XIV.—*Further Experiments and Remarks on the Measurement of Heights by the Boiling Point of Water.* By JAMES D. FORBES, D.C.L., F.R.S., Sec. R.S.Ed., &c., Professor of Natural Philosophy in the University of Edinburgh. (With a Plate.)

(Read 4th December 1854.)

In 1843 I presented a paper to the Royal Society of Edinburgh, giving an account of experiments made on the boiling point of water in the Alps, under various barometric pressures. My object was twofold: first, to describe an apparatus which I considered more practically available than those previously in use; and, secondly, to give a simple, and, as I believed, new formula for computing heights from such observations.

With reference to the second point, I became aware, some time after the publication of my paper, that Sir JOHN LESLIE had proposed to compute heights by the thermometer, assuming the change of the boiling point to be *exactly* in proportion to the height ascended. While cheerfully conceding to Sir JOHN LESLIE priority on this point, I submit that he did not bring forward experiments to justify its practical adoption.

Of late years both the instrument and the formula have been objected to by M. REGNAULT of Paris, and the latter by Dr JOSEPH HOOKER, who finds that it does not correctly represent his Indian observations. This has caused me to examine the whole subject, and also Dr HOOKER's observations on the boiling point, with the particulars of which he has kindly furnished me, and I proceed to lay the details before the Society.

In 1843, when I wrote, the method of determining heights by boiling water had fallen very much into abeyance, principally owing, as I believed, to the inconvenient instruments employed, and partly to the uncertainty of the deduction of heights. As the thermometric method is principally valuable when barometers cannot be safely transported, and must always be inferior in accuracy to good barometric results, my intention was to do a service to physical geography, by introducing a convenient and effective instrument, by means of which water could be certainly made to boil even in untoward circumstances, and the temperature ascertained, not to the illusory nicety of two or three decimals of FAHRENHEIT's degree, as Archdeacon WOLLASTON attempted, but to within about $\frac{1}{20}$ th of a degree, corresponding to about 25 feet of elevation, which I stated as the *utmost* degree of accuracy which I expected to attain, even in favourable

circumstances.* The formula of one degree of lowering of the boiling point for 550 feet of elevation, in an atmosphere at 32° , I stated to represent my observations quite sufficiently, and better than DALTON'S Table of the Elasticity of Vapour, which was the one then commonly in use. I refer to my former paper for a description of the boiling apparatus, consisting of a thin copper pan heated by a "Russian furnace," having a powerful jet of inflamed alcoholic vapour. The thermometer (contrary, I believe, to the usual practice) had its bulb in the water, not in the steam.

In 1844 M. REGNAULT published in the *Annales de Chimie* a table of the elasticities of vapour at moderate temperatures, and a comparison with some boiling water experiments in Switzerland and the Pyrenees. He also contrived (I am not sure at what date) a small apparatus for the use of travellers, somewhat resembling Archdeacon WOLLASTON'S.

In 1845 he published a second paper on the same subject in the same Journal, in which he quotes my observations, which he rejects as not conforming to his law of Elasticities of Steam, and attributes their discrepancy to faults in the boiling apparatus, and to errors of graduation of the thermometer.

The slightest comparison of M. REGNAULT'S paper with mine, shows, however, that the discrepancies complained of do not argue anything against the accuracy either of his Table of Elasticities, or of my mode of observing; but they disappear almost entirely when the correction for the index error of the thermometer I used is applied to the temperatures observed. This index error ($0^{\circ}62$ in excess) is given in my paper (page 414), and, of course, should have been applied when it was intended to compare *absolute* temperatures with *absolute* pressures, but had not been used by me when my object was merely to ascertain the *relative* variation of those quantities, as in page 412. When the index correction is applied, the deviations of my observations from M. REGNAULT'S Table fall, as will be immediately seen, considerably within those of M. MARIE quoted by him, of which he says that they "s'accordent avec la formule aussi bien qu'on peut le désirer."†

To the observations of 1842, made in the Alps, and published in my former paper, I have now added a fresh series, made in 1846, with the same apparatus and thermometer, which confirms them in a remarkable manner. These series are denoted by I. and II. in the following Table. They were projected in the manner described in my former paper, the ordinates being the temperatures, the abscissæ the logarithms of the corresponding observed barometric pressures, which numbers are proportional to the heights in an atmosphere of uniform temperature. Through the points a straight line may be drawn, with a very close approximation (See Plate III., fig. 1). The deviations of the observations from this

* Transactions of the Royal Society of Edinburgh, vol. xv., p. 411.

† *Annales de Chimie*, 3^{me} Serie, vol. xi., p. 332.

simple law are given in the 6th column of the following Table. Column (7) gives the temperature from M. REGNAULT's Table in the *Annales de Chimie*.

