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Transactions of the Actuarial Society of Edinburgh / Volume 3 / January 1896, pp 1 - 35
DOI: 10.1017/S204605620000032X, Published online: 22 April 2013

Link to this article: http://journals.cambridge.org/abstract_S204605620000032X

How to cite this article:

Henry R. Cockburn and Robert Murrie (1896). On the Interests of Heirs of Entail, and the Calculation of the Pecuniary Values. Transactions of the Actuarial Society of Edinburgh, 3, pp 1-35 doi:10.1017/S204605620000032X

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On the
Interests of Heirs of Entail
and the
Calculation of the Pecuniary Values

BY

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Part I. was composed by MR. COCKBURN, and Part II. by MR. COCKBURN and MR. MURRIE, and they were read before the Actuarial Society of Edinburgh on 11th December 1884 and 9th February 1888 respectively.

On the Interests of Heirs of Entail, and the Calculation of the Pecuniary Values.

PART I.

The Entail Acts in relation to the Pecuniary Interests.

ON account of the facility afforded by recent Acts of Parliament to the proprietors of Entailed Estates for disentailing, we trust that a paper dealing with the provisions of the Entail Acts which require actuarial calculations for giving effect to them, forms a suitable subject to be brought under the notice of this Society, any of whose members may be called upon to estimate the value of the pecuniary interests of the various heirs of entail. In treating the subject we beg leave to draw your attention briefly to the most important Statutes relating to Disentail, and to one or two decisions of the Courts on points affecting the actuarial calculations of the pecuniary interests of the heirs of entail. Before doing so, however, we might shortly refer to the origin of entails in this country. An entail is a deed by which the succession to heritable property is settled on a series of individuals or heirs of entail, who may be different from the ordinary legal heirs. More generally speaking, an entail or tailzie comprehends every deed by which the legal course of succession is altered and an arbitrary one substituted. Deeds of this nature have been known in most civilised nations, and their origin is doubtless due to the desire natural to man, to perpetuate his name and family in connection with his possessions, fostered by the State, for political ends which were specially suited to a bygone age. In Greece, Rome, and also in France, deeds of entail, restricting the line of succession to a special series of heirs, have existed from a very remote period, and in Great Britain they date from the time, at least, of Alfred the Great, in whose reign the first Statute relating thereto was passed. In Scotland—and that country alone is specially dealt with in the present paper—the Statute 1685, c. 22, is the basis of our Law of Entail, and its provisions regulated the mode

of constituting strict Entails by deed of conveyance containing prohibitory, irritant, and resolute clauses duly recorded in the Register of Entails. That Act declared that it should be lawful to his Majesty's subjects to tailzie their lands and estates, and to substitute heirs in their tailzies with such provisions and conditions as they should think fit, and to affect the said tailzies with irritant and resolute clauses, whereby it should not be lawful for the heirs of tailzie to sell or dispoise the said lands, contract debt, etc., or do any other deed whereby the same might be evicted from the other substitutes in tailzie, or the succession frustrated or interrupted. These clauses, sufficiently expressed, were made essential to the validity of the entail. The deed of entail required to be registered in the Register of Entails, and it was thus made accessible to all interested at any time. The law of entail in modern times having been found to be attended with serious evils, various Acts of Parliament have been passed with a view to remedy these, such as the Acts of 1848, 1853, 1868, 1875, 1878, and 1882.

Apart from these Acts, lands will be disentailed—

1. By prescriptive possession upon an adverse or fee-simple title. If any heir possess under a different title for the prescriptive period, the entail will cease by prescription.

2. By the failure of the last existing heir of tailzie entitled to take under the deed of entail, that is, when the estate devolves upon the last person called under the deed as an heir of entail.

3. By the succession of heirs-portioners—that is, when at any time before the destination is exhausted the succession opens to two or more female heirs.

Let us now consider the effect of the disentailing Statutes.

On first approaching this subject we thought it would be unnecessary to go over the whole of these Acts, and that we would only require to deal with the law as it now stands; but on reflection we came to the conclusion that a better idea would be obtained of the law, and a better understanding of Actuarial Reports on any disentail effected by means thereof, if each of the various Acts were briefly referred to.

The first and most important Act, authorising heirs in possession to disentail their estates, is the Act of 1848, commonly known as the **Rutherford Act**. It proceeds on the following preamble, viz.: "Whereas the law of entail in Scotland has been found to be attended with serious evils, both to the heirs of entail and to the community at large, it is expedient that the same be amended," and it confers on heirs of entail important powers and privileges which we shall now advert to. Under certain conditions the heir of entail may exercise the power to disentail, and the Act con-

templates and provides for several situations in which, under the said conditions, this power may be exercised, according as the deed of entail has been executed, and the heir of entail has been born, at a particular date or relative dates. Entails by this Act were divided into two classes:—

First, Entails dated before 1st August 1848, commonly called Old Entails.

The heir in possession under an old entail could disentail—

1. If born after 1st August 1848, and being of full age without any consents.

2. If born before 1st August 1848, with consent of the heir next in succession, being his heir-apparent, provided that the consenting heir be capax, born after 1st August 1848, and twenty-five years old (reduced by the 1875 Act to twenty-one years).

3. Irrespective of the date of his birth:—

(a) If he is the only heir in existence, and unmarried. (The condition as to his being unmarried was repealed by the 1875 Act.)

(b) Or if he had obtained the consents of the whole heirs, if less than three in existence at the date of their consents, and at the date of presenting the application.

(c) Or if he had obtained the consents of the three nearest heirs at the said dates entitled to succeed in their order successively immediately after the heir in possession.

(d) Or if he had obtained the consents of the heir-apparent under the entail, and the heir or heirs, not less than two, including such heir-apparent who in order successively would be heir-apparent.

It is provided by the Act in all the foregoing cases that the nearest heir consenting should be twenty-five years old (reduced to twenty-one by the 1875 Act), and not subject to any legal incapacity. The phrase heir-apparent, as used in the Entail Statutes, means the heir whose right of succession can only be defeated by his death before the heir in possession.

Second, Entails dated on or after 1st August 1848, commonly called New Entails.

The heir in possession under a new entail could disentail—

1. If born after the date of the entail, and of full age, without any consents.

2. If born before the date of the entail, with the consent of the heir next in succession, being heir-apparent, provided that the consenting heir be capax, born after the date of the entail, and twenty-five years old. (The age of the necessary consenting heir was reduced by the 1875 Act to twenty-one years.)

These were the conditions, so far as they affect the pecuniary interests of the heirs of entail, upon which an estate could be disentailed under the Act of 1848. The **Acts of 1853 and 1868** did not materially affect these interests—the former providing, however, in the case of a disentail, that the instrument of disentail might be executed before presenting the application to the Court—and the latter, that of 1868, dealing with provisions for children of the heir-apparent, to which we shall afterwards refer. The **Act of 1875** made, however, a few important alterations which deserve to be noticed. They are—

1. The nearest heir of entail may consent when twenty-one years of age, instead of twenty-five years of age as formerly.

2. In any application for authority to disentail lands held under an old entail, the consent of any of the heirs whose concurrence was required by the Act of 1848 might be competently given after the application had been presented. In the event (and this is most important) of any of the foresaid heirs, except the nearest heir for the time, declining to give, or being legally incapable of giving, his consent, the Court may dispense with such consent in terms of the following provisions:—

(a) When any of the foresaid heirs, other than the nearest heir for the time, declines to give, or is legally incapable of giving, his consent, the Court shall on the petitioner's application, and after intimation to the heir or heirs so declining, *ascertain the value in money of the expectancy or interest* in the entailed estate, with reference to such application of such heir or heirs declining or incapacitated as aforesaid.

(b) Upon such value in money being paid into bank or secured on the estate, to the satisfaction of the Court, the Court shall dispense with the consent or consents of the heir or heirs, the value of whose expectancy or interest has been ascertained as aforesaid, and shall thereupon proceed as if such consent or consents had been obtained, provided always that nothing therein contained shall render it competent to dispense with the consent of the nearest heir for the time entitled to succeed to any entailed estate sought to be disentailed.

