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IX. *A new Theory of Terrestrial Magnetism.*

By PROFESSORS JOHN PERRY *and* W. E. AYRTON*.

IN the autumn of 1876, while experimenting on magnetic transparency, we designed an apparatus for testing whether a moving body having a definite electric charge would, like a current, deflect a magnet. While waiting for the conclusion of the rains, and the advent of the very dry season which accompanies a Japanese winter, in order to try our instrument in conjunction with an ordinary plate-glass electrical machine, we received the account, published in the Philosophical Magazine for September 1876, of the experiments just performed by Mr. Rowland in the laboratory of Professor Helmholtz, by which it had been conclusively shown that a charge of electricity mechanically moved had the properties of an ordinary electric current as far as the deflection of a magnet was concerned†.

* Read March 8.

† Additional confirmation has recently been given on this subject by the experiments described by Mr. Crookes in his paper on the "Illumination of the Lines of Molecular Pressure &c.," since he has shown that the stream of particles which is shot off from the negative terminal in a very perfect vacuum, and which produces the green phosphorescence, carries electricity with it and is deflected by a magnet. This may be regarded as a sort of converse experiment, since it proves that a magnet deflects a moving charge of electricity.

Until this point was settled, it was unlikely that attention would be directed to the electromagnetic effects that might arise from the rotation of a charged body like the earth. Shortly, however, after the execution of the experiments referred to we attempted (as described in our paper "On Rain Clouds and Atmospheric Electricity," which appeared in the *Philosophical Magazine* for March 1878) the solution of a new theory of terrestrial magnetism. This problem we have attacked in a variety of ways; and the following solution, to which we have at length been led, we beg to offer for the acceptance of the Physical Society.

The points near the surface of the earth have different linear velocities from those in the interior (although all the points have the same angular velocity of rotation round the earth's axis); therefore, if the earth had an initial electrical charge, residing of course, in accordance with the well-known electrical law, on its surface, the electrified particles would have velocities relative to the remainder; hence, as a direct consequence of the results of the experiments published by Professor Helmholtz, the interior of the earth would be a magnetic field, quite independently of its interior constitution. And precisely similar reasoning, of course, proves that outside the earth's surface there would also be a magnetic field. (*Vide* addition at the end of the paper.) To determine the strength of this field we have the following relationship to start with.

In § 526, Clerk Maxwell's 'Electricity,' it is shown that an element of current C , of length ST , acts upon a unit magnetic pole at a point P with a force

$$C \frac{ST}{PS^2} \sin PST,$$

in a direction at right angles to PS and ST . Combining this with the experiments referred to above, we may assume that if a charge of static electricity (measured in electromagnetic units) Q , at the point S , moves in a direction ST with a velocity v relative to a point P , it produces on a unit magnetic pole at P a force

$$\frac{Qv}{PS^2} \sin PST,$$

in a direction at right angles to PS and ST ; and this is the *only assumption* employed in the following investigation.

Now suppose the earth to have the uniform density of electricity σ over its surface, and let its radius be unity. Consider the force produced, by the rotation of the electricity at a point S on the surface having coordinates r, θ, ϕ , at a point inside the sphere having the coordinates r_1, θ_1, ϕ_1 . Then if the sphere be rotating with an angular velocity w round the axis of z , and if θ be the angle between this axis and a radius, while ϕ is the angle between the axis of x and the projection of a radius, the velocity of S relative to P will have for its coordinates

$$u \text{ or } -w (\sin \theta \sin \phi - r \sin \theta_1 \sin \phi_1) \text{ parallel to } x,$$

$$t \text{ or } w (\sin \theta \cos \phi - r \sin \theta_1 \cos \phi_1) \text{ parallel to } y;$$

also

$$PS^2 = l^2 + m^2 + n^2,$$

where

$$l = \sin \theta \cos \phi - r \sin \theta_1 \cos \phi_1,$$

$$m = \sin \theta \sin \phi - r \sin \theta_1 \sin \phi_1,$$

$$n = \cos \theta - r \cos \theta_1.$$

Now the direction-cosines of PS are proportional to l, m , and n ; and the direction-cosines of ST, the direction of motion of S *relatively* to P, are proportional to u, v , or to $-wm, wl$, and 0. Consequently PS is perpendicular to ST. Also, if λ, μ, ν are the direction-cosines of a line at right angles to PS and ST,

$$\lambda = \frac{nl}{PS\sqrt{l^2 + m^2}},$$

$$\mu = \frac{mn}{PS\sqrt{l^2 + m^2}},$$

$$\nu = \frac{-(m^2 + l^2)}{PS\sqrt{l^2 + m^2}},$$

where the negative sign must be given to the root. Now if F is the force at P due to the charge of surface-density σ on the elementary area $d\theta \cdot \sin \theta \cdot d\phi$ at S moving relatively to P, we know that it must be at right angles to PS and ST, and equal to

$$\frac{\sigma \sin \theta \cdot d\theta \cdot d\phi \times \sin PST \sqrt{u^2 + t^2}}{PS^2}.$$

