AIR-GAP AND CORE DISTRIBUTION.

THE MAGNETIC FLUX AND ITS EFFECT UPON THE REGULATION AND EFFICIENCY OF DYNAMO-ELECTRIC MACHINERY.--I.

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The predetermination of the regulation of any dynamo electric machine involves several important steps, and not the least among these, especially in the case of alternating current apparatus, is the determination of the air-gap distribution of the flux, for upon it depends the inductance of the armature winding, the wave form of the E. M. F. and of the current.

The contours of the pole-pieces and the armature teeth of a machine affect, therefore, to an important degree its commercial usefulness, as the air-gap distribution, and consequently the dynamo regulation, is directly dependent upon the shapes of these iron parts. In designing electrical machinery the mechanical details should always be given careful attention, and frequently a method of successive trial will have to be followed to the end of finding the best pole and tooth shapes to be used to fulfil the requirements of the specifications.

In developing a method for accurately calculating the *armature surface* density, let us consider first the simple conditions illustrated in Fig. 1. Here the armature surface is taken as a plane extending indefinitely to right and left. The pole-piece is a rectangular block extending indefinitely in a vertical direction above the armature, and having perfectly square corners. The length of the air-gap or the vertical distance from the pole-face to the armature surface is the same at all points. Both the armature and the pole-piece extend indefinitely at right angles to

FIG. 1.

the plane of the paper, and Fig. 1 is only a cross-sectional view of the apparatus assumed.

Suppose, now, that a M. M. F. is applied to the pole-piece to set up a magnetic field in the air-gap. Since the permeance of the pole-piece and of the armature is high, we can consider, for all practical purposes, that all points on the sides and face of the pole-piece are at the same potential relatively to the armature surface, and that all points on the armature surface are at the same potential relatively to the pole-piece, and that, therefore, a constant difference of magnetic potential exists between points on the pole-piece and points on the armature. That is, for all practical purposes, there is the same difference of potential between $(a-b)$ as there is between $(c-d)$ or $(e-f)$.

And, *the number of lines of force that pass between any pair of points on the pole-piece and the armature respectively (as $c-d$, in the plane of the paper) will be inversely proportional to the distance¹ between them. And the strength of the magnetic field at the surface of the armature at any point (as d) will be proportional to the sum of the reciprocals of the distances of the point (d) from all the points on the perimeter of the pole-piece made by a section-plane passing through the point (d), as in Fig. 1.*

That these assumptions represent a close approximation to the conditions actually existing in dynamo machinery, will be evident when it is remembered that in any dynamo from 60 to 90 per cent. of the M. M. F. impressed upon the magnetic circuit is expended in forcing the flux across the air-gaps, in spite of the fact that these air-gaps are seldom as much as one per cent. of the length of the path traversed by each useful line of force. The drop in M. M. F. is therefore from 600 to 2000 times as great per unit of length in an air-gap proper as it is in an iron portion of the circuit.

The curve Λ in Fig. 1 represents the distribution of the magnetic flux at the *surface of the armature* as determined by the foregoing method; for the length of any ordinate ($h-d$) at any point (h) on this curve is equal to the sum of the reciprocals of the distances of the foot of the ordinate (d) from all points (g to e) indicated on the pole-piece. The curve Λ extends indefinitely to right and left, and is asymptotic to the surface of the

1. In measuring these distances it must be remembered that the most direct path *in air* must be taken, as $(d-a-g)$ Fig. 1, *not* $(d-g)$ direct.

armature. The curve indicates that the strength of the magnetic field does not diminish as rapidly as is usually supposed after the limits of the working air-gap proper are passed.

Fig. 1 illustrates the field strength at the surface of the armature when but one pole is acting upon it. Suppose we bring up another pole, similar in every respect to the first, except that it is of opposite polarity. It will tend to set up the field density at the surface of the armature that is represented by curve *B* of Fig. 2; but since this magnetic field is opposed to that of the pole *N*, a field will result that is equal to the difference of the superimposed fields, or to the field strength represented by the curve *C*, Fig. 2. As a result of the subtraction, the flux density is shown to be zero at any point on the surface of the armature that is midway between the two poles, and that the sign of the

FIG. 2.

field is reversed at such point. In this case the strength of the field *at the surface of the armature* is greatest at the center of the poles and diminishes to right and left. The reduction in the strength of the field under the pole corners is marked, and the form of the fringing field is clearly outlined. It is to be further remarked that the contour of the curve *C* of flux distribution changes its form as the distance between the poles *N* and *S* is varied.