3. The restriction in section 3 of Act 1848, that the heir of entail in possession, being the only heir of entail in existence for the time, should be unmarried when he exercises the powers conferred upon him by the section of that Act, is by this Act repealed, but this Act shall not in any way affect the provisions of section 8 of the Act of 1848. These

provisions were that if an heir under an old entail had secured by obligation in any marriage contract the descent of the estate upon the issue of the marriage, it should not be competent to the heir in possession to disentail until there should be born a child of such marriage, capable by himself or his guardian of consenting to the disentail, or until such marriage should be dissolved without such child being born, unless the trustees named in the contract shall consent to the disentail.

We shall now proceed to consider what alterations—and they were many and important—the **Act of 1882** made upon the law of entail and the powers it conferred on the heirs in possession in connection with the disentail of their estates. The Act provided—

(1) That it should be lawful for an heir of entail in possession of an entailed estate held under an entail dated on or after 1st August 1848, to disentail the estate and acquire it in fee-simple, by applying to the Court in the manner provided by the Entail Acts, if he should be the only heir of entail in existence, or if he should obtain the like consents as are required by the Act of 1848, in the case of entails dated prior to the said date—that is to say, that heirs under new entails should have now the same powers as were given by the Act of 1848 to heirs under old entails.

(2) In any application under the Entail Acts to which the consent of the heir-apparent or other nearest heir was required, and such heir or the curator *ad litem* appointed to him in terms of this Act should refuse or fail to give his consent, the Court should ascertain the value in money of the expectancy or interest in the entailed estate of such heir, with reference to such application (the application may be other than to disentail), and should direct the sum so ascertained to be paid into bank in the name of the said heir, or that proper security therefor should be given over the estate, and should thereafter dispense with the consent of the said heir, and should proceed as if such consent had been obtained, and the provisions as to valuing the interests of the remoter heirs (sect. 5 and 6 Entail Amendment (Scotland) Act 1875) should apply to the nearest as well as to other heirs, and should apply to new as well as to old entails.

(3) With regard to settlements by marriage contracts, this Act (1882), sect. 17, which, making no distinction between old and new entails, repeats the enactment in the 8th section of the Act of 1848 above mentioned, with this difference, that in this Act we have the words “entered into prior to the passing of this Act,” which would seem to imply that a marriage

settlement made after the passing of this Act would be no barrier to a disentail.

Certain powers are by the Act of 1882 conferred on creditors of heirs of entail in possession, enabling them to exercise the power of disentailing, which do not fall far short of those conferred on heirs in possession themselves, which it is not necessary for our purpose to enter upon.

We have been speaking of disentailes, but there is another subject akin to it, actuarially of much importance, namely, the powers of heirs of entail to grant provisions to wives or husbands, and younger children, which we shall now notice. We use the words "younger children" for convenience—the expression used in the Acts is "children who shall not succeed to the entailed estate."

The first Act relating to this matter, and, indeed, the only one (with the exception of the 1868 Act, which confers somewhat similar powers on the heir-apparent), was that of 1824, commonly called the **Aberdeen Act**. The subsequent Acts, in so far as they deal with the matter, provide only for cases where there had been special provisions secured by marriage settlements. Although not at first applicable to all entails, the Aberdeen Act was subsequently declared to be so by the Act of 1868, excepting in cases where the operation of the 1824 Act is specially excluded by the deed of entail. The preamble of this Act bears: "And whereas sundry entails of lands and estates in Scotland contain no powers in regard to the granting of provisions for wives and husbands and children of the proprietors thereof, and in many other entails by reason of the change in the value of money, the improved value of lands and estates in Scotland, and other causes, the powers of granting provisions to the wives, husbands, and children of the proprietors of such estates, have been entirely inadequate for these purposes." The Act provided:—

1. That it shall be lawful to every heir of entail in possession of an entailed estate, under an entail already made or hereinafter to be made, in Scotland, under the limitations and conditions after mentioned, to provide and infest his wife in a liferent provision out of his entailed lands and estates by way of annuity, provided always that such annuity should not exceed *one-third* part of the free yearly rent of the said lands and estates where the same should be let, or of the free yearly value thereof where unlet, after deducting public burdens, liferent provisions, and the yearly interest of debts and provisions, including the interest of provisions to children, and the yearly amount of other burdens of what nature soever affecting the said lands and estates, or the yearly rents or proceeds thereof, and diminish-

ing the clear yearly rent or value thereof to such heir of entail in possession, all as the same might happen to be at the death of the granter.

2. That it should be lawful for any heir-female in possession of an entailed estate to provide an annuity to her husband to the extent of *one-half* of the free yearly value of the lands as aforesaid, but if the lands were already burdened with a prior existing annuity to a wife or husband, the annuity to be granted to the husband should not exceed *one-third* part of the yearly rent.

Where there were two liferents to wives or husbands subsisting at any one time, it was not to be competent to grant a third annuity to take effect till an existing one shall fall in.

3. The provisions to children should be as follows :—

One child—One year's free rent.

Two children—Two years' free rent.

Three or more children—Three years' free rent.

Such provisions to be valid and effectual only to such child or children as should be alive at the death of the granter, except in the case of a special settlement by marriage contract. The provision granted to the child succeeding to the estate, in so far as not previously paid, should be extinguished. When provisions have been granted to children to the full extent, no further provision could be granted till and so far as the existing provisions were diminished either by the death of the child or payment of the amount.

Where the provisions granted to a wife or husband, or to a child or children, under the authority of this Act, were in excess of the proportions which have just been mentioned, the Court had power, on the petition of any of the heirs of entail, to restrict the provisions to the extent provided in the Act. It is important to remember that the provisions of this Act may be excluded by the deed of entail in terms of section 8 of the 1868 Act. On the other hand, the Aberdeen Act expressly bears that it has no power to diminish the provision of the heir, provided such heir has not exceeded his power in virtue of the deed of entail, except that he must not deprive the succeeding heir of more than two-third parts of the free yearly rent. These are in the main the various provisions which heirs of entail are empowered to make for wives, husbands, and children.

We shall now, by way of illustration of the meaning and effect of the statutory provisions as bearing on the actuarial calculations dealt with in Part II. hereof, refer briefly to the decisions of the Court of Session and the House of Lords in two cases where the value of the interests of the heirs, and the principles by which the same are to be calculated and ascertained, have been the subjects of litigation.

The first case in point of time was **De Virté v. Wilson**, 19th December 1877. The questions raised were (1) the manner in which the actuary had estimated the values of the consenting heirs, and (2) the division of the surplus which arose from the way in which these values had been ascertained.

The facts were:—

A, heir of entail and married, husband alive.

B, next heir in succession, unmarried, and disentail proceeds on the assumption that she is the heir presumptive.

C, next heir in succession after B, and unmarried.

D, next heir in succession after C, married and has a family.

In this case the principles were discussed upon which the valuation of the “expectancy or interest” (you will remember these are the words used in the Act of 1875) of the heir whose consent is dispensed with should proceed. The following are a few extracts from the note appended to the Judge’s interlocutor on this point. The actuary ascertained the values of the “life interest” of the heir in possession and the three next heirs. These values were calculated on the footing that their respective rights were equivalent to the possession of, and succession to, an annuity of the same amount as the rents of the estate. The sum thus brought out was deducted from the fee-simple value of the estate, and after allowing for burdens, etc., the remainder or surplus is divided in different proportions among the three next heirs. A large part of the surplus is assigned to the next heir, B, because she could acquire the estate in fee-simple by executing a deed of disentail, and because the estate may fall to her own issue. No share of the surplus is given to the second heir, C, because she is unmarried, and there is no possibility of issue; and the balance of the surplus is assigned to the third heir who has children. The reason given for this latter apportionment is that she represents her son or other descendant not called to disentail. The Judge remarked that he considered it should have been the expectancy or interest, and not the “consent” which should have been valued. Before the Act of 1875 the consent might be legitimately valued, as it could have been withheld at the pleasure of an heir, but that Act must be held to have made an important change, when it enacted that the consent of the second or third heirs is not necessary, provided they are paid the value of their “expectancy or interest.” This expression he conceived to mean merely, their own individual expectancy or interest, or, in other words, the value of their own chance of succession. The interest of all the heirs after the third is cut off and cannot, it was thought, be protected or taken into account in fixing the expectancy or interest of those heirs, who, for the purpose of disentailing, are alone recognised by the Statute. By recognising the children of the third heir, the interest of a fourth

heir was practically taken into account. He was of opinion that the interest of the heir in possession was not equivalent to a mere annuity, and that the surplus should not belong to the next three heirs. If there was a surplus it should belong to the heir in possession. For, as he goes on to point out, the heir in possession was not merely a liferenter but proprietor, though subject to fetters, and when, by following the statutory conditions, he got rid of the fetters he became the proprietor in fee-simple, and must as a necessary consequence obtain all the advantage arising from the change, under deduction of these payments which he had made in order to effect it. It is to be observed that two very important points are determined here—namely, (1) that it is the “expectancy or interest” of the next heirs which is to be valued, and not their “consent,” and further, that the interest of all heirs after the third cannot be taken into account as affecting the interests of the heirs disentailing. And (2) that as the interest of the heir in possession is not to be measured by his life rental, chance of having heirs of his body, power to grant provisions, etc., as was formerly the case, but as the owner of the whole value of the estate, less the value of the interests of the heirs whose consents are dispensed with, therefore the surplus, when there is one, should belong to him, and should not be divided rateably among the subsequent heirs called to disentail.

