AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

New York, April 17th, 1895.

The 96th meeting of the INSTITUTE was held at 12 West Thirty-first Street this date, and was called to order by President Houston, at 8.15 p. m. Seventy members and guests were present.

THE PRESIDENT:—Gentlemen, before proceeding with the regular course of business, it is my sad duty, as President of the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS, to announce to you the sudden death of our Treasurer, Mr. George M. Phelps. There were few, if, indeed, any, of the members of the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS who were more closely identified with the growth and work of this institution than Mr. Phelps. He was one of our charter members. He was elected a Manager in 1885, and had, since that time, been continuously a member of Council. He was elected Treasurer of the INSTITUTE in May, 1887, and had held that office continuously until the time of his death. These facts, are in themselves, evidences of the very high esteem in which our late Treasurer was held by the INSTITUTE. My acquaintance with Mr. Phelps commenced with the beginning of our INSTITUTE in 1884. I was first brought into contact with him at the International Electrical Exhibition, held in Philadelphia, under the auspices of the Franklin Institute. I learned then to respect the gentleman for his many good qualities. While I have not had that intimate friendship and acquaintance with him, which some of you have had, yet I wish to testify to the many excellent traits of character he possessed, especially to the excellent and faithful services he has rendered me, as your presiding officer, during the two years in which I have had the honor to be connected with the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS as its President.

Genius is a rare gift, and if it be true that genius consists in the possession by one individual of some trait of character, not possessed in general by mankind, then I take it that the faculty called common sense is one of the most marked attributes

of genius since it is so very uncommon. Our late Treasurer, in my judgment, possessed an uncommon amount of that very uncommon trait, common sense. This was particularly noted in the council chamber, where we were always sure, if a knotty question arose, that needed a clear understanding and calm judgment, that we would get the best of advice from our late Treasurer.

I join with you in mourning over his sudden death. Death is at best an unwelcome visitor. But nature is merciful to us at least in this respect, that when death comes with old age, she kindly tempers our departure so that when we die, we practically have been dying for many years. But when a man is struck down in the vigor of his manhood, right at the height of his usefulness, as was our late Treasurer, then death indeed becomes a sad visitor.

I will not attempt to speak of the social characteristics of Mr. Phelps or of those traits which so distinguished him as a man; for, as I have said, you know them well. He was a dignified gentleman, and if we add, of the old school, it is certainly not to his detriment. He was a delightful companion, a sympathetic friend, a faithful brother, a loving father and husband, and I think that it can be said of him that he filled the measure of a useful life, in that he so lived that the world was better and brighter and happier that he had lived in it.

I felt that it would not be proper for me to open the meeting of the INSTITUTE without making some brief reference to the death of our Treasurer, and I trust that you will pardon the very imperfect manner in which I have made it.

MR. HAMMER:—In connection with this announcement permit me to read the following communication from the Council:

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

New York, April 17th, 1895.

The Council of the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS, desiring to express its sense of the loss to the INSTITUTE in the death of George May Phelps; at its meeting held Wednesday afternoon, April 17th 1895, at the INSTITUTE headquarters appointed a special committee of three to draft suitable resolutions which the undersigned as committee herewith respectfully present.

Resolved: That in the death of George May Phelps, the Council has suffered the loss of an energetic, faithful and capable member, and the INSTITUTE a most efficient officer; one who was constantly striving to uphold the standard of its work, and one who was ever watchful of its welfare. In his disinterested efforts to serve the INSTITUTE, he brought to bear upon its deliberations, his keen perceptions, intellectual ability, and eminent force of character.

Resolved: That a copy of this resolution be suitably engrossed and framed, and presented to Mrs. George Wallace Graham, his daughter, and the sole surviving member of his family.

(Signed)

WILLIAM J. HAMMER.
JAMES HAMBLET.
RALPH W. POPE.

MR. A. E. KENNELLY:—Mr. President, I beg to move that the INSTITUTE hereby cordially and unanimously adopts the resolution which has been placed before it.

PROF. ANTHONY:—Mr. President, I second that motion.

THE PRESIDENT:—It is moved and seconded as you have heard. Those in favor of the motion, will signify their assent by rising. [The motion was unanimously carried.]

THE PRESIDENT:—The Secretary will now read the minutes of the last meeting.

The Secretary read the minutes of the ninety-fifth meeting of the INSTITUTE held March 20th, 1895, which were approved.

THE SECRETARY:—At the Council meeting held this afternoon the following associate members were elected:

Name.	Address.	Endorsed by
BALDWIN, JAS. C. T.	Superintendent Chicago Telephone Co., 203 Washington St.; residence, 1350 Washington Boulevard, Chicago, Ill.	A. V. Abbott. S. J. Larned. A. S. Hibbard.
CABOT, FRANCIS ELLIOTT	Supt. of Inspection and Electrician, Boston Board of Fire Underwriters, 55 Kilby St.; residence, East Milton, Mass.	E. J. Houston. W. J. Jenks. A. E. Kennelly.
FORTENBAUGH, S. B.	Asst. Prof. of Electrical Engineering, University of Wisconsin, Madison, Wis.	D. C. Jackson. F. R. Jones. Harris J. Ryan.
HARRIS, W. C., JR.	Electrician, Harris & Williamson, Birmingham, Ala.	Geo. H. Harris. S. R. Gross. Richard Varley, Jr.
JONES, G. H.	Agent, General Electric Co., Casilla 18 D Santiago; residence, Iquique, Chili.	J. W. Kirkland. John B. Blood. Chas. P. Steinmetz.
LEBLANC, CHARLES	Chief Engineer Railway Department, Thomson-Houston Co., 27 Rue de Londres, Paris, France.	F. J. Sprague. S. D. Greene. D. C. Jackson.
LINDSAY, WM. E.	Chief Engineer and Electrician, Swift & Co., National Stock Yards, East St. Louis, Ill.	Fred'k Bedell. Ernest Merritt. Harris J. Ryan.
NUNN, PAUL N.	Consulting Engineer, San Miguel Cons. Gold Mining Co., Telluride, Colo.	C. P. Steinmetz. A. Jay Wurts. Chas. S. Bradley.
WINSLOW, GEORGE HERBERT	Electrical Engineer, Westinghouse Elec. & Mfg. Co., Pittsburg, Pa.	A. Jay Wurts. L. B. Stillwell. Chas. F. Scott.