Hence, if $\delta X, \delta Y, \delta Z$ be the resolved portions of the force F

parallel to the axes of coordinates, and if dS stands for $\sin \theta \cdot d\theta \cdot d\phi$, we have

$$\delta X = \frac{\sigma \cdot dS \cdot \sqrt{u^2 + t^2}}{PS^2} \lambda;$$

and similarly for δY , δZ .

Hence, if $\iint$ stands for the summation over the whole surface of the sphere, and if X is the total force at P in the direction of the axis of x ,

$$\begin{aligned} X &= - \iint \frac{\sigma dS w \sqrt{l^2 + m^2}}{PS^2} \frac{nl}{PS \sqrt{l^2 + m^2}} \\ &= - \iint \frac{\sigma w dS}{PS^2} n \frac{l}{PS}. \end{aligned}$$

Similarly,

$$\begin{aligned} Y &= - \iint \frac{\sigma w dS}{PS^2} n \frac{m}{PS} \\ Z &= \iint \frac{\sigma w dS}{PS^2} \frac{m^2 + l^2}{n} \frac{n}{PS}. \end{aligned}$$

Now the resolved part of the force along the axis of x is the same as would be the force in that direction due to a distribution of attracting matter of density $-\sigma w$ over the surface of the sphere. Similarly the resolved part of the force along the axis of y is the same as would be the force in that direction due to a distribution of attracting matter of density $-\sigma w$ over the surface of the sphere; and the force in the direction z is the same as would be the force in the same direction due to a distribution having a density $\sigma w \frac{l^2 + m^2}{n}$ or $\sigma w \left(\frac{PS^2}{n} - n \right)$.

Regarding the force as being due to such a distribution,

$$Z = \iint \frac{\sigma w dS}{PS^2} \cdot \frac{PS^2}{n} \cdot \frac{n}{PS} - \iint \frac{\sigma w n dS}{PS^2} \cdot \frac{n}{PS}.$$

Now the first integral is $\iint \frac{\sigma w dS}{PS}$, which we know has a value equal to the potential inside the sphere due to a uniform distribution of density σw over the surface, and is therefore a constant, $4\pi\sigma w$. So that the entire force in the direction z is $4\pi\sigma w$ minus the force in direction z due to a distribution of attracting matter of density $n\sigma w$ over the surface of the sphere.

Now it is easy to show that a distribution of attracting matter of a density proportional to n or to $A + C \cos \theta$ over the surface of a sphere will give

$$\begin{aligned} X &= 0, \\ Y &= 0, \\ Z &= \text{a constant}; \end{aligned}$$

therefore all that is necessary is to determine the value of this constant. We neglect the term A , because a uniform distribution produces a constant potential, or a zero force in all directions; the distribution $C \cos \theta$ being a zonal harmonic, produces a potential inside the earth,

$$\begin{aligned} V &= \frac{4\pi C}{3} r \cos \theta_1, \\ &= \frac{4\pi}{3} Cz; \end{aligned}$$

so that the force Z which equals $\frac{dV}{dz}$ is $\frac{4\pi}{3} C$. Thus for the distribution $C \cos \theta$ we have the force $\frac{4\pi}{3} C$; so that the constant force above mentioned, $4\pi\sigma w$, requires the distribution $3\sigma w \cos \theta$. From this we must subtract the distribution $n\sigma w$, or $\sigma w (\cos \theta - r \cos \theta_1)$, giving us for the total distribution of attracting matter over the surface of the sphere a density

$$2\sigma w \cos \theta + \sigma w r \cos \theta_1;$$

but the latter term means a uniform distribution, producing therefore no internal force, and may therefore be neglected. And the first term is a zonal spherical surface harmonic; therefore the electromagnetic potential due to the rotation of the electricity on the surface of the earth is

$$\frac{4\pi}{3} 2\sigma w r \cos \theta_1 \text{ inside the earth,}$$

and

$$\frac{4\pi}{3} 2\sigma w \frac{1}{r^2} \cos \theta_1 \text{ outside the earth,}$$

where w is the angular velocity of the earth on its axis, r the radial distance of any point from the earth's centre, θ_1 the co-latitude of the place, $4\pi\sigma$ the total quantity of electricity uniformly distributed over the surface of the earth measured electromagnetically, and the unit of length the earth's radius.