The calculation of distribution curves similar to curve *A* of Fig. 1 is most readily effected by an application of the calculus, although in cases where the contours of either the armature or pole-piece surfaces are very irregular, it may be necessary to space off the pole-piece perimeter and obtain the summation of reciprocals for any given point on the armature by first calculating each one separately.

A simple and most handy formula for use in this connection is that for the determination of the summation of the reciprocals of lines drawn to a series of points on a plane surface. Referring to Fig. 1, let y_e denote the value of the reciprocal ($d-e$) and c_o the value of the distance ($d-o$) and x_e that of the distance ($o-e$),—then,—

$$y_e = \frac{1}{\sqrt{c_o^2 + x_e^2}}$$

Therefore if y denotes the reciprocal of the distance of any point on the surface (g, a, m, e) from d ; x , the distance of any point on the line ($o-e$) from o ; z , the distance of any point on the line ($n-g'$) from n ; and c_n the distance ($d-n$), then,—

$$\sum_{x_m}^{x_e} y = \int_{x_m}^{x_e} \frac{dx}{\sqrt{c_o^2 + x^2}} = \log (x + \sqrt{c_o^2 + x^2}) \left[x_m \right]^{x_e}$$

and, since, as $(a-g') = (a-g)$, the reciprocal of the distance ($d-g'$) approximates the value of the reciprocal of the distance ($d-a-g$), then,—

$$\sum_{z_m}^{z_g} y = \int_{z_m}^{z_g} \frac{dz}{\sqrt{c_n^2 + z^2}} = \log (z + \sqrt{c_n^2 + z^2}) \left[z_m \right]^{z_g}$$

and the value of the ordinate from the point h , or

$$(d-h) = \sum_{x_m}^{x_e} y + \sum_{z_m}^{z_g} y.$$

In estimating the value of the theoretical case that has just been discussed as an aid in the practical design of electrical machinery, it may be well to bring out the points of difference between the assumed and the actual conditions. In the first place the dimensions of the pole-pieces are limited vertically and laterally as well as transversely, and therefore although the assumed condition of field strength distribution may exist in a plane passing through the center line of the side faces of the poles, it does not follow that these conditions will in any wise obtain at the outside edges of the poles. Then again the difference of potential maintained between the armature surface and the poles is not constant for all points on the polar surfaces. It is only constant for points below the magnet windings, while the difference of M. M. F.

between points on the yoke (and at the top of the magnet windings) and those on the armature is zero. In passing through the winding spools therefore the M. M. F. diminishes from a maximum to zero.

As will be shown later, the limitation of the dimensions of the poles in practice does not seem to materially affect the results when the outer ends of the armature coils do not cut the lines of force of the outside or end fringe of the field, and where they do, the strength of this fringe can be determined by applying the construction to the end surfaces.

The matter of the variation in the M. M. F. is not in general of great consequence owing to the fact that the distance reciprocals of points on the poles that are near the yokes are very small as compared with those of points on the working air-gap. It is ordinarily sufficient, therefore, to consider the potential constant to points *half way up to the field winding*, and to carry the summation only to these points. With this thought in mind the summation of reciprocals was only carried to *e* and *g* in determining the form of curve A of Fig. 1. In special cases where on account of modifications peculiar to the design it is deemed wise to take account of the variations in the M. M. F., it is only necessary to multiply each reciprocal by a number that is proportional to the M. M. F. acting on the path from which it is determined. In the application of the calculus to this case a special integration is necessary for the polar points lying within the winding spools.

In bringing forward experimental results to show in how far the method that has been discussed can be relied upon, I will first call your attention to the data that is given by Dr. S. P. Thompson in the second lecture of his "Electromagnet," which refer to the determination of "the relation of leakage to pull" in the case of a horse-shoe electromagnet. He says:—

"The amount of magnetism that gets into the armature does not go by a law of inverse squares, I can assure you, but by quite other laws. It goes by laws which can only be expressed as particular cases of the law of the magnetic circuit. The most important element of the calculations, indeed, in many cases is the amount of percentage of leakage that must be allowed for."

Dr. Thompson then gives the record of the number of lines of force entering the armature when the latter is placed at successive distances from the pole-pieces.

These give ratios of length of air-gap to width of pole-face of

1/13, 2/13, 5/13 and 10/13, and for this series of air-gaps, sets of readings were taken with exciting currents of .7, 1.7, 3.7 and 5.7 amperes. I have taken the data and multiplied each series by a constant which reduces the short air-gap reading of each to the same numerical value, and have plotted these results in Fig.