Total, 9.

The following associate members were transferred to full membership, their applications having been approved by the Board of Examiners, March 12th, 1895.

SCHMID, ALBERT	Superintendent, Westinghouse Electric and Mfg. Co., Pittsburg, Pa.
DODGE. OMENZO G.	Professor of Mathematics, United States Navy, Washington, D. C.
CARUS-WILSON, CHARLES	ASHLEY Professor of Electrical Engineering, McGill University, Montreal, Can.
LIGHTHIPE, JAMES A.	District Engineer, General Electric Co., San Francisco, Cal.

CHILDS, ARTHUR EDWARD	Electrical Engineer, Westinghouse Electric and Mfg. Co., Philadelphia, Pa.
WARING, JOHN	Warner Electric Co., Ovid, N. Y.
PUFFER, W. L.	Assistant Professor of Electrical Engineering, Mass. Institute of Technology, Boston, Mass.
RYAN, H. J.	Professor of Electrical Engineering, Cornell University, Ithaca, N. Y.
Total, 8.	

At the same meeting Mr. George A. Hamilton was elected Treasurer to serve for the unexpired term of the late Treasurer, George M. Phelps.

THE PRESIDENT:—The first business of the meeting is the reading of the paper by Mr. Kennelly and myself on "Resonance in Alternating Current Lines." Will Mr. Kennelly read the paper?

[The following paper was then read by Mr. Kennelly.]

RESONANCE IN ALTERNATING CURRENT LINES.

BY EDWIN J. HOUSTON AND A. E. KENNELLY.

In a paper, by one of the authors, on the subject of "Impedance," read before the INSTITUTE¹ on the 18th April, 1893, the drop of pressure in conductors carrying alternating currents and assumed to possess no capacity, was considered. In the present paper, it is proposed to consider the conditions of current and pressure in an alternating current circuit, possessing electrostatic capacity as well as inductance, resistance, and leakage.

The objects of the present paper are, therefore:

(1) To present the formula expressing the variation of electric current and pressure in an alternating current circuit under the most general conditions, and in its simplest form.

(2) To present calculations for the pressure and current in any alternating current circuit, by a graphical method.

(3) To investigate the effects of static capacity in an alternating current circuit.

(4) To investigate the effects of capacity and inductance on the delivery of energy at the distant end of a circuit.

In Fig. 1, let AB , be a length of $L = (L_1 + L_2)$ miles, or kilometres, of conducting circuit, supplied at A , with an E. M. F. of E volts, and connected to any desired apparatus at B . First assuming the line to be grounded at B , let E , be the pressure in volts, and i , the current strength in amperes entering the line at A . If the E. M. F. is alternating, the values of E and i , will be their effective values, such as are indicated by properly constructed voltmeters and ammeters. If the E. M. F. is continuous,

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the values of E and I , will be their ordinary numerical values. If the E. M. F. is alternating sinusoidally, E and I , will possess not only numerical magnitudes, but also direction in a plane. Let e , be the E. M. F. in volts, and i , the current strength in amperes at B, the receiving end of the line. When B, is grounded, we must have $e = 0$, and when B is insulated, $i = 0$. At any intermediate position C, in the line, distant L_1 , miles or kilometres from A, and $L_2 = L - L_1$ from B, let the E. M. F. be e , and the current strength i .

In any circuit, whether alternating or continuous, we assume that the pressure and current are known at one end of the line at least; that is to say, we know either E and I , or e and i . It is, therefore, required to determine e , and i , at all intermediate distances under the given conditions.

FIG. 1.

Under ordinary conditions we have to supply at the distant end of the circuit, a steady pressure of e volts, to a transformer, motor, or other translating device, and, when the load is known, i , is usually known.

In practice, lines are not usually grounded, a metallic circuit being employed. It is probably simpler, however, to discuss the phenomena as they occur on ground-return circuits, and then to bring metallic circuits within the treatment, by regarding them as composed of two parallel, grounded circuits, with the ground connection at the center of the generating E. M. F. at A, and at the center of the translating device at B. Each line will then have half the E. M. F. and half the impedance of the transmitter and receiver. (See Appendix III).

Regarding now the line A B, as being perfectly insulated be-

tween A and B, and as operated by a continuous E. M. F., we know that the pressure at any point is equal to the terminal pressure, with the addition or subtraction of the drop, according to the terminal considered, or

$$e = E - I L_1 r = e + i L_2 r \quad \text{volts} \quad (1)$$

and

$$i = I = i \quad \text{amperes} \quad (2)$$

where r , is the resistance of the conductor in ohms per mile or kilometre. The above simple relations between pressure, current, and distance, in a continuous current circuit, undergo comparatively simple variations in an alternating current circuit. Suppose an alternating E. M. F., E , of the sinusoidal type, and of angular velocity $\omega = 2 \pi n$, n being the frequency, be employed on the line at A, with a current I , which may or may not be in phase with E . The line may possess an inductance of l henrys per mile or kilometre, a capacity of c farads per mile or kilometre, and a leakage conductance (the reciprocal of insulation) of g mhos per mile or kilometre. Then formula (1) becomes, in the case where all these quantities are present.

$$e = E \cosh L_1 z - I y \sinh L_1 z = e \cosh L_2 z + i y \sinh L_2 z. \quad (3)$$

$$i = I \cosh L_1 z - \frac{E}{y} \sinh L_1 z = i \cosh L_2 z + \frac{e}{y} \sinh L_2 z. \quad (4)$$

It will be seen that formula (3) differs from formula (1) in that the pressures E and e , are multiplied by the hyperbolic cosine of the quantity $L_1 z$, or $L_2 z$, according to the end of the line considered, while the current strengths I and i , are multiplied by the hyperbolic sines of the same quantity, and also by the quantity y .

In other words, the pressure at any point on the line, instead of being equal to the pressure at some other point of reference, plus or minus the drop between them, is equal to that reference pressure multiplied by the hyperbolic cosine of a function of the intervening distance, plus or minus a drop $I y \sinh Lz$, which is also a function of the intervening distance, and these two quantities are not added numerically but geometrically.

Formula (4) is a more complex form of formula (2).