SERIES.	Barometer corrected and reduced to 32°.		Temp. Boiling Water Observed*	Linear Formula.	Difference.	REG-NAULT'S Formula.	Difference.	LOCALITY.
	Millim.	Eng. Inch						
I.	527·5	20·79	194°50	194°28	+ 0°22	194°18	+ 0°32	Col Collon.
II.	528·05	20·79	194·25	194·33	− 0·08	194·24	+ 0·01	Aiguille du Moine.
II.	569·0	22·40	197·90	197·94	− 0·04	197·78	+ 0·12	Breven.
I.	575·9	22·67	198·43	198·51	− 0·08	198·37	+ 0·06	St Bernard.
I.	588·1	23·15	199·45	199·52	− 0·07	199·39	+ 0·06	Tacul.
I.	593·2	23·35	199·95	199·94	+ 0·01	199·80	+ 0·15	Do.
I.	606·9	23·89	200·93	201·04	− 0·11	200·90	− 0·03	Prarayon
II.	609·3	23·99	201·15	201·24	− 0·09	201·07	+ 0·08	Montanvert.
II.	610·15	24·02	201·35	201·31	+ 0·04	201·15	+ 0·20	Do.
II.	612·3	24·105	201·30	201·47	− 0·17	201·33	− 0·03	Do.
I.	638·6	25·14	203·55	203·51	+ 0·04	203·36	+ 0·19	Naversch (Gressonay.)
II.	675·0	26·57	204·60†	...	...	...	...	Chamouni.
I.	723·6	28·49	209·47	209·54	− 0·07	209·53	− 0·06	Martigny.
(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)

* Corrected for Index Error + 0°·65.
† Evidently a mistake; the error amounting to nearly 1½°.

The following comparisons with the same apparatus and a different barometer, I find recorded as made at Edinburgh in January 1843.

Barom. corrected.	Boiling Point corrected.
27·760	208·57
29·040	210·72
29·879	211·95
30·064	212·18

If we project the Alpine observations alone (which are the most consistent with one another), in the manner exemplified in my former paper, as is done in Plate III, fig. 1, I find that they may be admirably represented by a straight interpolating line (giving the results recorded in column (5) of the preceding Table), and yield the following results.

The *uncorrected* reading of the thermometer, at the standard pressure of 30 inches, is 212°·75; at a pressure of 760 millimetres (29·922 inches), 212°·62, The Edinburgh observations (distinguished by the letter *e*), taken separately, give almost the same results.

The Alpine observations above give a uniform rate of ascent of 543·2 feet for 1° of fall in the boiling point. This is when we use LAPLACE's barometric coeffi-

cient for a standard atmosphere at 32° . It is about 6 feet less for 1° than I formerly deduced. If we include the *whole* of the observations (Edinburgh and the Alps), the rate of ascent will not be sensibly altered, though the squares of the errors will be somewhat increased.

The coincidence of the formula with observation, shewn in column 6, is highly satisfactory. In only three instances does it exceed a tenth of a degree of FAHRENHEIT.

The coincidence with M. REGNAULT'S Table, shown in column 8, is not less satisfactory (a very slight change in the index error* would neutralize the preponderance of positive errors).

From the mode of experimentally obtaining his results, M. REGNAULT'S Table is *in fact a table of boiling points* (see *Ann. de Chimie*, 1844), which was not the case with DALTON'S Table, which differs from it sensibly: and I have no doubt that M. REGNAULT'S are, on the whole, the most accurate numbers we at present possess.†

Hence,

1. Observations of height by the thermometer down to about 190° , or for elevations not exceeding 12,000 feet, may, within the usual limits of error, be reduced *indifferently* by M. REGNAULT'S Table, or by my arithmetical proportion.

2. The method of placing the thermometer in water instead of steam, and of using a powerful alcoholic furnace, which may be removed to one side until the escape of steam becomes uniform and moderate,‡ appears to give remarkably steady and consistent results. The graduation of the thermometer, as regards the length of the degree (which was entrusted to Mr ADIE, and on which M. REGNAULT throws doubts), is sensibly correct.

3. 543·2 feet per degree seems to express the observations better than 549·5, formerly given.

Let us now turn to Dr HOOKER'S observations. In his "Himalayan Journals," vol. ii., p. 456, speaking of his numerous observations on the boiling point at a

* The reason for this change in the index error, according to the two hypotheses, will be seen in the concluding paragraph of this paper.

† I still retain the doubt expressed in my former paper, as to whether the boiling point can be taken correctly to represent the temperature of steam whose elasticity is that of the atmospheric pressure at the time. This doubt is confirmed by the difference of M. REGNAULT'S and MAGNUS'S Tables of Elasticity, as also by experiments of a different kind. I take this opportunity of adding, that I have obtained true ebullition of water in an exhausted receiver at the low temperature of 46° ; the syphon-gauge then stood at 0·25 inch, being ·06 below the elasticity of vapour, at that temperature, as given by M. REGNAULT.

