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On Several Classes of Simple Groups. By G. A. MILLER.

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Let G be any primitive substitution group of degree kp , p being any prime number, and of order $g = pm$, m being prime to p . Since a primitive group cannot contain any intransitive self-conjugate sub-group, every self-conjugate sub-group of G that differs from identity must include all its substitutions of order p . It is easy to prove that these substitutions of order p generate a simple group.

If this group were compound, it would have to contain a self-conjugate sub-group whose order could not be divisible by p . This sub-group would therefore be intransitive, and hence it could not be self-conjugate in G . Hence the sub-groups of order p that would be contained in G would generate a characteristic sub-group of G which would not coincide with G . If this characteristic sub-group were compound, it would have to contain a system of largest self-conjugate sub-groups whose orders would not be divisible by p . As this is clearly impossible, we have the

THEOREM I.—*If a primitive group is of degree kp (p being any prime number) and of order mp (m being prime to p), all of its sub-groups of order p generate a simple group. When k exceeds unity this simple group must always be of a composite order.*

COROLLARY I.—*If the degree of a primitive group is divisible by 2, its order must be divisible by 4.**

COROLLARY II.—*The sub-groups of order p that are contained in any transitive group of degree p generate a simple group.†*

The quotient group of G with respect to the given simple sub-group has a 1, pn isomorphism with the group formed by all its substitutions that transform any one of its sub-groups of order p into itself, np being the number of these substitutions that are found in the simple sub-group. Hence we observe that G cannot have more than one composite factor of composition whenever k is less than 5. When $k = 1$ the given quotient group must be cyclical. A given simple group can therefore be self-conjugate in only one transitive group of degree p and of a given order. To determine all the transitive groups of degree p that contain a given simple group as a self-conjugate sub-group, it is only necessary to determine the largest sub-group of the metacyclic group which contains any one of the sub-groups of order p that is included in the given simple group, and transforms this simple group into itself.

It is known that a cyclical substitution of order p cannot occur in any primitive group that does not include the alternating group unless the degree of the group is p , $p+1$, or $p+2$.‡ The case when the degree is p is included in the preceding considerations. When

* Cf. Frobenius, *Berliner Sitzungsberichte*, 1895, p. 180.

† *Bulletin of the American Mathematical Society*, Vol. iv., 1898, p. 139.

‡ *Ibid.*, p. 143.

the degree is $p+2$ * the group must be triply transitive. We proceed to prove that all the substitutions of order p that are contained in such a group generate a simple group of composite order. If the triply transitive group which is generated by the substitutions were compound, it would contain a transitive self-conjugate sub-group whose order could not exceed $(p+1)(p+2)$, since every self-conjugate sub-group of a transitive group of degree p includes substitutions of order p .

Since the maximal sub-group of degree $p+1$ would be doubly transitive, it could not contain any intransitive self-conjugate sub-group. Hence the given transitive self-conjugate sub-group would be either of order $(p+1)(p+2)$ or of order $p+2$. In either case it would involve just $p+1$ substitutions of degree $p+2$. This is clearly impossible since the group contains substitutions of order p . Hence we have the

THEOREM II.—*If a primitive group of degree $p+2$ contains substitutions of order p , all of these substitutions generate a simple group whose order is divisible by $p(p+1)(p+2)$.*

When a primitive group of degree $p+1$ contains substitutions of order p these substitutions must generate either the entire group or a characteristic sub-group. Since this group is doubly transitive, it cannot contain an intransitive self-conjugate sub-group. If it is compound, it must therefore involve the regular group of order $p+1$ as self-conjugate sub-group. Since this regular group cannot contain any sub-group besides identity that is commutative to one of the given substitutions of order p , it must involve p conjugate sub-groups; i.e., $p+1$ must be a power of 2. Hence the

THEOREM III.—*If a primitive group of degree $p+1$ contains substitutions of order p , all of these substitutions must generate a simple group whose order is divisible by $p(p+1)$, whenever $p+1$ is not a power of 2. When $p+1$ is a power of 2 the group generated by these substitutions of order p cannot contain any self-conjugate sub-group except the regular group of order $p+1$ which contains p sub-groups of order 2 and identity.*

COROLLARY I.—*If a primitive group contains substitutions of degree and order p , it cannot have more than one composite factor of composition.*

COROLLARY II.—*If the substitutions of order p that are contained in such a primitive group do not generate the entire group, they generate a*

* In what follows we shall assume that $p > 2$.

characteristic sub-group with respect to which the entire group is isomorphic with a cyclical group whose order is a divisor of $p-1$.

Most of these results can be readily applied to the theory of abstract groups. If a group contains a maximal sub-group that does not include any self-conjugate sub-group of the entire group besides identity, and if the order of this maximal sub-group is the quotient obtained by dividing the order of the entire group by n , then it is possible to represent the entire group as a primitive substitution group of degree n .* Hence it follows from what precedes that the operators of order p which are contained in a group in which n may be equal to p or $p+2$ generate a simple group, and that such a group cannot have more than one composite factor of composition. Similar remarks can evidently be made in regard to most of the other results.

Finite Current-Sheets. By J. H. JEANS. Received May 30th, 1899. Communicated by G. T. WALKER, M.A., June 8th, 1899.

1. The theory of the currents induced in an infinite current sheet has been completely worked out by Maxwell (*Elect. and Mag.*, §§ 647-666). I have attempted to develop a similar theory for the case of a finite current-sheet.

As in the case of the infinite current-sheet, the mathematical discussion falls naturally into two parts: in the former it is necessary to determine the currents induced by an instantaneous change in the inducing field; in the latter to examine the dissipation of these currents under the action of resistance and self-induction.

In §§ 4 and 5 I have discussed these two questions for the particular case of a semi-infinite plane, but the method is quite general, as I have tried to show in § 8. § 7 contains the extension of former results to the two cases of an infinite strip with parallel edges, and an infinite plane from which such a strip has been removed. In § 6 Maxwell's theory for an infinite plane is deduced

* Dyck, *Mathematische Annalen*, Vol. xxii., 1883, p. 102.