These results, which we think are logical consequences of the experiment performed in Professor Helmholtz's laboratory, and referred to at the commencement of this paper, may now be applied in various ways.

For example, if the iron of the earth is arranged nearly in a hollow sphere, of external and internal radius a_2 and a_1 , then, since the potential given above is a zonal harmonic, we can at once apply Poisson's result; and we find that the potential due to magnetization of the hollow sphere is

$$\frac{4\pi\kappa(3+8\pi\kappa)(a_2^3-a_1^3)\frac{8\pi}{3}\sigma wa_2^3 \cos \theta}{9+36\pi\kappa+32\pi^2\kappa^2(a_2^3-a_1^3)r^2}$$

for all points outside the outer surface of the sphere; and hence, for points outside the surface of the earth, the total magnetic potential is

$$V = \left[\frac{4\pi\kappa(3+8\pi\kappa)(a_2^3-a_1^3)\frac{8\pi}{3}\sigma wa_2^3}{9+36\pi\kappa+32\pi^2\kappa^2(a_2^3-a_1^3)} + \frac{8\pi\sigma w}{3} \right] \frac{\cos \theta}{r^2},$$

where κ is the coefficient of magnetization.

Now Biot's approximation to the law of intensity of the force is

$$\sqrt{1+3\sin^2\lambda},$$

where λ is the latitude of the place; and we understand that this approximation is generally considered, for rough purposes, as a fairly accurate one.

Our equation for any point at a distance r from the centre and having a colatitude θ is

$$V = M \frac{\cos \theta}{r^2};$$

and

$-\frac{dV}{d\theta}$ is the force directed towards the north,

$-\frac{dV}{dr}$ is the force directed downwards towards the earth's
centre;

therefore, if I be the magnetic intensity,

$$\begin{aligned} I^2 &= \left(\frac{dV}{d\theta}\right)^2 + \left(\frac{dV}{dr}\right)^2 \\ &= \frac{M^2 \sin^2 \theta}{r^4} + \frac{4M^2 \cos^2 \theta}{r^6}, \end{aligned}$$

or, putting r equal to unity (that is, for a point on the earth's surface),

$$\begin{aligned} I &= M\sqrt{1+3\cos^2\theta} \\ &= M\sqrt{1+3\sin^2\lambda}, \end{aligned}$$

which is Biot's expression.

Now this is a result which could not have been anticipated, and speaks well for our new theory of terrestrial magnetism.

It is well known that many forms of distribution of iron inside the earth may be found which, with the existing potential given above, will produce Gauss's distribution of potential over the surface of the earth; and it would be very interesting to try how close an approximation to the real potential would be obtained by considering the iron of the earth to form a hollow ellipsoid, one of its principal axes coinciding nearly, but not quite, with the earth's axis. This calculation would be comparatively easy; but we prefer at present merely confining ourselves to simple illustrations of our theory.

Thus, let us, for simplicity, assume that the magnetic matter of the earth is iron, with everywhere a coefficient of magnetization

$$\kappa = 30^*;$$

then the terms involving κ^2 will be large compared with the others; so that if, for a very rough approximation, we assume a_1 to be nought, and a_2 equal to unity, or the whole earth to consist of iron, we find

$$V = \frac{16\pi}{3} \sigma w \frac{\cos\theta}{r^2}.$$

Now Gauss gives for the magnetic moment of the earth,

$$3.3092 n^3,$$

in millimetre-milligramme-second units, and where n is the number of centimetres in the earth's radius. Consequently, since the dimensions of a magnetic moment are

$$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1},$$

* κ for ordinary iron is probably between 20 and 30; and in our ignorance of the internal state of the earth, or what effect the great heat or pressure may have on the coefficient of magnetization, we are compelled to use this value; but it is possible, of course, that the real value of κ may be very different.

the earth's magnetic moment becomes

$$\cdot 00033092 n^{-\frac{5}{2}} n^3,$$

the units being the earth's radius, gramme, second.