‡ The superabundance of heating power, and the mass of liquid in ebullition, I consider very important to the good result. No other portable apparatus that I am aware of, gives so ready means of adapting the force of the flame to the circumstances of the case. With a common spirit-lamp in fixed position below a boiler, it is next to impossible to regulate the rate of boiling, especially in an exposed situation. Mine is also the only instrument, so far as I know, which can be used in a gale of wind.

great range of elevation, he states, that having deduced the heights by my formula, he finds that it “is certainly not applicable to the Sikkim Himalaya,” but that Captain BOILEAU’S Tables, founded on M. REGNAULT’S, give, at all ordinary elevations, a “very close approach to accuracy, on the mean of many observations.” We find, however, from the results which Dr HOOKER has himself given at page 458, the average difference of the barometric and thermometric results is far from showing a close agreement, and seems to throw a doubt on the sufficiency of the observations for testing one formula rather than another.

The *average* error (without respect to sign) of each observation is about 140 feet, or $0^{\circ}26$ of a degree, whilst it occasionally rises to half a degree, or even a whole degree; and this by the formula preferred by Dr HOOKER. The *average* deviation of my observations from the formula is only $0^{\circ}08$.

Desiring to arrive at the exact truth, and to test the alleged inapplicability of a linear formula to the Indian observations, I wrote to Dr HOOKER, requesting him to send me as large a selection as possible of *direct* comparisons between the barometer and the boiling point at different heights. With this request he most kindly complied, sending me (with much trouble to himself) a long series of comparisons, which he had drawn from his original note-books. From the extent and range of these, I consider them well worthy of preservation, and accordingly transcribe them in the following Table.

Sikkim, Himalaya, Boiling Points.

STATION.	Boiling Point.		Barom. Corrected.
	S.*	L.*	
Great Rungeet River, .	210·8	...	29·211
Bhomsong,	210·2	...	28·559
Gt. Rungeet Guard-house,	208·4	207·8	27·972
Do.,	208·4	207·7	27·781
Choongtam,	202·5	202·6	24·697
Dengha,	200·6	...	23·726
Mr MULLER’S (Dorjiling),	...	199·0	23·372
Do.,	199·6	199·5	23·358
Dr CAMPBELL’S (Dorjiling),	200·1	200·1	23·369
Mr HODGSON’S (Dorjiling),	199·5	199·3	23·030
Sinchul,	197·0	...	21·892
Lachoong Village, .	196·4	...	21·928
Do.,	196·4	...	21·751
Lamteng,	196·3	...	21·654
Zemu Samdong, . .	195·9	196·1	21·605
Do.,	196·3	196·2	21·596
Do.,	196·3	196·1	21·633
Mainom,	193·4	...	20·480
Junction of Zemu & Thlonok,	192·6†	192·6	20·212
Tallum Samdong, .	191·4	191·6	19·758
Do.,	192·1	192·2	19·881
Yeumtong,	191·1	191·2	19·490
Do.	191·8	...	19·505

* S means small and L large Thermometer.

† 193·6 in my “Journals,” by error apparently; for I find 192·6 in my Note-book and Calculation-book.—J. D. H.

Sikkim Himalaya, Boiling Points—(continued.)

STATION.	Boiling Point.		Barom. Corrected.
	S.	L.	
Zemu River, . . .	190·6	190·2	19·386
Tungu, . . .	189·5	...	18·869
Do., . . .	189·5	...	18·974
Do., . . .	190·1	...	18·952
Jongri, . . .	188·8	...	18·356
Zemu River, . . .	188·5	...	18·507
Lachee-pia, . . .	185·7	...	17·267
Do., . . .	186·1	186·2	17·317
Momay Samdong, . . .	186·0	...	17·221
Do., . . .	185·9	...	17·215
Do., . . .	186·5	186·2	17·230
Palung, . . .	185·6	185·4	17·062
Do., . . .	185·3	185·2	17·091
Kongra Lama, . . .	184·1	...	16·959
Snow-bed above Yeumtong,	184·6	...	16·881
Tunkra Pass, . . .	184·1*	...	16·817
Yeumtso, . . .	183·2	183·1	16·385
Donkia, . . .	182·4	...	16·235
Mt. above Momay, . . .	181·9	...	16·106
Sebolah Pass, . . .	181·9	...	15·928
Kinchin-jhow, . . .	181·0	...	15·919
Donkia (W. flank), . . .	180·6	...	15·376
Do. (S. flank), . . .	179·9	...	15·442
Bhomtso, . . .	...	181·2	15·548
Donkia Pass, . . .	181·0	181·2	...
Do., . . .	181·6	181·3	15·489

* 164·1 by misprint in my Journals.—J. D. H.

Khasia Mountains Boiling Points.