The second case, and in some respects the more important one, is that of *M'Donald v. M'Donalds* commonly called the **Dalchosnie** Disentail case, 16th January 1879.

The facts were:—

A (male), heir of entail in possession and unmarried.

B (male), next heir in succession after A, married, no children.

C (female), next heir in succession after B, unmarried.

D (female), next heir in succession after C, unmarried.

The only other heir in existence was E (female), unmarried.

As this case has already been fully reviewed in the *Journal of the Institute of Actuaries*, vol. xxiii., it will perhaps be sufficient, without entering into details, to allude only to those points which demand special notice. One of the most interesting points decided may be explained as follows. We quote the words of the Journal:—“C and D had reversionary interests in an entailed estate on the death of A and B without male issue, and A was entitled to have the estate disentailed on paying to C and D the calculated values of their interests. The latter alleged that B had serious illnesses which reduced his expectation of life greatly below the average, and the value of their reversionary interests was in consequence greatly increased. They maintained accordingly that regard should be had to these circumstances in calculating the value of their

interests. On the contrary, A maintained that the state of the health of B was not to be taken into account, but that he must be assumed to be an average life, with the expectation of life corresponding to his real age. On this point the opinion of the Judges was much divided. Lord Adam, before whom the question first came, was of opinion that the state of the health should be taken into account, and this view was acquiesced in by a majority of the Judges of the Inner House, on the assumption that B would consent to a medical examination. It turned out, however, that B would not so consent, and the Judges of the Inner House then decided that the calculations should be made on the assumption that he was an average life. This decision was reversed on appeal to the House of Lords, which decided that B's health should be taken into account, but that it was for C and D to substantiate their allegation that the expectation of life was below the average; in other words, that in the absence of any allegation and proof to the contrary, B must be considered an average life for his age, and that the *onus* of proving that he was not so lay on C and D."

Another important point decided was as follows:—The actuary's report was objected to "in respect that the actuary had estimated the value of the chance of the second and third heirs acquiring the fee-simple of the estate by surviving all the other heirs of entail, and has thereby valued their interests as expectant fee-simple interests, instead of confining the inquiry to the value of their expectant interests as heirs of entail. It is upon this principle the actuary has allowed to these heirs for this 'chance' a resulting value more than double what on a proper interpretation of the Entail Statutes, and in particular of the Entail Amendment Act, 1875, sect. 5, the said heirs are entitled to, as the value in money of the expectancy or interest in the entailed estate of such heirs; and (2) in respect that the actuary, by estimating the value of the expectancy of the fourth heir, and taking her existence, age, and probability of issue into account, has introduced a fourth heir into the calculations, contrary to the true intent and meaning of the Entail Acts."

The Lord Justice-Clerk said he had the greatest difficulty in deciding as to this latter objection, but he came to the conclusion that to take into account the chance of the fourth heir was not a legitimate element in the inquiry, and that the heir in possession is only bound to satisfy the interests of the limited number of the heirs-substitute in regard to the succession under the entail prescribed by the Act.

Lord Ormidale, in commenting on the fact that the actuary has valued the chance of the second and third heirs acquiring the fee-simple, said that he was unable to resist the conclusion that the chances or probabilities of the second and third heirs coming to be

fee-simple proprietors of the estate did not form legitimate elements in valuing their interests in the entailed estates. It must be borne in mind that it was not the value of the heirs' *consent* to the dis-entail, but the value of their expectancy or interests as heirs of entail, which fell, in terms of the Statute, to be valued or estimated.

Lord Gifford said he had come to be of opinion that the interest to be valued was the interest in the entailed estate, and not the interest in a possible fee-simple estate, which could only emerge after the entail had expired and come to an end. The possibility that the entail might expire while the estate was in the possession of one or other of the ladies, was not a right under the entail, but a right at common law, which would only arise when there was no entail in existence. The Statute did not intend that the chances of the entail failing altogether, and the value of a *spes successionis* at common law, should be estimated. The thing which was to be valued was the "expectancy or interest in the entailed estate," not the expectancy of getting a fee-simple estate as the heir *alioqui successurus* at common law. The chance of being heir at common law the ladies would still have, notwithstanding the dis-entail, only that right would be defeasible as a fee-simple succession, and Lord Gifford was inclined to think that this was what the Statute intended. To give the heirs more than the value of the entailed succession would be to give them the value of a common law, *spes successionis*, which they had not as heirs of entail, but as heirs-at-law of the granter. The petitioner was the absolute proprietor of the estate, subject only to the deed of entail. He was not, as had sometimes been contended, a mere liferenter of the estate with certain powers, or one of a series of liferenters; on the contrary, he was the fiar of the estate—strictly and properly fiar—subject only to the disabilities created by the entail. Accordingly when the law gave him the power of dis-entailing, it simply enabled him to enlarge his radical right of fee by removing the disabilities, and the whole benefit of the enlargement accrued to him and not to the consenting heirs. They only got the value of their entailed interest. It was on this principle that De Virte's case was decided, and this decision altered what it appeared was the practice or understanding of actuaries previous to its date. The actuarial view was, that after deducting the life interest of the heir in possession, and the life interests and the values of the powers of the consenting heirs, the surplus value, as it was called, should be divided rateably among the heir in possession and the consenting heirs, but this view was negatived. The whole surplus went, not to the heirs rateably, but solely to the heir in possession, and the heirs whose interests were to be valued only got the value of their interests and no more. The whole value of the estate went to the heir in posses-

sion, subject only to deduction of the value of the interests of the first, second, and third heirs of entail. In this case there was a fourth heir ; but under the Statute she was to get nothing, and it was not right to take her into account at all to enhance the shares of the second and third heirs. He thought the Statute meant to draw the line at the end of the three next heirs, and to exclude more distant heirs altogether, either as to their chance of liferent or fee.

The Court sustained both objections, and the respondents appealed to the House of Lords, maintaining that regard should have been had to their contingent chances of succeeding to the fee-simple of the estate.

The Lord Chancellor was unable to concur in the view of the Court of Session, that in valuing the interests of the second and third heirs their interests should be treated as those of liferenters only, and that the possibility should not be taken into account that when the survivor of them came into possession, the fourth heir might be dead without issue. Supposing the Act of 1875 had not been passed, what would have been the interest which the appellants would have enjoyed in specie, supposing they had refused their consent to disentail. It cannot be doubted that what the surviving appellant would have enjoyed would have been an interest equal to a liferent or equal to a fee, just according as the fourth heir was or was not dead without issue at the time. This was the amount of the interest which could have been enjoyed in specie, and this, as it appeared to him (the Lord Chancellor), was the interest that must be valued.