The quantities y and z , in formulas (3) and (4), depend on the impedances of the conductor and of the dielectric. The impedance of the conductor may be graphically represented as follows: Let the line r (Fig. 2), represent to scale, the resistance of the conductor per mile (or kilometre), and let $l \omega$, be the conductor

reactance per mile, or the product of the angular velocity of the E.M.F. and the inductance in henrys per mile. Then the vector sum will be $(r + j l \omega)$ where $j = \sqrt{-1}$, will be the conductor impedance, shown by the dotted line C. I., and hereafter contracted C. I., expressed in ohms per mile at the angle α .

The impedance per mile of the dielectric has next to be found. Here we have a capacity in parallel with a leakage. The joint admittance of these quantities will be the sum of their separate admittances $g + j c \omega$, Fig. 3. The length of the line g , representing the leakage conductance per mile in mhos, and $c \omega$, the dielectric-susceptance per mile, obtained by multiplying the capacity in farads per mile by the angular velocity ω . Then the line D. A. will

FIG. 2.

FIG. 3.

FIG. 4.

FIG. 5.

FIG. 6.

be the dielectric-admittance, hereafter contracted D. A., in mhos per mile, at the angle β . The dielectric-impedance D. I. will be the reciprocal of this admittance at the same angle $-\beta$, but negative, or below the line, as represented in Fig. 4.

The quantity z , is the geometric mean of the conductor impedance and the dielectric admittance, or

$z = \sqrt{C.I. \times D.A.}$, in the most general case a vector at an angle $\frac{\alpha + \beta}{2}$ as shown in Fig. 5. This quantity z , will hereafter

be described as the mean C. I. D. A.; *i.e.* the geometrical mean of the conductor-impedance and the dielectric-admittance.

$y = \sqrt{C.I. \times D.I.}$, in the most general case, a vector at an

angle $\frac{\alpha - \beta}{2}$, as shown in Fig. 6. This quantity y , will hereafter be described as the M. C. D. I. (geometrical mean of conductor and dielectric impedances). When the line is very long, the apparent resistance of the line from the sending end is y , so that the current strength entering the line will be $I = \frac{E}{y}$ amperes.

To illustrate, suppose a line whose conductor has a resistance of 2 ohms per mile, and an inductance of 20 millihenrys per mile, a perfect insulation ($g = 0$), and a capacity of half a microfarad per mile ($c = 0.5 \times 10^{-6}$). Let $\omega = 2000$, corresponding to a frequency $n = 318.3$ then,

$$l\omega = 0.020 \times 2000 = 40 \text{ ohms, and } c\omega = 10^{-3} = \frac{1}{1000} \text{ mho,}$$

then the conductor impedance, c. i. = $40.05 \angle 87^\circ 08'$ ohms. The dielectric-admittance D. A. = $10^{-3} \angle 90^\circ$ mho. The dielectric-impedance D. I. = $1000 \angle 90^\circ$ ohms.

$$\text{Then } z = \text{M. C. I. D. A.} = \sqrt{0.04005 \angle 177^\circ 08'} = 0.2001 \angle 88^\circ 34'.$$

$$y = \text{M. C. D. I.} = \sqrt{40.05 \times 1000 \angle 2^\circ 52'} = 200.1 \angle 1^\circ 26' \text{ ohms.}$$

The quantities $L_1 z$ and $L_2 z$, are prolongations of z ; *i.e.*, vectors obtained by multiplying z , by L_1 and L_2 and setting off the product at the angle of z , in this case $88^\circ 34'$.

In order to determine e and i , we require to find the hyperbolic sine and cosine of the quantities $L_1 z$ and $L_2 z$. If $L_1 z$ and $L_2 z$, were mere numerical magnitudes, their hyperbolic sines and cosines could be determined from tables of those functions, but since $L_1 z$ and $L_2 z$, are vectors whose angle is $\frac{\alpha + \beta}{2}$, the hyperbolic sines and cosines are vectors and are tedious to compute. This labor can be avoided by the use of Plate I, which will give by inspection¹, the value of any hyperbolic sine and cosine within a certain range of magnitude, sufficient for most practical purposes.

When the plate is employed to determine hyperbolic cosines, it is held so that the letters R, appear in their correct position to the observer. To determine hyperbolic sines, the plate is held at right angles to this position.

If we have a point p , Fig. 7, situated in a plane, and we know its rectangular coordinates x and y , we may readily determine its polar coordinates r and θ , by calculation. Thus, if $x = 1$

1. Plate I, on an enlarged scale, may be obtained from the Secretary.

and $y = 1$, $r = 1.414$ and $\theta = 45^\circ$. We may either transform the coordinates by computation in each case, or we may carry out the process graphically. This could be done by drawing lines parallel to x and y at suitable distances apart, intersecting everywhere at right angles, and marking off from the point $x = 0$, $y = 0$, as center, a series of concentric circles, and radii intersecting all these circles at successive angular distances at right angles. We could then find directly from the diagram, by inspection, the polar coordinates of any point whose rectangular coordinates are given.

Plate I is intended for the purpose of giving by inspection the polar coordinates of a vector point, representing the hyper-

FIG 7.—Determination of the Hyperbolic Cosine of the Plane Vector whose coordinates are $X = 0.2$ and $Y = 0.7$ —Result $0.5 / 21^\circ 15'$.

bolic sine or cosine of a given vector, whose rectangular coordinates x and y are known. A series of ellipses represent values of x , and a series of hyperbolas, everywhere intersecting the ellipses at right angles, represent the values of y . At the intersection of any ellipse value of x , with the hyperbola value of y , will be the radius and angle, *i. e.* the polar coordinates, of the hyperbolic sine or cosine, according to the direction in which the plate is held by the observer.

If, for example, we take the coordinates $x = 0.2$ and $y = 0.7$, and enter the plate for hyperbolic cosines, we will find, as in Fig 7, that starting from the horizontal line 00 , or the

major axis of the ellipses at the right hand extremity, and counting γ , counter-clockwise, we find a hyperbola $E R F$, Fig. 7. This hyperbola is marked $\gamma = 0.7$. We then find the ellipse $x = 0.2$, $C R D$, and, at the intersection of these two lines will be the point whose polar coordinates give the hyperbolic cosine of the vector sought; namely $R = 0.5$, $/21^\circ 15'$, for R , lies on the circle of radius 0.5, at an angle, counted counter-clock-wise from $o D$, of approximately $21^\circ 15'$.