FIG. 3.

3; each series is marked and its points connected by a black line. The initial or short air-gap density of the first series is only .2 of the initial density of the last series, and yet the curves practically coincide, showing that for any given position of the armature relative to the poles, the strength of the field is directly

proportional to the impressed M. M. F. and that the field distribution of the flux remains constant, as theory requires.

Now supposing the armature of this electromagnet to be a plane surface, and calculating the relative amount of flux that will enter the armature for the several air-gaps, by the method already developed, the data represented by the upper curve of dots were obtained. The ordinates of these points are, therefore, proportional to areas enclosed by curves like curve *c* of Fig. 2, determined for each position of the armature. Supposing the

FIG. 4.

armature to be of horse-shoe form, the data obtained gave the lower curve of dots. The result for the armature actually used would fall between these two curves of dots. We have here evidence to show that a careful application of this method of *reciprocal summation* will give results within the bounds of experimental accuracy.

An extended series of experiments carried on by Messrs. J. L. Roe and S. R. Fox, under my direction, upon alternating and direct current electromagnets confirm the above.

I am further much pleased that the opportunity is afforded me of calling your attention, at this time, to a set of especially clear and instructive curves recently published² from the University of Nebraska. In Fig. 4 I have reproduced in the small circles the graphical presentation of the data from which one of these was plotted. The small circles represent the experimental exploration of the flux distribution in the air-gap of an Edison bipolar dynamo. The curve shown by the black line in Fig. 4 is the calculated curve of flux distribution for this machine obtained by the method of reciprocal summation. It agrees closely with the experimental work.

FIG. 5.

The dotted curve of Fig. 4 outlines the flux distribution that is usually *assumed*. Its ordinates are inversely proportional to the length of the air-gap of the dynamo, and it departs noticeably from the real and calculated values through that important space represented by the fringing field.

To give you further evidence of the probable accuracy of results calculated on this basis I have applied the method to the

2. "Armature Reactions in a Continuous Current Dynamo," by C. A. Bessey, *The Electrical Engineer*, vol. xxv, p. 511, May 12, 1898.

determination of the flux distribution in the air-gap of a 50-K. w. bipolar Jenney dynamo. The design of this machine is such as to admit of a considerable range of adjustment in the mean length of the air-gap. Under normal working conditions the double air-gap of this machine, which has a smooth core armature, is 1.25" at the center of the pole-faces, and 1.52" under the leading and trailing pole-tips. The diagram of Fig. 5 presents the conditions existing with this arrangement. Subtracting curve B from curve A, curve C is obtained, and represents the calculated

FIG. 6.

distribution of the flux in the air-gap. The dotted curve D is plotted by taking the reciprocal of the length of the air-gap at successive points, and plotting the results so obtained above corresponding air-gap positions on the diagram. The ordinates of the curve D are therefore proportional to the armature surface density in the air-gap, if this density is taken as inversely proportional to the length of the air-gap. The star-points on the diagram represent the results obtained by exploring the air-gap field by the "pilot brush" method. As the star-points follow the curve

c much more nearly than they do the curve d, it seems probable that the calculated values are reasonably accurate.

In Fig. 6 is illustrated the result of imposing another set of conditions upon the same machine. The curve c represents the calculated distribution of the flux in the air-gap when the length of the double air-gap has a constant length equal to 1.98" at all points. The dotted curve d shows what the relative value of the flux density would be if the strength of the air-gap field were inversely proportional to the length of the air-gap. Since the ordinates of the c curves and the d curves of Figs. 5 and 6 are respectively proportional to the quantities they represent and to

FIG. 7.

one another, they can be compared, and to facilitate this are plotted together in Fig. 7. It is noticeable that the c curve of Fig. 6 represents relatively a very much stronger field than does the d curve. In fact the ratio of the mean ordinates of the c curves of Figs. 5 and 6 is .899, whereas the ratio of the mean ordinates of the d curves is only .699. We see therefore that the rate at which the strength of the air-gap field decreases as the length of the air-gap is increased, falls far short of being inversely proportional to the rate of increase in the mean length of the air-gap, supposing that the c curves represent the true condition.

This fact is also brought out by the curves of Fig. 3, which indicate that the length of the air-gap had to be increased five-fold before the strength of the field was reduced one-half.