Lord Hatherley, after reciting the provisions as to consents to disentail given by the Rutherfurd Act and the Act of 1875, remarked : " It appears to me probable that what the Legislature was aiming at was the entailed estate only as created by the deed of entail, and the interest now in question is an interest which is beyond the entail, and indeed after the interest in the estate tail has passed into the second or third heir, and is only arrested at the decease of the third heir by the fact of the next or fourth heir or her issue not being in existence to receive it, and that the interest which the third heir may acquire and dispose of by reason of the termination of the estate tail taking place immediately on the death of the fourth heir without issue, is the original fee-simple, which has been allowed to remain fettered, so as to carry it in its fettered state to the third heir in succession ; but which has become free from the fetters after it reached the third heir, who founds her claim not as interested under the entail, but as the unfettered fiar of the estate from the failure of the entail. We have the singular result of an interest in the third heir which is to be valued under the 1875 Act, and which results from the failure not of prior but of subsequent limitations to the fourth heir which

would not be the subject of any compensation if vested in that heir herself." Lord Hatherley felt great difficulty in coming to such a conclusion. He felt the words of the Act, "expectancy or interest," very strong; and though he should, from the nature and purport of the two Acts of Parliament, be led to the conclusion that so extended a meaning ought not to be given to them, yet the words used would be sufficient to include all the interest in the estate coming to the third heir by the results of the entail and taken from her by the Act. He went on to say that "if I came to the other conclusion, I should have to do so upon a very forced construction, because although it is not the entail itself which causes the reversionary interest to fall into possession, still it is a consequence of the reversionary interest that estate tail cannot be barred, for the Court is not authorised to allow it to be barred, except upon the condition of having the whole 'interest or expectancy' of the person against whom they allow it to be barred valued, according to the position which he or she may occupy at the time, when the disentailing deed is proposed to be executed. I think this interest in the estate is part of the interest or expectancy in the entailed estate directed to be valued." Lord Blackburn, in connection with the second point, as to introducing a fourth heir into the calculation, remarked that the heir in possession of an entailed estate in Scotland was the *fiar* fettered as far as the provisions of the *tailzie* fetter him, and no more. If there was no one in existence who could force these fetters he had the absolute fee. From this it followed that if it could be ascertained that all the heirs in existence would die without issue, the last surviving of the existing heirs would have the fee. A person who was in a *tontine* had a calculable interest in his expectancy of being the last survivor. Ought the analogous expectancy of one of the three heirs of entail to be the last survivor to be valued in this case, where there were four existing heirs after the heir in possession? The actuary had in this case valued it, and found that it more than doubled the sum to be paid to the second and third heirs. On objection, the Lord Ordinary thought he had done right, but all three Judges of the Second Division were of a contrary opinion. Lord Blackburn had doubt and hesitation on this point, and if it rested solely with him he was not prepared to reverse their decision that this should not be valued. The *Rutherford Act* proceeded on the assumption that the interests of the fourth heir under the fetters of the entail were too remote to make it proper to consult him as to the propriety of disentailing. This interest of the second and third heirs, arising only after the fourth heir had died without issue, was still more remote. The words used in the amending Act of 1875 were not such as to make him believe that those who used them had the point present to their minds; but the object of the Act being to facilitate

the disentailing of estates, if the words used were such as to be capable of bearing the construction that the only thing to be valued was the expectancy under the fetters of the entail while they exist, and not the expectation of the fee that might arise after they had expired, we should put on the Act of 1875 that construction which would forward the object of the Act. Lord Blackburn was not prepared, however, to dissent from what he understood to be the opinions of both the noble Lords who heard the argument, that the words of the Act were not such as to be capable of bearing that construction. The following Order was thereafter accordingly pronounced: "That it be referred to the Court of Session in Scotland to ascertain anew the value in money, as at the date of the instrument of disentail, of the expectancy or interest of the appellants in the estates, within the meaning of the Entail Amendment (Scotland) Act 1875." At this stage of the litigation the parties came to terms, and the claim was compromised by the heir in possession paying to the second and third heirs a sum somewhat less than the value of their life-interests and expectancy to the fee-simple, the deduction from the full sum being the estimated expenses of the litigation.

These are the only two cases which we think it necessary to bring under your notice, but as there are some rather important points in the latter case, which we do not allude to, we commend the whole matter as you will find it reported in the Journal referred to, and especially the opinions of the Judges in both Houses, to your careful study.

It would appear therefore that in calculating the value of the interests of these heirs of entail whose consents are required in disentailing, the following leading elements are to be taken into account in the calculation, namely—

(1) The rental and the value of the estate, and the burdens affecting it.

(2) The ages of the heir in possession, and of the necessary consenting heirs.

(3) The state of their health; if under average, the burden of proving it to be so lying upon him who makes the allegation.

(4) Whether they are married, and the ages of their wives, and the probability of issue.

(5) The powers of the heir in possession, and the consenting heirs, to grant provisions to wives, husbands, and children.

By all or any of these circumstances it is evident that the value of an estate to an expectant heir may be affected, but as his consent may be dispensed with if refused, no value is now to be put on his consent as a voluntary act. The values of the interests of the expectant heirs being calculated on the basis above mentioned, the rest of the estate belongs wholly to the heir in possession.

PART II.

The Calculation of the Values of the Pecuniary Interests.

WE now proceed to show how the values of the various interests may be calculated ; but, before doing so, it is desirable that we should shortly refer to the leading principles on which these interests are to be valued, which, as stated, have been laid down by the Court.

In the first of the cases referred to—*De Virte v. Wilson*—it was held that it is the expectancy or interest of the substitute heirs which should be valued, and not their consent ; for consent, if refused, will be dispensed with, and the Court will order the value in money of the expectancies or interests to be ascertained, and the amount paid into bank on deposit in the name of such heirs. This case also decided that the interest of the heir in possession is not to be measured by his liferent, chance of having heirs of his body, power to grant provisions, etc., as formerly, but as the whole value of the estate, less the value of the interests of the heirs to whom compensation has to be paid.

In the second case, the *Dalchosnie* Disentail case, it was decided that all the heirs are to be assumed to be of average health, the *onus* of proving them not to be so lying with the parties who make the allegation. Also, that the existence of an heir not called to the disentail is to be taken into account, if thereby the interests of the preceding heirs are affected ; but that, as such heir is not called to the disentail, no value is to be assigned to him.

Bearing these important decisions in mind, let us proceed to explain the calculations of the interests themselves. The elements required to be taken into account are as follows :—

- (1) The rate of interest.
- (2) The table of mortality.
- (3) The tables of probability of marriage and issue.
- (4) The provisions allowed by the Aberdeen Act, or deed of entail, to wives, husbands, and children.
- (5) Improvement debts on the estate.

(1) With regard to the **Rate of Interest** to be employed in the calculations, we find Mr. W. T. Thomson, in his well-known

book *On the Pecuniary Interests of Heirs of Entail*, stating that, while he “does not wish to advance an opinion as to the value of land, present or future,” he “has little hesitation in recommending the adoption of four per cent. in ordinary cases as an equitable foundation in the first instance for all calculations connected with disentail”; but he modifies this assumption by the subsequent statement, that “if complete accordance between the value of the property and of the heirs’ interest therein is wished, a calculation could be made according to the rate of interest which the price of the estate affords in perpetuity.” It may perhaps be considered a matter of opinion which of these two principles is the correct one, but we incline to the latter, and we believe that it is the one generally adopted. In order to employ it, it is necessary that the actuary should have particulars furnished to him of the fee-simple value, and of the net rental, of the estate. The net rental is generally ascertained without difficulty, and the fee-simple value is a matter for agreement between the heir in possession and the heirs to whom he has to pay compensation; or, in default of agreement, the Court will, on the presentation of the petition for disentail, make a remit to a valuer to ascertain the value of the estate.

Even in a matter so comparatively simple as the fixing of the rate of interest to be employed in the calculations, the actuary will frequently meet with complicated points that will require careful consideration. He must not, for instance, lose sight of the fact that the heir in possession has power, without consulting the subsequent heirs, to sell the entailed estate under direction of the Court, and thereafter have the price invested in consols or other securities specified in the Act of 1882. The effect of this may be illustrated by an example.