Turning Plate I through 90° , and again starting in this position, from the horizontal line 00 , or the line of minor axes of the ellipses, we enter the plate for the hyperbolic sines of the same vector whose coordinates are $x = 0.2$ and $\gamma = 0.7$. We travel counter-clockwise from the right over the same ellipse as previously employed, and, at its intersection R , Fig. 8, with the hyperbola $C R D$, of $\gamma = 0.7$, we have the point whose polar coordinates give the hyperbolic sine required. This point lies between the circles $R = 0.9$ and $R = 0.95$, but nearer to the former and approximately at a radius $R = 0.92$ and an angle of approximately $84^\circ 15'$.

It will be observed that the ellipses are carried by successive steps of 0.05, as far as $x = 1.25$, so that the diagram is limited in the range of abscissas to this amount. Since, however, counting counter-clockwise, γ passes from 0, at the right hand side to 1, at the vertical, 2 at the left hand side, 3 at the lower vertical, and 4 at the starting point on the right hand, and so on, 1 to every 90° , or 4 per revolution around the diagram, any value of γ , can be accommodated.

The diagram, therefore, becomes practically, to a moderate degree of approximation, a graphic table of hyperbolic sines and cosines of any vector quantity contained between the rectangular coordinates $x = 0$, $\gamma = 0$ and $x = 1.25$ $\gamma = \infty$. This range is sufficient for most problems that present themselves. Formulae for computing hyperbolic sines and cosines beyond this range are given in Appendix I.

In order, however, to obtain this extended range of γ , by repeating its value in successive cycles of 4, it has been necessary to alter the numerical values of the ordinates γ . Since the natural period of revolution is 2π , or 6.2832, corresponding to $\gamma = 4$ in the diagram, each unit ordinate of the diagram corresponds to $\frac{2\pi}{4}$ or 1.5708. In entering the plate, the ordinate of the vector $L_1 z$.

FIG. 8.—Determination of the Hyperbolic sine of the Plane Vector whose coordinates are $X = 0.2$ and $Y = 0.7$. Result $0.92 / 84^{\circ}15'$

or $L_2 z$ must be reduced in the ratio of 1 : 1.5708 or be multiplied by 0.6366. Thus, if $L_1 z$ happens to be $1.118 / 79^\circ 42'$, we should find, as in Fig. 9, that $x = 0.2$ and $y = 1.1$. Multiplying y , by $\frac{2}{\pi}$ or 0.6366 we have (Fig. 9), $y = 0.7$ and the hyperbolic sine and cosine of $x = 0.2$ and $y = 0.7$ are already given in Figs. 7 and 8.

Plate II. affords a graphical means of accomplishing this reduction of the y coordinate by inspection. If, for example, we look for $y = 1.1$ along the axis of abscissas, we find $y = 0.7$ as the corresponding ordinate. So that having found the abscissa x and ordinate y , corresponding to a given vector $L_2 z$, we find from Plate II. y , the transformed value of y , and enter Plate I. with x and y .

FIG. 9—Showing Analysis of $L_2 z$ into Rectangular components to

$$Y = \frac{2y}{\pi}.$$

FIG. 10—Diagram of pressure commencing at insulated distant end $e = 1000 / 0^\circ$

Returning to the general equations (3) and (4), if the distant end of the line B, be insulated, the current at B, is zero, or $i = 0$, and, therefore, $e = e \cosh L_2 z$, and $E = e \cosh L_2 z$. We only require therefore, to multiply the terminal E. M. F. at B, Fig. 1, by the hyperbolic cosine of the quantity $L_2 z$, to determine the pressure at any point along the line under these conditions.

Plate III., at A, represents a line 100 miles in length, for which the quantities r , l , c , g , ω , z and y , have already been determined above. If this line be considered as insulated at the distant end, on the left hand side of the plate, at 0 miles, and that a pressure of 1000 volts effective is maintained there by suitable adjustment of the sinusoidal E. M. F., generated at the sending end on the right of the diagram, 100 miles away, the pressure falls rap-

PLATE II. for transformation of y coordinates before entering Plate I.

idly along the line marked *volts*, until it reaches a minimum of about 50 volts, eight miles from the receiving end. That is to say, if the line were only eight miles long, and a sinusoidal generator, of angular velocity 2000 radians per second, were inserted at this point, it would only be necessary to maintain a pressure of about 50 volts effective at the generator terminals, in order to maintain the pressure of 1000 volts effective at the insulated receiving end. At a distance of about 16 miles from the receiving end, the pressure again rises to 1000 volts, and continues to rise and fall rhythmically in stationary waves, the length of which is approximately 16 miles. It will be observed that the amplitude of the waves diminishes toward the sending end, while the average value of the pressure increases, so that if the length of the line were made sufficiently great, the pressure would rise very considerably at the sending end, and the waves or fluctuations in the pressure would practically disappear. In other words, $\cosh L_2 z$, and $\sinh L_2 z$, approach more and more nearly circular paths, since the ellipses of x , show less and less ellipticity as their numerical value increases.

Similarly by formula (4), for an insulated line, z , being equal to 0, we have $e = \frac{e}{y} \sinh L_2 z$, and the curve of *amperes* at A.

Plate III, represents the corresponding series of currents for this line. This current commences at 0, at the receiving end, where the line is open, and in the next eight miles increases to five amperes, falling again in the next eight miles to about 0.5 ampere, the wave length of the stationary waves, being also approximately 16 miles, and the current strength increases, steadily rising towards the sending end, the amplitude of the fluctuations diminishing.

The maxima, or crests of the current waves, coincide with the minima, or troughs, of the pressure waves, and *vice versa*. Moreover, the pressure and current coincide in phase at each pair of maxima and minima, so that the phase of the current and pressure coincide four times in each wave length.

Following the line of volts from 0, at the receiving end of the line, and taking the pressure of 1,000 volts there received as the standard of phase, the pressure advances steadily in phase as we approach the sending end. At eight miles from the receiving end, the phase is 90° or a quarter cycle, in advance; at 16 miles 180° ; at 32 miles, 360° , or one complete cycle; at 94 miles, three

complete cycles. In other words, if we consider the pressure at the sending end (100 miles) as at standard phase, the pressure at the receiving end will lag more than three complete cycles. Similarly, the current along the line of amperes will at 0 mile be of strength 0, but $\frac{1}{4}$ cycle ahead of the pressure, and will steadily advance in phase towards the sending end, and at 100 miles the current will be nearly $3\frac{1}{4}$ cycles ahead of the current at the receiving end.

