our object is to ascertain whether equal increments of white stimulus, from some circle (*a*) near the circumference of the disc to some circle (*f*) near its center will afford evenly-graded shading from black, beyond the outer circle, to white within the inner circle. The black disc is marked out into three equal sectors by radii making angles of 120° with each other. One of such sectors is shown in Fig. 1. Arcs

FIG. 1.

of circles are drawn at the required distances *a* and *f*; and four more arcs are drawn through points *b*, *c*, *d*, *e* on any radius, the distances *ab*, *bc*, ..., *ef* being equal. We have now marked out the portion of the disc with which we are to deal by six evenly spaced arcs, and we desire to introduce white in equal increments, thus:

at <i>a</i> ,	0%
" <i>b</i> ,	20
" <i>c</i> ,	40
" <i>d</i> ,	60
" <i>e</i> ,	80
" <i>f</i> ,	100

Bisecting the sector we measure off on each side of the bisecting radius the angles required to give these percentages. Since our measurements deal with one-sixth of the total amount of white to be introduced on the completed disc, and since the angular value of 1% on such a disc is $\frac{360^\circ}{100}$ or 3.6° , the value of 1% in each of our half sectors of 60° will be $\frac{3.6^\circ}{6}$ or $.6^\circ$. Hence the angles we measure off are as follows:

at <i>b</i> , 12°	to give total of	20%
" <i>c</i> , 24°	"	40
" <i>d</i> , 36°	"	60
" <i>e</i> , 48°	"	80
" <i>f</i> , 60°	"	100

By sweeping a curve through points determined by these angular measurements an evenly graded increase of white from 0% to 100% is marked out as indicated by the boundary between black and white in Fig. 1. In practice the area thus limited is cut out (after tracing from a carefully prepared model) from white paper which is affixed to the black disc and if necessary accurately trimmed to give the true values of the percentages. All three sectors carry a similar white 'leaf.' By increasing the number of circles the number of measured points is increased and the curve may be drawn more accurately; and by checking the angular measurements, made with a protractor, by measurements of the chords, determined by calculation, yet further accuracy is secured.

On setting a disc so prepared in rapid rotation it is found *not* to give an evenly graded effect in sensation. Its inner central part is too white; the middle third much too light a gray to appear a good mean between white and black; while the outer third is again too light and ends off too sharply against the outer ring of black. Nor is the shading more satisfactory when similar areas are filled in with black on a white disc. Plate I., *B*, is from a photograph¹ of this disc in rotation. The central portion is too black and shades too rapidly into a light gray.

¹The photographs from which the figures in the plates are reproduced were taken by Mr. Frank Holmes, of Clifton, Bristol. They serve to indicate, though they do not quite faithfully reproduce the effects of the eye.

In the paper above mentioned Dr. Kirschmann states that a logarithmic curve for stimulus affords with a rotating disc a means of exemplifying in a class experiment the truth of the Weber-Fechner formula, according to which the stimulus must increase in geometrical progression in order that sensation shall increase by evenly spaced increments. A disc prepared in strict accordance with the instructions given in Dr. Kirschmann's paper (save that black 'surface paper' was substituted for black velvet) gives an increase of white stimulus in geometrical progression from circumference to center. But it does not afford satisfactory grading in visual sensation. The inner third is too uniformly light, the peripheral portion too uniformly dark; and there is too sudden a transition from light to dark. There is no gentle and even passage from white into black through lighter and lighter shades of gray. Plate I., *A*, is from a photograph of this disc in rapid rotation.