Suppose (1) it is proposed to disentail a certain estate, of which the net rental is, say, £2500. The estate is valued for the purpose of the disentail calculations, and the valuer reports that, on account of certain amenities, or perhaps from the fact of its possessing valuable sporting capabilities not productive of rental at the time, its value is £100,000 (forty years’ purchase). One of the heirs—say B—to whom compensation has to be paid, would, if he succeeded to the estate, be entitled to disentail without paying compensation to any one. If we followed our general rule, we should value B’s interest at $2\frac{1}{2}$ per cent., being the rate of interest yielded by the net rental on the capital value of the estate. It may, however, be fairly contended by the heir in possession, that, as he has power to sell the estate and have the price invested (as prescribed by the Act), the calculations should be made at, say, $2\frac{3}{4}$ per cent., or even a higher rate of interest, the effect of which would be to make the value of B’s interest smaller than if a $2\frac{1}{2}$ per cent. rate

had been used. The difference in the rates of interest employed would have been of less importance if B's interest had been a life-reversionary interest only, in which case, if the calculations are made at $2\frac{3}{4}$ per cent., the income must be taken as £2750.¹ We may now consider the converse of the first case.

Suppose (2) that an estate produces £5500 a year, and is valued for the purposes of the disentail at the low amount of £100,000. The rate of interest shown here is $5\frac{1}{2}$ per cent., an abnormally high rate, and doubt may arise whether it is the correct rate to use in the calculations. On considering the matter, however, we shall probably find that the reason why the valuation is so low is because the income is supposed to be of a precarious nature. It may, for example, be made up largely of sporting or mining rents. If, then, the heir to be compensated has simply a reversionary life-rent in the estate, it seems equitable that his expectancy should be discounted at a high rate of interest, bearing in mind the possibility of a reduction of the rental before he comes into possession. If, on the other hand, we are dealing with the interest of an heir who would succeed to the fee-simple, it seems equitable that his expectancy also should be discounted at a high rate, because, if the income has suffered diminution before he succeeds, the capital value of the estate will no doubt be similarly affected.

We shall now consider two cases which illustrate a different aspect of the question.

Suppose (3) an estate valued at £100,000 produces a net rental of £3500, and it is desired by the heir in possession to borrow upon it a sum of £50,000 (out of which he is to pay compensation to the subsequent heirs in respect of the sum borrowed). It seems evident in this case that the rate of interest to be adopted is not the rate yielded by the estate, but the rate which is to be paid on the loan of £50,000—say $3\frac{3}{4}$ per cent. In fact, we must look upon the property to be disentailed as £50,000, producing an income of £1875 a year, because these sums are the capital and income respectively of which the substitute heirs are deprived.

It may be observed in passing that, in a case of this kind, the compensation to be paid must be calculated on the whole sum to be charged on the estate, and not on the portion of the loan taken by the heir in possession. Thus, if the heir in possession wishes to borrow such a sum as will give him £50,000 in addition to the compensation he has to pay, such compensation, which we may call *S*, must be calculated, *not* on £50,000, but on £50,000 *plus S*.

¹ Since the above was written we have been informed that there was a case in the Outer House in which the Lord Ordinary decided against this view, though it is difficult to see on what grounds the decision was arrived at. It would seem, therefore, that the heir in possession in such circumstances may consider whether it would not be more advantageous first of all to obtain the consent of the Court to a sale of the estate and an investment of the price, and thereafter proceed with the disentail.

Suppose (4) that the heir in possession in case (3), after borrowing £50,000, should desire to disentail the estate entirely. We have the capital value of the estate £100,000, net rental £3500, and deducting from these respectively the debt of £50,000, and the annual interest upon it, £1875, there remains the capital value of £50,000 and the net rental of £1625. A little consideration shows that, theoretically, we ought first to calculate at $3\frac{1}{2}$ per cent. the value of the interest of each subsequent heir in the whole estate, and then deduct from such value the value calculated at $3\frac{3}{4}$ per cent., of the subsequent heir's interest in the debt. For, suppose the extreme case of a disentail taking place immediately after the £50,000 has been borrowed: If the estate had been disentailed before being burdened, the interests would have been valued at $3\frac{1}{2}$ per cent., but, as compensation has already been paid at $3\frac{3}{4}$ per cent. on the amount of the debt, the compensation now to be paid should be the difference between these two. It will generally, however, be found to be sufficiently accurate to base the calculations on the net value of the estate, and the net income after deducting debt and annual interest; that is—

Capital value,	£50,000
Income,	£1625
Rate of Interest,	$3\frac{1}{4}$ per cent.

(2) **The Table of Mortality.**—The table almost invariably used is the well-known “Carlisle,” mainly, it is presumed, on account of the fact that it is a very satisfactory exponent of the rate of mortality found to prevail among unselected lives. The following tables may also be employed, and may be considered preferable, as they are based upon observations of a class of lives similarly circumstanced to the majority of persons dealt with in disentail calculations :—

- “Peerage Males.”—*J.I.A.* vol. xxi. 432.
- “Peerage Married Men.”—*J.I.A.* vol. xxviii. 377.
- “Peerage Females.”—*J.I.A.* vol. xxviii. 378.

(3) **Tables of Probability of Marriage and of Issue.**—The estimating of these probabilities is more difficult than those of death and survivorship, the death of a man being a certain event, while the probability of his marriage, and of his having issue, are affected by various circumstances which are to a great extent under his own control. It has, however, been shown that a law of marriage and of issue does exist, and it has been clearly enough defined for ordinary practical purposes. The actuary has now been furnished with a very considerable collection of statistics (almost wholly drawn, however, from the experience of the British Peerage), and

valuable tables have been deduced therefrom, principally by Mr. Sprague and Mr. Day, which will be found in the Journal of the Institute of Actuaries. Further information on the subject will be found in the Actuarial Reports on the Widows' Funds of the Faculty of Advocates, Writers to the Signet, Clergymen of the Church of Scotland, Schoolmasters, etc. It is not necessary that we should review these statistics in detail, but we may say that, in Mr. Sprague's papers on the subject, the following will be found, which furnish the actuary with various probabilities constantly required in disentail calculations :—

1. The probability that a bachelor of a given age will marry and have issue.—*J.I.A.* vol. xxviii. 376, also vol. xxi. 436.

2. The probability of remarriage among widowers.—*J.I.A.* vol. xxviii. 361-375.

3. The probability that a marriage which has subsisted for a given number of years will hereafter become fruitful.—*J.I.A.* vols. xxii. 117, and xxv. 160.

4. The probability that a marriage will be fruitful.—*J.I.A.* vol. xxviii. 360.

And many other useful tables, too numerous to mention here.

(4) **The Provisions which the various Heirs are entitled to make for Wives, Husbands, and younger Children.**¹—We will show here how the amount of these (not their value at the date of the disentail) are estimated, so as to prevent unduly complicating the calculation of the pure interests of the heirs themselves when these come to be dealt with. As the various provisions under the Aberdeen Act are fully discussed in Part I., we shall very briefly refer to them now. They are as follows :—

(1) To the widow, an annuity of one-third of the free rents after deducting public burdens, liferent provisions, and the yearly interest of debts and provisions or other burdens including the interest of provisions to children, all as the same may happen to be at the death of the granter.

(2) To the husband (where the estate is held by an heir female) an annuity of one-half of the free rents, but if the estate is already burdened with a prior existing annuity to a wife or husband, the annuity to be granted to the husband shall not exceed one-third of the free rents.

Where there are two liferents to wives or husbands subsisting at any one time, it is not competent to grant a third annuity to take effect till an existing one shall cease.

¹ Compare pp. 8 and 9.

(3) The maximum provisions to younger children are as follows :—

One child—One year's free rent.

Two children—Two years' free rent.

Three or more children—Three years' free rent.

Such provisions to be valid and effectual only to such child or children as may be alive at the death of the granter, except in the case of a special settlement by marriage contract, where on the death of the child the provision does not lapse, but shall remain and be effectual as if such child had survived the granter of the provision. When provisions have been granted to children to the full extent, no further provision can be granted until those existing are diminished either by the death of the child or payment of the amount.

Where the provisions to wives, husbands, and children are in excess of the proportions of the free rent which have been stated, the Court have power, on petition, to restrict them to the values set forth in the Act. It is to be remembered, however, that the provisions authorised by this Act may be expressly excluded by the Deed of Entail (1868 Act, sect. 8). Where the heir of entail grants provisions in virtue of his power under the deed of entail, he must not deprive the succeeding heir of more than two-thirds of the free yearly rent.