Assuming Biot's distribution of magnetic force over the surface of the earth, which is also what our theory has led us to, we then get from Gauss's expression for the moment the result that the magnetic potential on the earth is

$$0\cdot33092 \cos \theta n^{-\frac{1}{2}},$$

or

$$0\cdot00001311 \cos \theta \text{ nearly ;}$$

so that roughly we have, for a point on the earth's surface,

$$\frac{16\pi}{3} \sigma w = 0\cdot00001311.$$

But

$$w = \frac{2\pi}{24 \times 60 \times 60} ;$$

$\therefore$ the density

$$\sigma = 0\cdot0107 \text{ unit of electricity,}$$

or the total charge

$$= 4\pi \times 0\cdot0107,$$

the fundamental units of space, mass, time being the radius of the earth, the gramme, and the second. But the dimensions of σ are $M^{\frac{1}{2}} L^{\frac{1}{2}}$; so that, in order to express σ in C.G.S. units, we must multiply by $\sqrt{n}$; therefore the total charge

$$= 4\pi \times 0\cdot0107 \sqrt{\frac{2,000,000,000}{\pi}} \text{ C.G.S. units}$$

$$= 4\pi \times 0\cdot0107 \sqrt{\frac{2,000,000,000}{\pi}} \times 10^7 \text{ microfarads.}$$

To get an idea of the electromotive force required to produce this charge, let us imagine one pole of a Daniell's battery connected with the earth and the other with all bodies in space. Then, since the capacity of the earth is 630 microfarads, this charge will be produced for each cell so employed; so that, if x is the number of cells necessary to produce our required distribution,

$$\begin{aligned} x &= \frac{4\pi \times 0\cdot0107 \times \sqrt{\frac{2 \times 10^9}{\pi}} 10^7}{630} \\ &= 54 \text{ million roughly.} \end{aligned}$$

We have therefore proved that if the earth be electrified, it must, from its very rotation, quite independently of all other bodies in the universe, be magnetic; and if it consist of a shell of iron, thick or thin, then that the law of distribution of magnetism produced by this electric charge in mechanical rotation will be identically that given by Biot; and, lastly, if the earth were wholly of iron, a difference of potentials of about fifty-four million volts between it and space would be sufficient to produce the necessary amount of charge.

Nôw, although fifty-four million volts is a large difference of potentials to be produced with a galvanic battery, there would not be the slightest difficulty in the earth having such a difference of potentials between it and space, seeing that the earth is surrounded by millions of miles of interplanetary vacuum, every inch of which is as good or better an insulator than a Crookes's vacuum; and it has been experimentally shown that many thousands of cells will not cause a discharge across even a comparatively thin film of such a vacuum.

But even without considering the highly insulating character of interplanetary space, we see from the experiments of Drs. De La Rue and Hugo Müller that the electromotive force of fifty-four million cells cannot, in all probability, initiate a spark between two points in ordinary air unless the distance separating them be less than four hundred feet. Consequently, if the electric charge which by its mechanical rotation produces the magnetism be on the earth itself and not in the air, it could not be discharged by sparking, unless another planet, having at least a potential nought relatively to the earth, came at least to within four hundred feet of its surface.

Next, as regards the sign of the electric charge on the earth's surface required to produce the earth's magnetic polarity, is it in accordance with the known phenomena of atmospheric electricity? To produce the earth's magnetism, we must have, in accordance with the known laws of electromagnetism, a *negative* current flowing from west to east, or in the direction of rotation of the earth. In the language of the new theory, therefore, the surface of the earth must be *negatively* charged; but Sir William Thomson has proved, by observations with his electrometer, that *all* the phenomena brought to light by atmospheric electricity, on a fine day,

could be produced by the sole agency of the earth having a *negative charge* and without any charge in the air itself. The negative charge, therefore, required for our explanation of the cause of terrestrial magnetism is sufficient to account for all the ordinary phenomena of atmospheric electricity.