A modification of this method of experiment serves to bring the investigation into closer touch with other psychophysical methods and with 'Masson's discs.' Instead of sweeping a continuous wave through, say, 10 determined points, white is added in step-like increments of the same proportional value. Thus to illustrate equal increments of stimulus the rotating disc gives ten rings in each of which there is a sudden increment of 10% of stimulus. This method of construction is illustrated to the right of Fig. 1. This disc is figured in Plate I., *E*, when it is in rapid rotation. Similarly a disc to illustrate a geometrical progression of stimulus gives also 10 rings which are figured in Plate I., *D*, when this disc is in rotation. In neither case do the rings give steps of equal value for sensation. By increasing the distance from which the rotating discs are viewed the steps become less marked to the eye until some of them disappear and seem to merge or shade into each other. But while some thus merge and lose their identity others remain easily distinguishable; which shows that they are not all of equal value for the eye. This method of experiment clearly brings the investigation into touch with that based on least perceivable difference. But a curious optical illusion somewhat distracts the judgment. Each ring is apt to appear double. Lying as it does between

a darker ring on the one side and a lighter ring on the other side it is peculiarly subject to the effects of contrast. Hence the part of it nearer the dark ring appears to be relatively lighter than the part near the lighter ring; and the physically uniform band differentiates in vision into a double band due to the effects of contrast.

Professor Kirschmann uses with his discs a small telescope for isolating small areas for comparison. Using tubes in a similar way I came to the conclusion that the shading of the disc as a whole affords a better means of judging of the grading than is obtained by examination of isolated areas in detail.

From many such observations as have been described, using both smooth and step-like increments of excitation, it appeared that neither an arithmetical nor a geometrical progression of stimulus affords an evenly graded series to the observer's eye.

The investigation here passed into the experimental stage. It was first necessary to ascertain what proportion of black and white would blend on a rotating disc so as to afford a sensation of 'mid-gray'—that is to say a shade which appears to the eye to be just mid-way between black and white. This was found approximately by means of discs with three concentric rings of about equal breadth, the inner black, the outer white, and an intervening ring with black and white sectors. The percentages for the papers affixed to the discs are from 25% to 28% of white, to from 75% to 72% of black, according to the brightness or other conditions of illumination. In a similar way it was found best to obtain a light gray of mean shade between this mid-gray and white. The proportions required are about 50% of white and black; and, to obtain a dark gray of mean shade between mid-gray and black, the requisite proportions are about 12.5% of white and 87.5% of black. Considerable difficulty was, however, experienced in judging a given ring to be an arithmetical mean strictly half way between those of higher and darker shades; and it was found that continuous curves yielded better results. After some experience the eye detects even slight departures from smooth and even grading in the sensory series.

It is here desirable to give in some detail the experimental results obtained with eight discs. They will serve to show the

NOTES.

- A. Stages 1-6 too white; 14-19 too dark; mid-region satisfactory.
- B. " 1-8 much too white; 14-19 better than A.
- C. " 1-8 very much too white; 14-19 better than A or B.
- D. " 1-6 good; 14-19 much too dark. Make new disc (E) combining 1-6 of D, 14-19 of C, and mean of intermediate stages.
- E. Good result; weakest 12-14, too dark. Try two new discs (F and G) with maximum + or - modifications to amount of 2 %.
- F. Not so good as E as a whole; 18-19 good; 12-14 worse rather than better. 1-8 rather too white.
- G. Not so good as E on the whole; 12-16 better than E or F; 18-19 much the same but not quite so good as E. Take results of E modifying 12-14 to accord with G.
- H. Satisfactory result A little too dark near 12. Accept as basis for theory.

From these results it will be seen that slight errors are more easily detected where the percentages of white are small than where they are relatively large. A study of the percentages will further show that whereas the first 5% of sensation is produced by the introduction of about 2% of white stimulus, the last 5% (from 95% to 100%) is produced by the introduction of 11% of white. Hence a small absolute addition of white to low percentages will be noticeable, whereas a much larger addition to high percentages will remain unnoticed.

Colors and shades were next dealt with, and experiments made with several pairs, such as red and black, blue and black, red and white, red and blue, purple and emerald green. The object here was to get a sufficiently wide series of data on which to base a discussion of the relation between stimulus and sensation. It was found that each pair gave a differing series of percentages. It will suffice to present the final results obtained with red on black, and blue on black. They were reached by methods similar to those already illustrated in sufficient detail. It should be stated that the results embodied in the accompanying Table (II.) were attained by an extended series of experiments and observations analogous to those given in Table I.