It will be observed that the provisions for a widow (or husband) and younger children mutually depend on one another, so that in estimating the amount of the annuity to be granted to the widow (or husband) the interest upon the younger children's provisions must be taken into account, and similarly in estimating the children's provisions, the annuity to the widow (or husband) must be taken into account. This problem has been investigated in *J.I.A.* vol. xix. p. 41, and we cannot, we think, do better than repeat the solution which is there given :—

“Suppose that a proprietor has charged his estate in favour of his widow and younger children to the full extent allowed by the Act, what interest will they actually take? Assume the net rental of the estate to be 1, and let i be the rate of interest that is payable on the portions of the younger children. Let x be the annual income which the widow will take, then $1 - x$ is the net income remaining after payment of this interest. The provisions for the younger children must not exceed three times this amount, and will therefore be $3(1 - x)$. The annual interest upon these provisions is $3i(1 - x)$, and the net rent after deducting the interest upon the younger children's provisions, but not of the widow's liferent, is $1 - 3i(1 - x)$. The widow's liferent cannot exceed one-third of this sum, so that we get the equation

$$x = \frac{1 - 3i(1 - x)}{3} \quad \therefore x = \frac{1 - 3i}{3(1 - i)}$$

The provisions for the younger children $3(1-x)$ are therefore equal to

$$\frac{2}{1-i}''$$

The alterations to be made on the above formulas, when the annuity to a husband has to be calculated, or when the provisions are applicable to a less number of children than three, are so obvious that it is unnecessary to deduce the formulas.

(5) **Improvement Debt.**—This may be either a fixed debt, in which case it will be treated in the same way as any other bond upon the estate, or it may be repayable by a terminable annuity. In the latter case we have to bear in mind the provisions in the Act of 1882 whereby such loans can be constituted permanent burdens for three-fourths of their original amount. It may frequently be assumed (if the heir in possession be not a very old man) that the debts will be so constituted during the lifetime of the heir in possession.

We are now in a position to take up and discuss a few specimens of the calculations of the interests of the various heirs under an entail, but before doing so we wish it to be clearly understood that we disclaim any attempt to lay down fixed rules for making such calculations. It will be seen that it is only in the very simplest cases that this can be done, and the actuary must decide, in each particular case that comes before him, what methods of calculation should be used.

After due consideration, we have come to the conclusion that we cannot do better than select as illustrations the cases adopted by Mr. Thomson in his book *On the Pecuniary Interests of Heirs of Entail*, modifying and supplementing the data used by him where, on account of the subsequent alteration of the law of Entail, that seems desirable. We assume in all the cases that there are a number of heirs in existence subsequent to those mentioned, so that there is no chance of an heir whose interest is to be valued becoming the last heir in existence.

The following Table may be useful as indicating at a glance the number of heirs whose interests require to be valued under the various conditions which may exist at the date of disentail:—

This Table is intended to show the heirs to whom compensation has to be paid. The additions in square brackets show the alterations by the 1875 Act, and the red ink those of the 1882 Act

An Old Entail is one dated before 1st August 1848 ; a New Entail one dated on or after 1st August 1848.

OLD ENTAILS. 1. Heir in possession, 2l, born on or after 1 Aug. 1848, without any consents.

2. Heir in possession, 2l, born before 1 Aug. 1848, with consent of heir-apparent born on or after 1 Aug. 1848 and 25 [21].

3. Heir in possession, 2l, { 1. If only existing heir and unmarried [although married].
2. With consent of (a) all existing heirs if less than three in existence

(b) three next

(c) heir-apparent, and heir or heirs in number not less than two, including such heir-apparent, who in order successively would be heir-apparent

provided that the first consenter is 25 [21]

consents subsequent to first may be valued and paid over

first consent may be valued and paid over.

NEW ENTAILS, 1. Heir in possession, 2l, born after date of entail.

2. Heir in possession, 2l, born before date of entail, with consent of heir-apparent born after and 25 [21].

3. Heir in possession, 2l, { 1. If only existing heir.

2. With consent of (a) all existing heirs if less than three

(b) three next

(c) heir-apparent, and heir or heirs in number not less than two, including such heir-apparent, who in order successively would be heir-apparent

provided first consenter is 21

all consents may be valued and paid over.

NOTE.—If first consenter be under 21, Curator may act for him (1882 Act, § 12).

CASE I.

Power to Disentail on paying Compensation to One Heir.—From the Table it will be seen that under

- (1) Old Entails, heir in possession aged 21, born before 1st August 1848, can disentail with consent of heir-apparent born on or after 1st August 1848, and 21 years of age (if the latter under 21, with consent of his *curator ad litem*);
- (2) New Entails, heir in possession aged 21, born before date of entail, may disentail with consent of heir-apparent born after, and 21 years of age (if the latter under 21, with consent of his *curator ad litem*).

The heir-apparent must be the son, grandson, or other direct descendant of the heir in possession—in other words, must occupy such relation to the heir in possession that his hope of succession cannot be defeated by the birth of a child intervening.

Take the following examples, in which we shall assume that the entail is an old one :—

CASE I.—EXAMPLE A.

A, the heir in possession, is aged 55. He is married, his wife being aged 45, and he has several children (including B, mentioned below).

B, eldest son of A, and heir-apparent, aged 25 (born after 1st August 1848).

Value of estate,	£50,000	Rental,	£2000
Entailer's debt,	£2000		
Improvement debt (permanent),	2000		
	<hr/> 4,000	Int. at 4 %/,	160
	<hr/> £46,000		<hr/> £1840
Children's provisions (created by former heir),	3,000	Int. at 4 %/,	120
	<hr/> £43,000		<hr/> £1720

The net income yielded by the estate, £1720, being thus 4 per cent. on the capital value, £43,000, we make the calculations at that rate of interest.

Aberdeen Provisions for Widow and Younger Children of A.—If the heir in possession pays off the existing children's provisions of £3000, he would then (unless restrained by the deed of entail)

be able to burden the estate with the usual Aberdeen provisions. The net income (after the £3000 has been paid off) being £1840, the provisions will be:—

$$\text{For Widow, } \frac{1-3i}{3-3i} \times £1840.$$

$$\text{For Children, } \frac{2}{1-i} \times £1840.$$

It used to be customary to take i as equal to .05, but this seems to us to be scarcely correct, as it assumes that 5 per cent. interest will have to be paid upon the children's provisions. It will be more correct to make i equal to say .04, which assumes that 4 per cent. only will be paid. At this rate of interest it is found that the children's provisions amount to £3833, and the widow's annuity to £562. As there are existing children's provisions of £3000 on the estate, the heir in possession can only charge the estate with the balance, namely, £833.

If B should succeed to the estate, he would, having been born after 1st August 1848, be able to acquire it in fee-simple, without paying compensation to any subsequent heir, but subject to the provisions for A's widow and younger children.

B's interest is therefore

$$\bar{A}_1 \times (43,000 - 833) = \text{Value of reversion on death of A, if B survives, to the estate, less provisions for A's younger children}$$

minus

$$a_1 \times 562 = \text{Value of annuity to wife of A, aged 45, after death of A (55), if B (25) survives A}$$

minus

the value of a similar annuity to a possible future wife of A.

It is, we think, not necessary to say anything as to the method of calculating the first two items, but we may show how the value of an annuity to a future wife of A may be approximately determined.

A's present wife, aged 45, may die in his lifetime, and it is necessary to allow for the chance of this and of his marrying again.

This can be readily done from the materials contained in Mr. Sprague's paper in *J.I.A.* xxviii. 350-392. The continuous formula for the general term is

$$(1 \div l_{45} l_{55}) \int v^n l_{45+n} \mu_{45+n} l_{55+n} dn (\Sigma w m_{[55+n]} \div w l_{[55+n]}).$$

If we multiply this general term by

$$d \frac{1}{55+n \cdot 25+n} | x$$

where x represents the probable age of the second wife that A would marry, if he married at age $55 + n$, we shall get an approximation to the benefit we are in search of as good as can be attained by means of the data at present available. The probable age of the second wife may be arrived at by a study of Table 3 in Mr. Chatham's paper in *J.I.A.* xxvii. 42.

The value of B's interest calculated approximately by the above formulas is £17,056.

CASE I.—EXAMPLE B.