Fig. 10 represents the phase variation of the pressure in the first 40 miles; o A, represents 1,000 volts, the received pressure at the insulated distant end, at standard phase; o B, represents 50 volts $/90^\circ$, the pressure about eight miles along the line; o C, 1000 volts $/180^\circ$ or negative, the pressure at about 16 miles; o D, 150 volts $/270^\circ$ or $150 \setminus 90^\circ$ at about 24 miles; o E, 1,000 volts $/0^\circ$ at about 32 miles; o F, 220 volts $/90^\circ$, at about 40 miles.

From this diagram it is evident that a complete wave of pressure, having reference both to phase and amplitude, is about 32 miles in length, but in relation to amplitude only, is about 16 miles. Similar reasoning applies to the current strengths, and it is evident that at points along the line differing in distance by 16 miles, the current is in opposite directions at any and every instant.

Plate III, shows at B, the conditions of current and pressure existing along the line when the distant end is grounded at 0 miles, and a current of one ampere is caused to flow to ground. In this case the pressure is zero at the distant end, and formula (3) gives

$$e = i y \sinh L_2 z; i = i \cosh L_2 z, \text{ with } i = 1 / 0^\circ \text{ amperes.}$$

The curve *amperes* commences at 1, and at eight miles falls to about 0.05 ampere $/90^\circ$; the pressure at the same distance being about 200 volts $/90^\circ$, so that the pressure and current are in phase. They then fall out of phase, until at 16 miles, when they come into phase again at values

$$i = 1.0 / 180^\circ \text{ ampere approximately.}$$

$$e = 20 / 180^\circ \text{ volts approximately.}$$

The same general conditions are maintained throughout the line along the B, curves; namely, the pressure and current come into step every eight miles. The phase advances 90° for each eight miles toward the sending end. The wave length, in reference to amplitude alone, 16 miles, and in reference to both

PLATE III.

amplitude and phase 32 miles. The pressure and current, on the average, rise towards the sending end, while the fluctuations diminish in amplitude, or are damped out.

At 94.5 miles the current strength is $1.12 / 0^\circ$ amperes.

At 94.5 miles the pressure is $100 / 0^\circ$ volts,

making the apparent resistance of the line at this point approximately 90 ohms. The actual resistance of the line is, from the data assumed, $2 \times 94.5 = 189$ ohms, or, roughly, twice as much, so that the current strength delivered to the line is about twice as great as that which would be delivered by a continuous-current pressure of the same amount. The current delivered at the distant end (1 ampere) is nearly double that which the line would deliver if operated by the same continuous current pressure.

Plate III, shows, at c, the conditions which are maintained between the pressure and current when a current of one ampere is delivered at the distant end of the line, under a pressure of 1000 volts, representing a resistance of 1000 ohms, such as a number of incandescent lamps in series to ground at the distant end, and a total activity is delivered of 1000 watts. In this case formulas (3) and (4) become

$$e = e \cosh L_2 z + i y \sinh L_2 z \text{ and } i = i \cosh L_2 z + \frac{e}{y} \sinh L_2 z.$$

FIG. 11. — Vector
Sum of Current
Strengths.

The *volt* curve is here the vector sum of the two volt curves in A and B. The *ampere* curve is similarly the vector sum of the two ampere curves in A and B. In other words, if a certain current strength is necessary at any point in the line as at A, Plate III, to maintain a pressure of 1000 volts, with no current, while another current strength is necessary at the same point, as at B, Plate III., to maintain one ampere of delivery with no pressure; then the sum of these two currents, at c, or $A + B$, will deliver one ampere and also maintain 1000 volts at the distant end.

For example, Fig. 11 represents by the line O A, the current strength ($3.35 / 79^\circ 30'$ amperes) in the wire at a distance of 35 miles from the receiving end, as at A, when 1000 volts and 0 amperes are delivered. The line A B, similarly represents the current strength ($0.79 / 8^\circ 10'$ amperes), in the wire as at B, when

0 volts and 1 ampere are delivered. The vector sum o b ($3.70 / 68^\circ$ amperes, represents the current strength at this point when 1000 volts and 1 ampere are delivered, as at c.

The general conditions c, being the sum of those at a and b, must be similar to those already traced, namely, the current and pressure are in phase at 0, 8, 16, 24, 32, 40, 48, 56, 64, 72, 79, 86 and 94 miles approximately. They both rise in value, on the average, towards the sending end, with fluctuations diminishing in amplitude, the phase advances by 90° , every eight miles (nearly). At 94 miles, the pressure is 1220 volts $/0^\circ$ and the current $3.6 / 0^\circ$ amperes.

The apparent resistance of the line at this point is, therefore, 339 ohms, and the activity $1220 \times 3.6 = 4392$ watts. The continuous-current pressure, which would have been required at this point, to supply one ampere at the distant end, would be 1188 volts, and the activity 1188 watts. We, therefore, require nearly four times as much activity in this line at a frequency of $318.3 \sim$ to yield the same power at 0 miles, as is required from a continuous source of E.M.F.

It is to be observed that while the pressure and current strength fluctuate in value as we pass along the line, the activity in the line does not fluctuate, but increases from 1000 watts at 0 miles, to 4392 watts at 94 miles. The rise is not uniform, being greater at the antinodes of current, or at the crests of the current waves, since the $i^2 r$ expenditure is greatest at those points; but the activity never ceases to rise as we advance toward the generator; for, it is evident that the activity in the line at any point, expressed by the co-directed product of pressure and current there existing, must, under the conditions here assumed, be equal to the terminal activity of 1000 watts, plus the total $i^2 r$ activity spent in the line up to the point considered.

The conditions of the line considered in Plate III. are not such as occur in practice. The frequency of $318.3 \sim$ is only known in telephony, being six or eight times as great as would be employed for long-distance power transmission. The capacity of 0.5 microfarad per mile would only be obtained in large buried cables, and the inductance of 20 millihenrys per mile is several times greater than usually occurs.