So far the results obtained are purely empirical. They were plotted in curves on the graphic method with a view to determining if possible the law which underlies them.

Before proceeding to a consideration of this law a criticism which may suggest itself must be met. It may be said that the

TABLE II.

Stage.	Sensation.	White on Black.	Red on Black.	Blue on Black.
20	100	100	100	100
19	95	89.	91.	94.5
18	90	79.	83.5	89.1
16	80	62.	69.	78.4
14	70	48.	55.5	67.9
12	60	35.8	44.5	57.6
10	50	27.	35.	47.5
8	40	19.5	25.5	37.6
6	30	13.	18.	27.9
4	20	7.9	12.	18.4
2	10	3.5	6.	9.1
1	5	1.8	3.	4.5
0	0	0	0	0

'black' and 'white' employed are very far from being absolutely black and white respectively; and that the colors used—surface paper and wall papers—are very far from being specifically pure. This is perfectly true. What is sought, however, is to obtain even grading between a sensation a on the one hand and a sensation x on the other hand; or, since neither visual sensation is pure, let us say a complex sensory effect a, d, g , on the one hand and another sensory effect s, v, z , on the other hand; and this may be attained no matter how complex either term may be. Experiment showed, for example, that a grading could as readily be obtained between 'black' and light gray as between 'black' and 'white'; the mid-point between the two being obtained by using 30% of the gray selected and 70% of 'black.' And so far as the colors are concerned the results appear to be due rather to the relative intensities of stimulation or excitation of the retina than to the effects of color as such. In any case there is no pretence that this investigation deals with pure colors or shades, though it may lead up to a further investigation of spectral colors.

Another point which calls for notice is the illumination. This is a matter of considerable importance. In a bright light the contrast between colors or shades of different intensities is heightened. A black and white disc which shows good shading for a medium illumination fails to grade smoothly in a strong light. But though the percentages and curve for any pair of

colors or shades varies with the conditions of lighting, the principle which it illustrates remains the same.

Proceeding now to a consideration of the law which appears to express the relation between stimulus and sensation it will be well to begin with a discussion of the curves for 'white on black,' 'red on black' and 'blue on black' which very closely accord with those which are plotted in Fig. 2. And the first

point to notice is that neither of these curves is throughout its whole extent logarithmic as it should be if the Weber-Fechner formula holds good. It is true that curves expressing geometric progression of stimulus can be found in each case nearly to fit that obtained by observation *in limited portions of its extent*. But the curves of Fig. 2 are so homogeneous that they seem to indicate some simple law applicable to all, and in each case throughout the whole range. And this law is certainly not that expression by the Weber-Fechner formula. It is well known how-

ever that this formula fails in cases of low and of high stimulation—a defect which is somewhat lamely explained by saying that it only holds good for the *optimum* region of sensation. It is generally admitted to be but an approximation to the true interpretation of the facts. A law which shall apply to the whole range of normal sensation is still a psycho-physical desideratum.

It will be remembered that the Weber-Fechner formula generally accepted runs as follows: In order to produce a series of just distinguishable sensations, giving an arithmetical progression of sensation, the determining stimuli must afford excitations in geometrical progression. But in the curves plotted in Fig. 2 the stimuli required to produce the sensation series 5%, 10%, 15% etc. are *not* in geometrical progression. This being so the question arises: Do they form an orderly series conforming to some simple law of progression? They do, and the law may be thus stated: *Equal increments of sensation are produced by increments of excitation in geometrical progression.*