If the heir B, whose interest we have to calculate, should be under twenty-one years of age, we would require to allow him only a liferent interest in the estate till he attained twenty-one, because he is not entitled to disentail and acquire the fee-simple till he reaches that age. Suppose, for instance, in Example A, that B had been eleven years of age (instead of twenty-five), the formula for the calculation of his interest would have been:—

1. Annuity of the net rents after deducting interest on £833 children's provisions (say £1687 net) during the next 10 years to B (11) after death of A (55)

minus

2. Annuity of £562 to commence on the death of A (55) and to continue during the joint lives of B (11) and A's wife (45) but to cease at the end of 10 years from the present time

plus

3. Reversion of B (11) 10 years hence to the fee of the estate (£43,000), less children's provisions (£833), contingent on B being then alive and A (55) dead

minus

4. Annuity of £562, to commence 10 years hence, to A's wife, (45 + 10) if B (11) is alive and A (55) dead (B would have to pay this annuity if he acquired the estate on attaining 21)

plus

5. Reversion of B (to £43,000 — £833 as before) after the next 10 years if he and A both live so long, contingent on B surviving A

minus

6. Annuity to A's wife payable by B, if he acquires the estate, at any time after attaining 21, by surviving A.

The various functions above described may be expressed symbolically as follows:—

1. $|_{10}a_{55}|_{11} \times 1687$
minus
2. $|_{10}a_{55}|_{11 \cdot 45} \times 562$
plus
3. $v^{10} \times (l_{21} \div l_{11}) \{ (l_{55} - l_{65}) \div l_{55} \} (43,000 - 833)$
minus
4. $v^{10} \times (l_{21} \div l_{11}) \{ (l_{55} - l_{65}) \div l_{55} \} (l_{55} \div l_{45}) \times a_{55} \times 562$
plus
5. $v^{10} \times (l_{21} \div l_{11}) (l_{65} \div l_{55}) \times \overline{A}_{\substack{1 \\ 65 \cdot 21}} \times 42,167$
minus
6. $v^{10} \times (l_{21} \div l_{11}) (l_{65} \div l_{55}) (l_{55} \div l_{45}) \times a_{\substack{1 \\ 65 \cdot 21 | 55}} \times 562$

To avoid complicating this case we have left out of account in the formulas the value of the provisions for a possible second wife of A.

Note.—It will be found in practice, however, that in many cases the difference between giving B the value of the fee contingent on his surviving the heir in possession and giving him only the rents until 21 and the fee thereafter, is inconsiderable.

In this case, the value of B's interest, if calculated approximately by the formulas given above, will be £17,548, while, if we assume that whenever he succeeds he will take the fee subject to Aberdeen provisions, the value comes out £18,141.

CASE I.—EXAMPLE C.

A, the heir in possession, is aged 60 ; married, his wife being aged 50 ; and he has a number of children, including B mentioned below.

B, eldest son of A, and heir-apparent to the estate ; aged 30 ; born after 1st August 1848.

Value of estate,	£125,000	Rental,	£5000
Deduct entailor's debt,	10,000	Interest 4 %,	400
	<hr/> £115,000		<hr/> 4600
Improvement debt,	3,000	Interest 4 %,	120
	<hr/> £112,000		<hr/> 4480
Children's provisions (created by former heir),	8,000	Interest 4 %,	320
	<hr/> £104,000		<hr/> £4160

The widow of a former heir enjoys an annuity of £1200. She is aged 70.

Provisions for Widow and Younger Children of A.—The widow of A will be entitled to an annuity of £800 after A's death, to be increased, after the death of the £1200 annuitant, to £1200, and A has power to burden the estate with a provision of £4000 for his younger children in addition to the children's provisions of £8000 created by a former heir. The above provisions are greater than would be allowed in terms of the Aberdeen Act, and it is therefore to be presumed that they are authorised by the deed of entail.

As in Case I., Example A, the interest of B is his chance of succeeding to the fee-simple, subject to the annuity to the lady of seventy, and to the provisions in favour of A's widow and younger children ; thus:—

1. Reversion on death of A (60), if B (30) survive him, to the estate, less provision of £4000 for A's younger children

minus

2. Value of an annuity of £1200 to the annuitant (70) after death of A (60) if B (30) survive A

minus

3. Value of an annuity of £800 to A's wife (50) after death of A (60) if B (30) survive A

minus

4. Value of further annuity of £400 to A's wife (50) after death of the survivor of A (60) and the annuitant (70) if B (30) survive A

minus

5. Value of an annuity to a possible future wife of A.

The symbolical expressions for the above functions are as follows:—

$$1. \bar{A}_{\overline{60:30}} (104,000 - 4000)$$

minus

$$2. d_{\overline{60:30}|70}^1 \times 1200$$

minus

$$3. d_{\overline{60:30}|50}^1 \times 800$$

minus

$$4. (d_{\overline{60:70}|50} \text{ provided 30 survive 60}) \times 400$$

minus

5. Value of annuity to future wife of A (see p. 26).

Omitting the fifth item, an approximate calculation of the value of B's interest comes to £46,571.

CASE II.

Power to Disentail on paying Compensation to Two Heirs.—From the Table it will be seen that under both old and new

entails an heir in possession aged twenty-one, irrespective of the date of his birth, can disentail with the consent (a) of all existing heirs, if less than three in existence. (If there were only two in existence the case we are about to consider would apply.) Or (b) with the consent of the heir-apparent and heir or heirs, in number not less than two, including such heir-apparent who in order successively would be heir-apparent. (If the heir-apparent and his eldest son were the parties to the disentail, the case we are about to consider would apply.)

Take the following example in which we assume the entail to be a new one:—

A, heir in possession, is aged 80; his wife is aged 70; and he has five children alive.

B, his eldest son (heir-apparent), is aged 50, born before the date of entail; his wife is aged 40; and he has four sons alive.

C, grandson of A, and eldest son of B, is aged 20; born after the date of entail, and not married.

Value of estate,	£75,000	Clear rental, £3000
Drainage debt, . £1000		
Children's provisions (created by former heir), . . . 8000		
	9,000	Int. at 4.°/., 360
	<u>£66,000</u>	<u>£2640</u>

Aberdeen Provisions.—These ought also to be taken into account, but as we have already indicated in cases I. (A and B) how they ought to be treated, we shall, in order to avoid repetition, leave them out of account here.

We proceed to calculate B's interest.

If B succeeds to the estate and has a son or grandson alive at the time, he will be entitled to disentail on paying one compensation. As he has four sons, it may be assumed that he will have a direct heir. An accurate calculation of B's interest, on this assumption, would have to take into consideration the various chances that the heir would be C, or any of B's other sons, or a son of one of the four. But for practical purposes it will be sufficient to fix an age which may be taken as that of the heir who will come next to B. It is more likely that the heir will be C than any one else, and, if not he, then the next eldest son of B, and so on. It would seem sufficient therefore, on the whole, to assume an age not greatly differing from that of C—say, age nineteen.

B's interest is therefore the reversion of the estate, expectant

on the death of A if B survives, burdened with the compensation which (it is assumed) B will have to pay to an heir-apparent now aged nineteen. This can scarcely conveniently be expressed in algebraic form, but its value may be calculated as follows:—The chance of B succeeding in the n^{th} year is

$$\frac{d_{80+n-1}}{l_{80}} \times \frac{l_{50+n-\frac{1}{2}}}{l_{50}};$$

and if he then succeeds, the compensation to be paid to his heir-apparent, omitting Aberdeen Provisions, will be

$$\bar{A}_{\frac{1}{50+n : 19+n}} \times 66,000.$$

The present value of B's chance of receiving a payment at the end of the n^{th} year is

$$v^n \times (d_{80+n-1} \div l_{80}) (l_{50+n-\frac{1}{2}} \div l_{50}) \\ \times (66,000 - \bar{A}_{\frac{1}{50+n : 19+n}} \times 66,000).$$

We may either calculate the value of this function for each year of age, or approximate to the sum of these values by a formula of summation.¹

C's Interest.

We have now to calculate the value of C's interest:—

C, if he succeeds, will be entitled to disentail without any consent. It may therefore be assumed that he will take the estate in fee if he survives A and B; for it is to be observed that although B may disentail, he can only do so on paying C (if alive) the full value of his interest.