Nevertheless, we have assumed these extreme conditions, partly for the purpose of obtaining conditions favorable to the devel-

opment of resonance in the line, and partly to disprove the claim that has been made as to the economy offered by resonant lines for the transmission of power. It is evident that if a conductor, intentionally loaded with inductance and capacity so as to be resonant to the frequency employed, offered economical advantages for the transmission of power, there would, probably, be no great difficulty in artificially loading and adjusting the line resonance. For this reason the investigation of purely hypothetical conditions is not devoid of practical interest.

It is clear, from an examination of the conditions at B, as already described, that the resonance of the line enables a greater current strength to be supplied through the grounded distant end than would otherwise be possible. In other words, by heavily loading the line of 2 ohms per mile, with inductance and capacity until its inductance-reactance is 40 ohms per mile, and its capacity-reactance 1,000 ohms per mile, we can deliver, roughly, twice as much current to ground through the distant end, over any length between eight and 24 miles, as would be possible with an unloaded line. Under these circumstances, however, the current delivers no energy. In order that it may deliver energy, the pressure at the distant end must be raised. When this is done, as at C, the advantage, at first gained, by increasing the possible current strength, is soon more than lost by the additional expenditure of power in the line.

In any alternating-current circuit, if there is no loss of energy, by leakage and hysteresis, in the dielectric surrounding the conductor, that is to say, if the line is aerial, and is perfectly insulated, the activity delivered to the line at the sending end must be equal to the power delivered at the receiving end, plus that wasted in the line as $i^2 r$. Any method, therefore, of supplying a given number of volts and amperes at the distant end, which entails variation from the mean or average value of the current strength, *i.e.* any method which induces fluctuations in the current strength, must unduly increase the total expenditure of $i^2 r$, and entail either a greater loss in transmission, or a greater expenditure in copper, except where the current to be delivered is great, and its delivery pressure small, conditions opposed to long distance transmission. In the case represented at C, Plate III, the activity absorbed by the line is 4,392 watts at 94 miles, of which only 1,000 watts are delivered at the distant end, making a loss of 3,392 watts in the line, or an average loss of 36.1

watts per mile, due entirely to $i^2 r$, and such as would be produced by a steady current of 4.25 amperes, the actual current fluctuating between 1 and 5.95 amperes. Continuous-current transmission, or alternating-current transmission at a low frequency, over the unloaded line, would have supplied 1,000 watts at a steady current of 1 ampere, and with a loss of only 188 watts.

Were it possible to make an alternating-current transformer, or an alternating-current motor, resonant; *i.e.*, if it were possible to tune sympathetically into resonance, not only the line, but also the energy translating device at the distant end, it might be possible to derive an advantage from the use of a loaded circuit, in spite of its greater $i^2 r$ loss. But an examination of the conditions of resonance shows, that where resonance exists, very little work is being done. A highly resonant condenser has its pressure and current in quadrature, and, generally, the mere existence of resonance implies an elastic reflection of energy with the minimum absorption. In other words, the conditions requisite for the maximum delivery of work to such a device, are inconsistent with the conditions requisite for its resonance.

While, therefore, in the case of a telephone circuit, it is possible that the current strength through the distant telephone, may, at some particular frequency, or range of frequencies, be increased, by loading the line, since the amount of work done in a telephone is small, yet when power has to be transmitted over a circuit on a large scale, it is wasteful to load the circuit.

And now as to the conditions favorable to the existence of marked resonance in an alternating-current circuit. When an alternating E. M. F., of E volts, is connected to a condenser through an inductant resistance of R ohms, the current strength in the circuit will attain the maximum value of $\frac{E}{R}$ amperes, when the

inductance-reactance $l \omega$, is equal to the capacity-reactance $1/c\omega$. Similarly, a conducting line tends to vibrate electrically in sections such that their inductance-reactance and capacity-reactance are equal. The complete wave-length will be $2 \pi \times$ the length of conductor in which these reactances are equal. With reference to amplitude only, the wave-length will be $\pi \times$ length of equal reactances, and the length in which the maximum amplitude of vibration is attained, $\frac{\pi}{2} \times$ length of equal reactances. For ex-

ample, with the line considered in Plate III, the inductance-reactance is 40 ohms per mile, and the capacity-reactance 1,000 ohms per mile. In five miles, the inductance-reactance and capacity-reactance will both amount to 200 ohms, and the complete wave-length will, therefore, be $2\pi \times 5 = 31.42$ miles. As the resistance of the conductor becomes more prominent, relatively to the reactances, not only will the wave-length be interfered with, but the vibrations themselves will be damped and obscured. So that, if a line had very small resistance, and the reactances of short lengths of the line were equal and great, large and frequent vibrations would be set up with very little damping towards the sending end; or in other words, marked resonance would be developed.

Evidently, therefore, the controlling factor in the development of resonance is the vector z , or the geometrical mean c. i. d. A. If the length of z , is small, and its angle large, the vibrations will be large, but of small wave-length. If its angle be small, and its length great, the resonant vibrations will either be small or entirely absent.

When the angle of z is 45° , the maximum possible dip in pressure						= 0 %	or maximum possible rise in pressure			0 %.
"	"	"	"	50° ,	"	"	2.5 %	"	"	2.6 %.
"	"	"	"	60° ,	"	"	23.4 %	"	"	30 %.
"	"	"	"	70° ,	"	"	45 %	"	"	82 %.
"	"	"	"	80° ,	"	"	63 %	"	"	270 %.
"	"	"	"	85° ,	"	"	87 %	"	"	670 %.

Therefore, no fluctuating rise in pressure in an alternating-current circuit is possible, unless the angle of the mean c. i. d. A. is over 45° , and marked resonance is not to be expected until this angle is over 60° . The most favorable condition for development of resonant effects is when the line is either insulated at the distant end, or is grounded through a large impedance, the condition represented at A, Plate III., z , having in that instance an angle of $88^\circ 34'$, and the rise in pressure being about 1900 per cent. or the dip 95 per cent., at 8 miles.

When a well insulated conductor has negligible dielectric hysteresis, the angle of the mean c. i. d. A. must be over 45° , and, consequently, when the distant end is insulated, the pressure must rise along any such alternating-current circuit, unless the line is long, and, in case of marked resonance, it will rise and fall rhythmically. In practice, however, this rise must be very small.