Let us fix our attention on curve A in Fig. 2, which represents the relation of stimulus to sensation for a graded series from 'white' through gray to 'black.' Now experiment shows that 27% of 'white' stimulus affords the sensation midway between 'white' and 'black'—in other words it corresponds to 59% of sensation. Let us take this single observed fact as the *datum* for a calculation based on the suggested law, and then see how far the percentages that result from this calculation accord with those obtained in the experimental work above described. Dividing the whole range into 10 equal sensation-increments of 10% each we have to ascertain the related stimulus-increments. It is clear that the sum of the first five stimulus increments gives the observed number 27% while the sum of the completed series of 10 stimulus-increments gives the total of 100%. Hence

$$100 = \frac{a(r^{10} - 1)}{r - 1} \quad (1)$$

$$27 = \frac{a(r^5 - 1)}{r - 1} \quad (2)$$

where a is the first step and r is the factor to give a geometrical series; from which the value of a ($= 3.49$) and

that of r ($= 1.22$) are readily obtained,¹ and these being known the value of each of the ten terms which make up the complete series may be calculated. The following Table (III.) gives the increments of sensation and their sums, the increments of stimulus and their sums, and (for comparison) the percentages obtained by experiment.

TABLE III.
WHITE ON BLACK.

Stage.	Sensation.		Stimulus.		Observed Percentage of Stimulus.
	Increments.	Sums.	Increments.	Sums.	
20	10	100	20.90	100	100
18	10	90	17.13	79.10	79.
16	10	80	14.03	61.97	62.
14	10	70	11.51	47.94	48.
12	10	60	9.43	36.43	35.8
10	10	50	7.73	27.	27.
8	10	40	6.33	19.27	19.5
6	10	30	5.20	12.94	13.
4	10	20	4.25	7.74	7.9
2	10	10	3.49	3.49	3.5
0	0	0	0	0	0

In Fig. 3 the curve for 'white on black' is plotted on the graphic method and the manner of its construction with step-like increments in geometrical progression shown.²

$$100 = \frac{a(r^{10} - 1)}{r - 1} \quad (1)$$

$$27 = \frac{a(r^5 - 1)}{r - 1} \quad (2)$$

Hence by division

$$\frac{100}{27} = \frac{r^{10} - 1}{r^5 - 1} = r^5 + 1$$

$$\therefore r^5 = \frac{100}{27} - 1 = \frac{73}{27}$$

$$\therefore 5 \log r = \log 73 - \log 27.$$

$$\text{Whence } r = 1.22009$$

Substitution in (2) gives $a = 3.488$.

The series of increments for 10 stages will be as under :

Stage	1.	2.	3.		9.	10
a.	a.	ar.	ar ² .		ar ⁸ .	ar ⁹ .

²The following note is kindly furnished by my colleague, Professor F. R. Barrell, to whom I am much indebted for valuable criticisms and suggestions.

Denote intensity of sensation by x ,

" " " stimulus by y ,

y must be some function of x , say $f(x)$.

Let three equal increments h be given successively to x and let $\kappa_1, \kappa_2, \kappa_3$, be the corresponding increments of y

$$\therefore y = f(x) \\ y + \kappa_1 = f(x + h)$$

$$= f(x) + f'(x)h + f''(x)\frac{h^2}{1.2} + f'''(x)\frac{h^3}{1.2.3} + \dots$$

$$y + \kappa_1 + \kappa_2 = f(x + 2h) \quad [\text{by Taylor's theorem}]$$

$$= f(x) + f'(x).2h + f''(x)\frac{4h^2}{1.2} + f'''(x)\frac{8h^3}{1.2.3} + \dots$$

$$y + \kappa_1 + \kappa_2 + \kappa_3 = f(x + 3h)$$

$$= f(x) + f'(x).3h + f''(x)\frac{9h^2}{1.2} + f'''(x)\frac{27h^3}{1.2.3} + \dots$$

$\therefore$ By subtraction

$$\kappa_1 = h\{f'(x) + \frac{1}{2}f''(x)h + \frac{1}{6}f'''(x)h^2\}$$

$$\kappa_2 = h\{f'(x) + \frac{3}{2}f''(x)h + \frac{7}{6}f'''(x)h^2\}$$

$$\kappa_3 = h\{f'(x) + \frac{5}{2}f''(x)h + \frac{13}{6}f'''(x)h^2\}$$

Now the hypothesis is that $\kappa_1\kappa_2\kappa_3$ are in G. P.