In the preceding investigation we have supposed the electric charge to be uniformly distributed over the earth, and so have arrived at a law of magnetic intensity merely varying with the latitude. But the sun and other members of the solar system may very likely have potentials so different from that of the earth that we can hardly conceive the amounts; consequently we should expect the static electric distribution of the earth would undergo periodic changes corresponding in time with those of the ocean-tides. But alteration in the static distribution of electricity on the earth's surface means, as we have shown, alteration in the law of magnetic intensity; consequently we should expect that this magnetic intensity would vary somewhat as do the ocean-tides; and this is known to be the case. But it is also evident that, besides these regular changes, every time a great mass of vapour is suddenly formed and condensed on the earth, and whenever great changes are occurring in the solar atmosphere whereby the lines of *electrostatic induction* from the sun to the earth are altered, we should find corresponding changes in terrestrial magnetism such as we now know as magnetic storms. And not only this, but as the planets are charged bodies, their motions relatively to the sun ought to cause motions in the sun's atmosphere such that, for instance, the allineation of a number of planets and the sun, or the near approach of any planets, if the aligned or approached planets have potentials nearer that of the sun than many of the other bodies of the solar system, ought to diminish the storms in the solar envelope, and ought to alter the electrostatic distribution on the earth. But it has been shown that the near approach of a planet to the sun both affects the sun's spots and terrestrial magnetism.

And, lastly, since the iron in the earth may, from its great pressure, possess great coercive force, we should expect (as we know to be the case) that magnetic changes would lag behind the astronomical influences accompanying them.

Addition, April 17th.—Since the reading of this paper before the Physical Society, several criticisms have appeared of this proposed explanation of terrestrial magnetism. Some of the writers have failed to realize that the various parts of a rotating sphere have relative motions one to the other, of such a nature that if some were electrified the others would become magnetized. Their difficulty seems to have arisen from the fact that the motion of a particle of a rotating rigid sphere consists of a rotation round the axis of the sphere combined with a rotation of the particle round its own axis; so that if two particles be looking at one another in one position of the sphere, they are looking at one another in all positions, just as the same side of the moon is always turned to the earth. But if this rotation of the particles round their own axes could be stopped, if, in fact, the motion of the particles became what is commonly known as “sun-and-planet motion” similar to that of the bobbins in the machinery used in sheathing telegraph-cables to prevent torsional strain being put into the iron wires as they are lapped on the core), then probably these writers would have no difficulty in seeing that the interior particles would be magnetized by the more rapidly moving electrified ones. Now the motion of an *unelectrified* particle round its own axis cannot in any way prevent it becoming magnetized by electrified particles revolving round it; for if it could, it would be equivalent to saying that, if the electrified particles were at rest and the *unelectrified* one revolving, the mere revolution of the latter would magnetize itself oppositely to the way it would be magnetized if it were at rest and the electrified ones only in motion—a result not only without experimental proof, but one also highly improbable. Consequently, if the particle has the two motions together (as it has in a rigid revolving sphere), it will still become magnetized if the surface of the sphere is electrified.

In fact, so little can the motion of a mass of iron prevent its becoming magnetized by a moving charge of electricity, that it has been suggested to us, within the last few days, by Mr. G. F. Fitzgerald, of Trinity College, Dublin, that probably a mass of iron would become magnetized by a static charge of electricity if both had rapid absolute motion in space, even although in the same direction in parallel lines and with

the same velocity—in fact, that *relative* motion is unnecessary. If this assumption (which has not yet been experimentally tested) be true, then not only will the mathematical analysis required in the investigation of the problem contained in this paper be much simplified, but, in addition the charge of electricity on the earth's surface necessary to produce, by its mechanical rotation, the earth's known magnetic moment will be considerably less even than the charge calculated above. We hope to put this idea of Mr. Fitzgerald's shortly to an experimental test; but in the meantime we give no further indication of the reasoning by which it has been arrived at, nor of the way in which our equations would be modified, preferring to leave the investigation in a perfectly rigid form as it now stands, rather than to introduce any assumption which might appear problematical, even although such an introduction would both add weight to our theory, and might explain, from the velocity *in space* of a place at midnight being greater than at midday, the cause of the solar-diurnal magnetic variation.

X. *The Maintenance of Constant Pressures and Temperatures.*

By FREDERICK D. BROWN, *B.Sc.*

[Plate IX.]

THE great majority of the results obtained from physical experiments vary with the temperature at which the observations are made. The measurements of the density of a substance, for example, of its refractive index, of its electric conductivity, of its elasticity, of the maximum tension, and of the latent heat of its vapour, all require that the temperature should not vary during the observations. Hitherto many of these measurements have been confined to temperatures differing little from that of the atmosphere; such temperatures are easily maintained constant by means of a bath of water or other liquid; but when we try to make observations at higher temperatures, the means at our disposal fail us, and we find that, except at certain points (such as 100°), we cannot keep up the same temperature long enough to make at leisure accurate readings of our instruments.

Many attempts have been made to obviate this difficulty,