For example, consider a uniphase circuit of two No. 8 A. W. G.

copper wires (diameter 0.325" or 0.825 cm.) supported on poles at an interaxial distance of two feet. The inductance of each wire in this metallic circuit will be 1.69 millihenrys per mile (3.38 millihenrys per double mile or loop-mile), and the capacity of each wire 0.0179 microfarad per mile (0.00895 microfarad per loop-mile). At the high commercial frequency of $133 \sim$ or $\omega = 835.6$, the inductance-reactance will be 1.412 ohms per mile, and the capacity-reactance 68,840 ohms per mile. The length of line in which these two reactances would become equal is 217.6 miles, making the wave-length, in regard to amplitude, 683.6 miles, and in regard to both amplitude and phase, 1,367 miles. For this line z , the geometrical mean c. i. d. A., is $0.004745 / 79^\circ 53' 30''$, indicating by the preceding table, a maximum possible rise of nearly 270 per cent. If the loop were 336 miles long, the calculated rise in pressure is 250 per cent., with the distant end insulated, so that 1,000 volts at the generator would produce 3,500 volts between the insulated ends 336 miles away. In existing practice, however, a more reasonable length would be 10 miles, and over such a length, the rise of pressure would only be from 1,000 volts at the generator, to 1,001 volts at the insulated distant ends, a quite insignificant amount, and when even a small load is thrown on at the distant end, the drop in the line, due to that load, would neutralize and reverse this rise first produced.

With reference to subterranean conductors in practical use, the Ferranti mains may serve as an example. The particulars for these mains are given by Dr. Fleming in his interesting paper on "Some Effects of Alternating-Current Flow in Circuits having Capacity and Self-Induction," as follows: (see Appendix II) $r = 0.034$ ohm, $l = 0.2857$ millihenry, $c = 0.367$ microfarad, $n = 67 \sim$. The inductance-reactance is, therefore, 0.1203 ohms per mile, and the capacity-reactance, 6472 ohms per mile; so that the length in which these reactances are equal is 231.9 miles. The amplitude wave-length would, therefore, be nearly 729 miles.

Owing, however, to the very small inductance of this line, the angle of the mean c. i. d. A. is only $55^\circ 53'$, representing only a small rise of pressure under the most favorable conditions, and over such a great length as 360 miles, the resistance of the conductor would practically damp out all resonance. In a length of 11.49 miles, the maximum length tested by Dr. Fleming, the rise of pressure at no load should be nearly 0.3 per cent., or near-

ly 30 volts in 10,000. Observations 13 and 14 of Table I., on p. 391 of his paper, show this rise, although other observations show a greater rise. The readings of the voltmeters are stated to have necessarily been subject to some little inaccuracy.

Formulas (3) and (4) are capable of including the effects of dielectric hysteresis when measurements of the same have been made; for, the effect of hysteresis in a dielectric, is to permit the expenditure of energy in it, as though leakage occurred across it, and the leakage conductor g , has only to be increased until $e^2 g$, is the total wasteful activity in watts in the dielectric per mile due to hysteresis and leakage combined.

The formulas are also applicable to triphase transmission when the proper inductance and capacity of the three conducting lines are employed. See Appendix III.

Formulas (3) and (4) are in the most general form and assume simpler forms in special cases. Thus if $l = 0$ and $c = 0$, we have the case of a leaky continuous-current circuit. In this case y and z , become numerical, instead of vector, quantities. Consequently $\sinh Lz$, and $\cosh Lz$, are numerical quantities, capable of being obtained from trigonometrical tables of these functions. The pressure in this case never rises towards the distant end of the line. When the distant end of the line is insulated, the pressure falls along a curve which is a simple catenary.

When $g = 0$ and $c = 0$, we have the case of an alternating-current circuit with negligible leakage and capacity. The formulas then become

$$(3) \quad e = E - I L_1 (r + j l \omega) = e + \frac{1}{2} i L_2 (r + j l \omega).$$

$$(4) \quad i = I = i$$

Here $(r + j l \omega)$, is the impedance of the conductor per mile, and the current strength remains the same throughout the line. As an example, suppose that the line consists of a pair of No. 0 A. W. G. aerial wires, each one mile long, at an interaxial distance of 2 feet, the frequency being 140 $\sim$. If a transformer be supplied at the distant end, with a pressure of 2000 volts at standard phase, and a current strength of $50 \sqrt{10^5}$ amperes, required the pressure and current at the generator. The impedance factor for such a pair of wires is 3.08, and the angle whose secant is 3.08 is $71^\circ 3'$, so that the impedance per mile is $1.602 / 71^\circ 3'$ ohms. The pressure at the sending end on each line is, therefore, $1000 + 50 \sqrt{10^5} \times 1.602 / 71^\circ 3' = 1000 + 80.1 \sqrt{61^\circ 3'}$, and the vector sum of

these is $1041.5 / 3^{\circ} 52'$. The E. M. F., at general terminals will therefore be $2083 \text{ volts} / 3^{\circ} 52'$. The current at the generator is $50^{\circ} \sqrt{10^{\circ}}$ amperes.

Although, as we have seen, a sinusoidal E. M. F. at ordinary commercial frequencies, has very little power to produce resonant effects in ordinary transmission lines, yet, where prominent harmonics exist in the E. M. F. of the generator, we may expect, since the frequency of these harmonics is some odd multiple of the fundamental frequency, that they will be able to excite resonance when the fundamental practically fails to do so. It should be remembered, however, that in practice, the amplitude of the harmonics in an E. M. F. usually diminishes with the order, *i. e.*, the frequency of the harmonic, and, consequently, that although a high harmonic may be capable of developing powerful resonance in a line by reason of its frequency, yet by reason of its small initial amplitude the effect of such resonance on the total pressure will be necessarily reduced. It is possible, however, under special conditions, that an E. M. F., differing markedly from the sinusoidal type, may give rise to a considerable elevation in pressure at the distant end of a line, and this is an argument in favor of the sinusoidal type of E. M. F. wave in generators for long distance transmission.

We have not yet had the opportunity of verifying the rise in pressure, or resonant effects here described, in an actual alternating-current circuit. As already pointed out, a circuit in order to manifest such effects in marked degree should have considerable length, good insulation, low conductor resistance, large capacity, large inductance and a high frequency.