$$\therefore \kappa_2^2 = \kappa_1\kappa_3.$$

Substitute the above values for $\kappa_1, \kappa_2, \kappa_3$, we find that the absolute terms and the coefficients of h are identical on both sides of the equation; the coefficients of h^2 give

$$\frac{1}{6}f''' + \frac{1}{4}f''^2 + \frac{1}{6}ff''' = \frac{7}{3}ff''' + \frac{2}{3}f'^2$$

$$\therefore f'f''' = f'^2$$

$$i. e., \frac{dy}{dx} \frac{d^2y}{dx^2} = \left(\frac{d^2y}{dx^2}\right)^2.$$

The solution of this differential equation is

$$y = Ae^{bx} + C.$$

$$\left. \begin{array}{l} \text{But sensation } x = 0 \\ \text{when stimulus } y = 0 \end{array} \right\} \therefore C = -A$$

$$\therefore y = A(e^{bx} - 1) = A(10^{bx} - 1) = A(\lambda - 1)$$

where A, B, b, λ are constants whose values are found by making curves pass through the point (100, 100) and any other one point given by experiment.

[In the body of the paper sensation is represented by y and stimulus by x $\therefore x$ and y should be interchanged in above equations.]

For the curve 'white on black' $A = 15.85$

$$b = .0086392$$

for the curve 'red on black' $A = 40.835$

$$b = .005377.$$

F. R. B.

A disc prepared in accordance with the percentages given by the theory affords good and even shading (Plate I., C). Another disc in which the increments in 10 stages are introduced step-fashion gives a series of rings which on the whole grade well, though the effects of contrast have to introduce a disturbing element. When such a disc is viewed from progressively increasing distances, all the rings merge into a continuum together.¹ Some of the rings do not disappear earlier than

others as in the case to which allusion has previously been made. Plate I., F, is from a photograph of this disc when it is in rapid rotation.

It will be noticed that in the theoretical series,² as in the

¹ This does not hold good of the accompanying photographic reproduction, which is so far inaccurate.

² Alike in the experimental series and in the theoretical interpretation of the results 27% of white is accepted for mid-gray. Under the conditions above

closely similar series of percentages reached by experiment, a much larger absolute addition of 'white' is required to give say 10% increment of sensation at one end of the scale than is required at the other end of the scale. It is well known that a smaller amount of white is just perceivable on a black ground, than of black on a white ground. This may be illustrated by means of rotating discs. It will, for example, be found that a gray ring produced by 5% of white on a black ground is more marked than a gray ring produced by 10% of black on a white ground, notwithstanding the fact that the amount of white in the more obvious ring is but half that of black in the less readily distinguishable gray circle. More careful experiments showed that, whereas the difference between 99% and 100% of white (that is, 1% of black on a white ground) is just perceptible, from .3% to .4% of white was just perceivable on a black disc, the exact amount varying with the brightness of the illumination. Now the increment of white on black which corresponds (in accordance with the law here suggested) to 1% of black on white is .38%. Too much stress should not be laid on the accordance of the two results; but it may be given for what it is worth.

It has already been stated that the curve founded on observation, on which the theoretical curve is based, differs from a logarithmic curve. This may be illustrated by Fig. 4. The firm line gives the theoretical curve based on the principle of increments of stimulus in geometrical progression. The broken line shows the logarithmic curve which passes through the percentages at stages 6 and 14. Although this is one of the best logarithmic curves which can be found for purposes of com-

mentioned, with a medium illumination this to my eye gives good shading. With a brighter light, or at a smaller distance, the percentage of white has to be diminished. For though the eye is capable of some amount of adaptation to the amount of light received, adaptation effected through changes in the size of the pupil, there remain outstanding differences. For the distance at which the photographs for the accompanying plate were taken the percentage to give mid-gray was reduced to 25%. Some of my friends consider my mid-gray too light; a few regard it as too dark. It seems that the personal equation, and the sensitiveness of the individual retina must be taken into consideration. I would therefore lay no special stress on the percentage which for my eye gives mid-gray; that on which I do lay stress is that, when this datum is determined, for any given individual under given circumstances, the relative percentages have values in accordance with the law of progression above set forth.

parison, it will be seen that the divergence is far in excess of any probable errors of observation.

