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On the Collapse of Boiler Flues. By A. E. H. LOVE.

Read May 11th, 1893.

Abstract.

The problem consists in discovering the conditions of a collapse of a thin cylindrical shell under external pressure, when the ends are constrained to occupy fixed positions. Since all problems of collapse depend on the geometrical possibility of finite displacements being accompanied by only infinitesimal strains, it appears at the outset that, unless the shell can receive a displacement of pure bending without stretching of the middle surface, collapse is impossible. The assumed condition of no terminal displacement is equivalent to closing the ends of the shell, and, since a closed surface cannot be bent without stretching, this condition apparently precludes the possibility of collapse. On the other hand, it is well known that, if the external pressure exceed a certain value, an infinitely long cylindrical shell of given small thickness and given diameter will collapse under the pressure. The critical pressure has been determined by Bryan, and Basset, who find the same result. It is therefore to be expected that, if the cylinder is of sufficient length, the extensional displace-

ment which must be superposed upon the displacement of pure bending in order to satisfy the end conditions will be practically unimportant except in the neighbourhood of the ends. The problem is thus reduced to discovering the order of magnitude of the length of the shell, in order that it may be treated as infinite when the thickness is small. For this purpose, consider the case where the pressure is just equal to the critical pressure, and the displacement of pure bending in the infinite cylinder is consequently of the form

$$u = 0, \quad v = \frac{1}{2}A \cos 2\phi, \quad w = A \sin 2\phi,$$

where A is a small arbitrary constant. (The displacement u is parallel to the generator, v is along the circular section, and w along the radius outwards.) By means of displacements of this form the equations of equilibrium can be satisfied, but the boundary conditions at the ends cannot. Now take the case of an infinite cylinder with an end $x = 0$, at which v and w must vanish, and seek a displacement involving both flexure and extension of the middle surface to be superposed on the displacement given by the above forms, such displacement to satisfy the equations of equilibrium, and the boundary conditions:—(1) that the new v and w are equal and opposite to those above given at $x = 0$, (2) that the new u, v, w vanish at $x = \infty$. The required solution can be determined, and is of the form

$$u = e^{-mx} (A_1 \cos mx + B_1 \sin mx) \sin 2\phi,$$

$$v = e^{-mx} (A_2 \cos mx + B_2 \sin mx) \cos 2\phi,$$

$$w = e^{-mx} \frac{m^2 d^2}{4(2+\sigma)} (B_2 \cos mx - A_2 \sin mx) \sin 2\phi,$$

in which B_2 and A_2 can be determined so as to satisfy the conditions at $x = 0$, σ is the Poisson's ratio of the material of the shell, and

$$m = [12(1-\sigma^2)]^{1/2} / \sqrt{dt},$$

t is the thickness, and d the diameter of the shell. If σ be taken equal to $\frac{1}{2}$, the reciprocal of m is about $\cdot 546$ of the mean proportional between the thickness and the diameter, and it follows that whenever x is great compared with this quantity the influence of the end is unimportant, and the displacement approximates to one of pure bending. To make the tendency to collapse occur in practice, it would be necessary that the half-length of the flue should be great compared

with m^{-1} , and the practical conclusion would be that for a flue of length l stability would be secured if

$$\frac{1}{2}l < n/m, \quad \text{or} \quad l < N\sqrt{dt},$$

where N is a considerable number. It is customary in stationary boilers to make the flues in detached pieces connected by massive flanged joints, so that the effective length of the flue is the distance between consecutive joints. If the number N be taken equal to 12, we have the rule that the distance between the joints must be not greater than twelve times the mean proportional between the thickness and the diameter. The value $N = 12$ accords well with what has been found safe in practice, but the rule as to spacing the joints is new.

On some Formulæ of Codazzi and Weingarten in relation to the application of Surfaces to each other. By Prof. CAYLEY.

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An extremely elegant theory of the application of surfaces one upon another is developed in the memoir, Codazzi, "Mémoire relatif à l'application des surfaces les unes sur les autres," *Mém. Pres. de l'Institut*, t. xxvii. (1883), No. 6, pp. 1-47; but the notation is not presented in a form which is easily comparable with that of the Gaussian notation in the theory of surfaces. I propose to reproduce the theory in the Gaussian notation.

Codazzi considers on a given surface two systems of curves depending on the parameters t, T respectively; the curves are in the memoir taken to be orthogonal to each other, but this restriction is removed in the general formula given p. 44, *Addition au Chapitre premier*. For a curve of either system, he considers the tangent, the principal normal, or normal in the osculating plane, and the binormal, or line at right angles to the osculating plane (say these are $\tan$, prn and bin). For a curve of the one system, that in which t is variable