C's value is therefore:—

Reversion to £66,000, expectant on the death of the survivor of A (80) and B (50), provided C (20) be then alive. The symbolical expression for which is

$$\bar{A}_{\frac{1}{80 \cdot 50 : 20}} \times 66,000.$$

Omitting, as above explained, the Aberdeen provisions, we value the interests at the following sums:—

B	£27,132
C	£26,613

¹ Since the paper was written, doubts have been expressed as to whether or not a substitute-heir, who, after succeeding to the estate, would have to pay compensation before disentailing, should be allowed the value of his reversion to the fee-simple of the estate less such compensation, or only to a reversionary life-interest therein. If the latter is decided to be the correct method of ascertaining his value, any actuary who has followed the methods of calculation we have adopted would have little difficulty in making the necessary alterations in the formulas.

CASE III.

Power to Disentail on paying Compensation to Three Heirs.—

From the Table it will be seen that under both old and new entails an heir in possession (if aged 21), irrespective of the date of his birth, can disentail with the consent of the next three heirs.

Take the following example, in which we assume the entail to be a new one:—

A, the heir in possession, is aged 50, and unmarried.

B, the first heir (not heir-apparent), aged 30, unmarried; born after the date of the entail.

C, the second heir, aged 60; married, his wife's age being 55, and he has a number of children. He was born before the date of the entail.

D, the third heir, son of C, aged 20; born after the date of the entail.

Value of the estate, £50,000.

Clear rental of the estate, £2000.

Aberdeen Provisions.—It is to be observed that it is only in the event of A, though he marries, *having no issue*, that we are concerned with the annuity for his widow, and we are not concerned at all with the provisions he could make for children, because what we have to calculate is the values of the interests of B, C and D, and these would be defeated entirely by the birth of issue to A. We have to calculate the probability of A marrying, but having no issue, and this is of course the difference between the chance of his marrying and the chance of his having issue.

Probability of marrying for bachelor of fifty, per table <i>J.I.A.</i> xxviii. 376,	·12459
Probability of marrying and having issue—same table,	·06075
Difference, being the probability of marrying but having no issue,	<u>·06384</u>

The Aberdeen annuity to a childless widow of A will be
 $\frac{1}{3}$ of £2000 = £666, 13s. 4d.

We are not concerned with the annuity to a possible widow of B, because, if B succeeds to the estate, he will take it in fee-simple, and the interests of the subsequent heirs will be entirely defeated.

We ought, strictly speaking, to take into account, in calculating the interest of D, the provisions that C could make for his wife and younger children, if he should succeed; but having regard to the age of C's wife, and the remote contingency of his succeeding by the deaths of A and B before him without leaving issue, we think we may conclude that the provisions in question need not be taken into account.

B' Interest.

If B should succeed to the estate, he would be able to acquire it in fee-simple, subject only to the annuity to A's widow, if he should leave one. The value of B's interest is thus:—

1. Reversion on death of A (50) without issue, if B (30) survives him, to £50,000, the value of the estate

minus

2. Value of an annuity of £667 to a wife of A, if he should marry but have no issue.

Or, expressed symbolically:—

1. $\bar{A}_{50:30} \times 50,000 \times \text{probability that A will have no issue (= .93925)}$

minus

2. $a_{50:30|2} \times 667 \times \text{probability that A will marry but have no issue (= .06384)}.$

(We may assume the age of A's possible wife to be, say, twenty-five years less than his own.)

In these formulas it is assumed that an outside factor is used for the probability of issue to A, but a more exact calculation could be made if we had the deaths without issue in each successive year among a certain number of bachelors living at the age of 50. The table already referred to—*J.I.A.*, vol. xxi. p. 436—can be used for this purpose, by a process which is rather too lengthy to be conveniently explained here.

The formula for $\bar{A}_{50:30}$, provided 50 leaves no issue, is

$$\sum v^{n-\frac{1}{2}} \delta_{49+n} l_{29\frac{1}{2}+n} \div (bl)_{50} l_{30},$$

where δ signifies the deaths without issue. (See formulas in Mr. Sprague's paper in *J.I.A.* vol. xxiv. page 327.)

Using the latter formula for the reversion, we value B's interest at about £17,500.

C's Interest.

The interest of C would be defeated, not only if he should die before A and B, and if either of them should have issue, but also if B should succeed to the estate by the death of A without issue; because, in that case, B would be in a position to disentail without paying compensation to C, who would thus lose all his interest in the estate. As C has a number of children, we may assume (as we did in Case II.) that, if he succeeds, he will be able

to disentail on paying compensation¹ to an heir-apparent, aged, say, 19 at present. The value of C's interest is therefore :—

1. Reversion to estate on death of A (50), if B (30) be then dead and C (60) be alive, provided A and B have no issue

minus

2. Present value of the compensation which C would have to pay to his son if the former should succeed to the estate

minus

3. Value of an annuity to a possible widow of A, which would be a burden on C's interest if he should succeed to the estate.

The above functions may be expressed symbolically as follows² :—

$$1. \left(\bar{A}_{\substack{2 \\ 50:30:60 \\ 1}} \text{ provided 50 and 30 have no issue} \right) (50,000) \quad -$$

minus

$$2. \left[\sum v^{n-1} \left\{ \frac{\delta_{49+n}(l_{30}-l_{29\frac{1}{2}+n})(l_{59\frac{1}{2}+n})}{(bl)_{50} l_{30} l_{60}} \right\} \times \left\{ \begin{array}{l} \text{Probability that 30 will} \\ \text{have no issue.} \end{array} \right. \right. \\ \left. \left. \times \bar{A}_{19+n.60+n} \frac{1}{l_{60+n}} \right\} \right] \times 50,000$$

minus

$$3. \left(\bar{a}_{\substack{2 \\ 50:30:60|x \\ 1}} \text{ provided 50 marries, and both he and 30 have no issue} \right) \times 667.$$

This brings out the value of C's interest as £134 approximately.

D's Interest.

D's interest is deferred to the death of the survivor of A, B, and C, and will be defeated entirely if A or B should have issue, or if A should die before B. If, however, D should succeed to the estate after attaining the age of 21, he will be entitled to acquire it in fee-simple. His interest is thus :—

Reversion to the estate on death of survivor of A (50), B (30), and C (60), if D (20) is then alive, and if B (30) has died before A (50), and both A (50) and B (30) have had no issue

minus

provisions for a possible childless widow of A, and for widow and younger children of C, if he should succeed to the estate.

¹ See footnote, page 31.

² δ represents the deaths without issue in the case of A (50). We have used an outside factor for the probability of B having no issue, as the deaths without issue cannot conveniently be calculated from any published tables. The probability that B will have no issue (*J.I.A.* xxviii. 376) is .50821.

Or, expressed symbolically :—

$$\left\{ \bar{A}_{\frac{(50 \cdot 30) \cdot 60}{21} : 20} \text{ provided 50 and 30 have no issue} \right\} \times 50,000$$

minus provisions as above mentioned.

As before stated, we do not think the provisions for C's widow and younger children need be taken into account, and we are further of opinion that the provision for A's widow, if he should leave one, will not greatly affect the value of D's interest. We shall therefore only calculate the value of the above reversion. Omitting, in the first instance, the question of issue to A and B, the continuous formula for the reversion would be

$$\int \left[v^t \{ l_{50+t} \mu_{50+t} (l_{30} - l_{30+t}) l_{60+t} l_{20+t} \bar{A}_{\frac{1}{60+t \cdot 20+t}} \} \div \{ l_{50} l_{30} l_{60} l_{20} \} \right] dt$$

$$+ \int \left[v^t \{ l_{50+t} \mu_{50+t} (l_{30} - l_{30+t}) (l_{60} - l_{60+t}) l_{20+t} \} \div \{ l_{50} l_{30} l_{60} l_{20} \} \right] dt$$

To give effect to the condition that A shall die without issue, we may substitute

$$\frac{1}{2} (\delta_{49+t} + \delta_{50+t}) \quad \text{for} \quad l_{50+t} \mu_{50+t}$$

$$\text{and} \quad (bl)_{50} \quad \text{for} \quad l_{50}$$

The condition that B shall die without issue will be given effect to by multiplying the value of the reversion by the chance that B will not have issue, which we have previously found to be .50821.

By the above formula the value of D's interest comes out £1600.