Through the courtesy of the Pennsylvania R. R. Co., through its Division Operator, Mr. W. F. Taylor, we were able to make some measurements of current strength and pressure over various lengths of telegraph circuits, both with continuous and alternating currents, employing with the latter a frequency of $125 \sim$. The resistance of a telegraph line is so large, relatively to its inductance and capacity, that at this frequency, the angle of the mean C. I. D. A. was only 54° , and, consequently, no marked resonant effects could be expected. There were, however, marked differences between the measurements with continuous and alternating currents. The results were in agreement with theory within the limits of uncertainty as to the magnitude of the inductance, capacity, and leakage of the lines.

The effect of inductance in an alternating-current circuit, possessing negligible capacity, is to increase the drop in the conducting lines, without sensibly increasing the energy expended in the lines by a given current strength. Consequently, for any given activity supplied at the receiving end, at a certain pressure and current, the pressure at the generating end is raised above that which would be needed if the circuit could be operated by continuous currents. There is, therefore, a certain percentage of increased capacity in the generating plant, *i. e.*, an increased pressure for the same current strength, and the cost of the generating plant, therefore, tends to increase, owing to the inductance in the line, although the expenditure of energy in the line is not sensibly increased. The effect of capacity in the circuit is opposite to that of inductance, the pressure tending to rise at light loads along the line under their combined influence. At the same time the current, which is employed in charging and discharging the line, expends some energy in the conductor as i^2r . The cost of the plant is, therefore, reduced, as the pressure at the generator is reduced, but more power is expended in the conductor. The benefit to be derived in this manner from capacity in the line is, however, limited. Beyond a certain capacity-reactance, the loss of power in the line, owing to its resonance, costs more than the saving in generator capacity.

We conclude, therefore, from the foregoing, that in an alternating-current circuit:

(1) The combined effect of capacity and inductance is to produce a tendency to resonance.

(2) The tendency to resonance is favored by high frequency, high insulation, high inductance, high capacity, and high conductance.

(3) When the line is free at the distant end, the resonant effects are more pronounced at that end than at the generator.

(4) The determining factor for the production of resonance is the angle of the vector geometric mean C. I. D. A., and rapidly increases with the same.

(5) The complete wave-length in a resonant line is 2π times the length, in which the reactances of the inductance and the capacity are equal, the maximum difference of pressure or current strength being exhibited in one-quarter of this wave-length.

(6) When resonance occurs, the current and pressure in the circuit are not in phase except at their maximum and minimum values, *i. e.*, at the crests and troughs of the waves.

(7) In resonant circuits, the lag of the pressure and current is 90° between any successive pair of maximum and minimum values.

(8) In ordinary lines, worked sinusoidally at ordinary frequencies, resonant effects are practically negligible.

(9) Prominent upper harmonics in the E. M. F. of the generator may sometimes give rise to appreciable resonant effects, although the resonance to the fundamental frequency may be inappreciable.

(10) A certain amount of capacity tends to increase the plant efficiency of the generator, and is, therefore, economical, but marked resonance in a line not only ceases to be economical, but is actually wasteful.

(11) Although, on a large scale, less energy can be delivered at the distant end of a powerfully resonant line, than on a non-resonant line, yet a greater current strength can in some cases be delivered over a powerfully resonant line.

APPENDIX I.

Useful Additional Formulas.

Vectorially, $\cosh(a + j b) = \cosh a \cos b + j \sinh a \sin b$. (5)

“ $\sinh(a + j b) = \sinh a \cos b + j \cosh a \sin b$. (6)

Numerically, $\cosh(a + j b) = \sqrt{\cosh^2 a - \sin^2 b}$. (7)

“ $\sinh(a + j b) = \sqrt{\sinh^2 a - \sin^2 b}$. (8)

The apparent resistance of a line A B, from the sending end A, when insulated at B, is $y \coth Lz$ ohms;

“ grounded at B, is $y \tanh Lz$ ohms.

When L , is great, these converge towards y ohms.

The apparent resistance of a line A B, at the receiving end B, when grounded at B, and supplied with E. M. F. at A, is $y \sinh Lz$ ohms.

Example I. Required the hyperbolic cosine of the vector $1.118 / 79^\circ 42'$ whose rectangular coordinates are $x = 0.2$ and $y = 1.1$,

$$\begin{aligned} \text{by (5) } \cosh(0.2 + j 1.1) &= \cosh 0.2 \times \cos 1.1 + j \sinh 0.2 \times \sin 1.1. \\ &= 1.0201 \times 0.4536 + j 0.2013 \times 0.8912. \\ &= 0.4627 + j 0.1794. \\ &= 0.4963 / 21^\circ 12'. \end{aligned}$$

Example II. Required the hyperbolic sine of the same vector
by (6) $\sinh(0.2 + j 1.1) = \sinh 0.2 \times \cos 1.1 + j \cosh 0.2 \times \sin 1.1$.
 $= 0.2013 \times 0.4536 + j 1.0201 \times 0.8912$.
 $= 0.09130 + j 3.909$.
 $= 0.9136 / 84^\circ 16'$.

APPENDIX II.

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APPENDIX III.

- I. In the case of a conducting line with ground-return circuit,

r	is the res. in ohms per mile or kilometre of the conductor ;
l	ind. in henrys " " " "
c	cap. to ground in farads " " "
E	effective E. M. F. at A, Fig. 1 ;
e	" " " " B, Fig. 4.
- II. In the case of a pair of conducting lines overhead, such as a uniphase aerial metallic circuit, consider each line separately, as being supplied with an effective E. M. F., E equal to half the effective E. M. F. of the generator, and an effective E. M. F., e , equal to half the effective E. M. F. e of the motor, transformer or translating device.

r	= res. in ohms per mile of one wire ;
l	= ind. in henrys " " "
c	= twice the capacity in farads per mile as measured between the wires ;

g = twice the conductance in mhos per mile measured between the wires.

When the conductors are buried, and each is surrounded with a metallic sheet, *e.g.*, a lead covered cable, c , is the capacity in farads per mile of each conductor to ground, and g , the conductance (and hysteretic coefficient) in mhos per mile of each conductor to ground.

III. In the case of three triphase conductors, consider each line as a single conductor operated to ground by $\frac{1}{\sqrt{3}}rd$, of the effective E. M. F. measured between wires at each end, with inductance and capacity as modified by the presence of the two other active lines.