Passing now to the curve for 'red on black' (which is shown graphically in *B*, Fig. 2) the following Table (IV.) gives the application of the law in a manner similar to that in the previous Table (III.) for 'white on black.'

Since the excitation effect of the red employed is less than that of 'white' a larger amount of stimulus (*viz*, 35% as compared with 27%) is required to give a sensation midway between red and black. The curve, therefore, passes through this point instead of through 27%. The value of a (the first of 10 steps) is here 5.382 and that of r 1.1318; but the law still holds good. Equal increments of sensation are produced by increments of excitation in geometrical progression.

It is unnecessary to give details with regard to the curve for dark blue on black in the experimental work for which I have

TABLE IV.
RED ON BLACK.

Stage.	Sensation.		Stimulus.		Observed Percentage of Stimulus.
	Increments.	Sums.	Increments.	Sums.	
20	10	100	16.40	100	100
18	10	90	14.49	83.60	83.5
16	10	80	12.80	69.11	69.
14	10	70	11.31	56.31	55.5
12	10	60	10.	45.	44.5
10	10	50	8.83	35.	35.
8	10	40	7.80	26.17	25.5
6	10	30	6.90	18.37	18.
4	10	20	6.09	11.47	12.
2	10	10	5.38	5.38	6.
0	0	0	0	0	0

been assisted by Miss Alice Worsley. It will suffice to give the percentages of blue stimulus ($a = 9.124$; $r = 1.02022$).

It will be observed that the progression is nearly arithmetical. But a strict arithmetical progression does not give satisfactory results. Even when 48.5% instead of 47.5% was taken for the mid-point, the shading was distinctly less satisfactory.

BLUE ON BLACK.

Stage.	Increments.	Percentage of Stimulus.
20	10.926	100
18	10.708	89.074
16	10.495	78.366
14	10.287	67.871
12	10.084	57.584
10	9.884	47.500
8	9.688	37.616
6	9.496	27.928
4	9.308	18.432
2	9.124	9.124
0	0	0

In the cases considered in this paper we have positive stimuli—excitations due to the white, red or blue on a background of black, which may be regarded as incapable of affording any appreciable amount of positive stimulation to the retina.¹ When we pass, however, to cases of shading where two positive stimuli are concerned (such as white into red, red into blue, orange into red) we obtain curves which differ in type from those given

¹ I do not think that the *Eigenlicht* modifies the results in any appreciable degree.

above. I propose to deal with these in a subsequent paper. The experimental results and their discussion are well advanced; and the principle involved appears to support the conclusions already reached. It must here suffice to say that the interaction of different stimuli and the effects of contrast render the problem more complex and the curves less simple.

Leaving on one side therefore the interpretation of the results of observation on the mutual shading of colors, or of a color and white, which undoubtedly indicate some more or less complex mode of interaction, attention may be recalled, in conclusion, to that which it was the main purpose of this investigation to determine—the relation of sensation to stimulus in the relatively simpler cases of white, red and blue on a black, and probably negative, background. Observation seems to show that for the phenomena under consideration the Weber-Fechner formula cannot be accepted as it stands in the text-books. The law that the observations here recorded seem to indicate is that *equal increments* of sensation are produced by *increments of stimulus in geometrical progression*. Still, as we have seen, the Weber-Fechner formula is approximately applicable to limited portions of the curves in Fig. 2, though in none of the cases, even approximately, to its whole extent. But since the majority of observations which accord best with the Weber-Fechner formula have been made within a *limited* optimum range of sensation, they would probably not be widely divergent from the requirements of the formula which results from my research. Hering's pupils came to the conclusion that, near the threshold of visual sensation the stimulus increases in arithmetical progression. Now a glance at the curves of Fig. 2 will show that the progression near the threshold differs so little from arithmetical that it would be difficult by experiments on least observable difference to determine the departure therefrom. These observations also may therefore be held to be in substantial accord with the psycho-physical formula which is here suggested.

No attempt can be made in this paper to correlate the results obtained with those reached by previous observers in the field of vision.