STATION.	Boiling Point.		Barom. Corrected.
	S.	L.	
Churra, . . .	204·3	204·4	25·596
Amwee, . . .	205·1	...	25·981
Nurtiung, . . .	205·0	...	25·913
Nunklow, . . .	203·9	...	25·083
Kala Panee, . . .	202·1	...	24·492
Do., . . .	202·5	...	24·559
Do., . . .	202·0	...	24·556
Do., . . .	202·6	...	24·668
Myrung, . . .	201·9	...	24·453
Do., . . .	202·0	...	24·386
Syong, . . .	201·8	...	24·219
Do., . . .	201·9	...	24·319
Moflong, . . .	201·0	201·1	23·936
Do., . . .	201·5	...	24·009
Do., . . .	201·7	...	24·133
Do., . . .	202·0	...	...
Chillong, . . .	201·2	...	23·727

Upon carefully projecting Dr HOOKER's results in Plate III., fig. 2, in the same manner as my own, that is, by exhibiting the logarithms of the pressure (or the heights) in terms of the temperatures of boiling water, I found, in the first place, large deviations among the results, increasing also at great elevations. The breadth of the space is so considerable over which the individual observations are distributed, that it seems impossible, from the observations only, to assign any one curve as particularly indicated by them; and for the most part they are as well represented by a straight line as by any curve not absolutely sinuous. I must, however, note that below the temperature of 187° , or at heights above 13,600 feet, something like a dislocation occurs in the continuity of the observations. Dr HOOKER was aware of this circumstance, and ascribes it to "the metal of the kettle, and consequently of the thermometer, getting heated above the temperature of the boiling water." Whatever may have been the cause, this part of the series, the most important for testing a formula, can hardly be relied on for that purpose.

There is no doubt that M. REGNAULT's numbers represent, as well as any numbers can be expected to do, the main features of Dr HOOKER's observations.* But the differences between M. REGNAULT's numbers and my approximate formula are trifling, compared to the latitude of error which the projection of the observations themselves discloses. Balancing the errors as nearly as possible, the observations between 212° and 190° are well represented by a line which gives 538 feet of ascent for a fall of one degree in the boiling point, which it will be seen differs only by $\frac{1}{100}$ th part from the corrected result of my Alpine observations.† I beg to observe, that this coincidence is the more striking, because, from the method of projection used, it was *impossible* to guess at the numerical result until the interpolating line had been fixed upon.

In criticising Dr HOOKER's results, I do so with every feeling of courtesy and respect, in the same spirit, in short, in which I am sure he found it necessary to state his objection to my formula. The whole of his *barometrical* observations appear to have been made with the greatest care and fidelity, and, judging by the results, with great success. From not knowing his thermometric apparatus, I am unable to determine why these observations are of less value. I should attribute it rather to the boiler, or to the mode of using it, than to the thermometers; for Dr HOOKER speaks of a coincidence in the readings of different thermometers so exact as to be unusual. Dr HOOKER states that he finds the errors by actual calculation considerably less, if reduced by M. REGNAULT's numbers than by mine;

* A few points marked by the letter *r*, calculated from REGNAULT's formula, are inserted in the figure for the sake of comparison.

† The entire series of Dr HOOKER's observations is best represented by 548 feet for 1° , when we include the (somewhat doubtful) highest observations. This agrees almost exactly with my earlier determination.

This rigorously represents M. REGNAULT'S Tabular numbers up to a height of 20,000 feet, and probably much farther.

The distinction between the results of this formula and the one given in this paper

$$k \equiv n\mathbf{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{B})$$

(where n is 540 nearly),* is easily specified, and their practical agreement within certain limits shown, as follows :—

(A) is an equation to a parabolic arc PQ (Plate III., fig. 3), referred to a line PR parallel to a tangent at the vertex; (B) is an equation to a straight line $p q$, not necessarily passing through P (if we assume a small correction $P p$ for the standard boiling point), but which shall represent the parabolic arc PQ as nearly as may be. It may be shown that the greatest deviation of the line $p q$ from the curve at any point need not exceed *one-eighth* of the value of T^2 in formula (A). If we suppose the range of the boiling point to be 20° , (corresponding to an elevation of 11,000 feet), T^2 is 400 feet, and the greatest error of the linear formula at any height inferior to 11,000 feet is 50 feet, corresponding to *one-tenth of a degree of FAHRENHEIT*, thus confirming the results previously arrived at.

EDINBURGH, 4th December 1854.

* If we aim at representing M. REGNAULT'S Table only, between the temperatures 212° and 192°, the coefficient should be only 535. The difference in the kind of glass used in constructing thermometers would alone account for the variation. The reason of the different coefficient in the formulæ (A) and (B), arises from the different inclination of the tangent line PT, and the intersecting line $p q$, in Plate III., fig. 3, as explained in the succeeding paragraph of the text.