To determine how nearly the calculated results agree with the actual conditions existing in the case of the Jenney dynamo, the magnetic characteristics of the machine were determined both for the short and the long air-gap adjustments; and the ratios of ordinates corresponding to like field currents along the "straight" parts of these curves below the "knee" were found to vary between .824 and .864, the average being about .835. Although not conclusive, these ratios tend to substantiate the calculated results.

Up to the present the application of the method to machines having smooth core armatures has only been considered. But to be of value it should apply with equal force to all types of machines, and in fact the most important field for its application is to modern alternating current machinery in which the shapes

FIG. 8.

given to both the polar and armature projections are very varied in character.

The computations required to determine the flux distribution where the armature surface is slotted, are considerably more involved than are those for a smooth core machine. To accomplish a result that will be of value in determining the wave forms of the E. M. F. and of the inductance developed in the armature coils, the armature must be placed in a series of positions, each of which presents a different space position of the teeth to the poles. The complete armature surface distribution of the flux must then be determined for each position of the armature relative to the poles, the exact perimeter of the armature teeth being followed in each case and the measurements being taken from working drawings of the machine. When this has been accomplished a sufficient amount of information will have been obtained to admit

of the accurate determination of the rate of change of the flux through any coil on the armature, and consequently of the wave form of the E. M. F. developed in the coil as rotation takes place. In other words, the data will be sufficient to determine the *no load* E. M. F. waves developed in the armature, provided the hysteresis³ and eddy current disturbances in the armature iron are not sufficiently great to cause a lateral displacement of the wave.

In Fig. 8 is shown diagrammatically a section through the poles and armature of an 8-50-900-1150 two-phase General Electric alternator, the results of calculations in connection with which I

FIG. 9.

wish to report to you at this time. Each of the coils on the armature of this machine embraces three teeth as indicated by the single coil shown in Fig. 8 and in the discussion which follows, I will endeavor to show with what success the wave of E. M. F. developed in these coils has been calculated.

The curve *c* of Fig. 8 represents what the flux distribution in the air-gap would be if the armature core were smooth. Its ordinates are therefore proportional to magnetic density at the upper surface of the teeth. The number of lines of force enter

3. Armature Inductance, TRANSACTIONS, vol. xiv., p. 292.

ing the sides and bottoms of the slots had also to be determined by the method of reciprocal summation. The result of some of this work is shown in Fig. 9, where three of the air-gap distribution curves are plotted relatively to the perimeter of the armature developed into a straight line. The positions of the top of the teeth and the bottoms of the slots are indicated by the breaks in the line through the center of each curve. The curves are determined for positions of the teeth differing by one-third of the pitch of the teeth, and the gradual change in the distribution and amount of the flux that enters a tooth as it passes across the pole-face can be traced by following the small letters *a*, *b*, *c*, *d*, etc.

The areas of the curves of Fig. 9 that are separated off by the ordinates are proportional to the total number of lines of force

FIG. 10.

entering the upper surfaces of the teeth and the lower surfaces of the slots, respectively. In other words, the area between ordinates *q* and *r* is proportional to the flux entering tooth *b* of Fig. 8, within the surface between *q* and *r*; and the area between the ordinates *r* and *s* is proportional to the flux entering the bottom of the slot *e* between the limits *r* and *s*. The ordinates to the curves are drawn at points that are opposite the center of coils in the slots and represent the limit of the useful flux entering the coil in the slot considered. The coil shown in Fig. 8 rests in slots *c* and *i*, and therefore when the coil is in this position the total area between the ordinates *q* and *v* of Fig. 9 must be taken to be proportional to the total flux entering the coil when it is in this position.

In Fig. 10 is shown the curve *M*, which is the magnetization curve of the armature coils at no load, *i. e.*, its ordinates are proportional to the instantaneous values of the magnetism within the coils as they pass before the poles. The values of the ordinates of this curve were obtained from the distribution curves shown in part in Fig. 9, as already explained.

Now since the rate of change in the amount of flux within the coils is proportional to the E. M. F. developed in the coils, the wave of E. M. F. has been determined directly from the curve *M* by the method of tangents, and is shown by the broken line and stars of the curve *E*. The curve *E*, which is quite an irregular curve, is therefore and finally, the calculated curve of electromotive force developed by this alternator.

The points represented in Fig. 10 by small circles trace the wave of E. M. F. actually developed by this alternator as determined experimentally by the rotary contact method.

The method of calculating field densities which has been outlined can, I believe, be made to play an important part in connection with the design of electrical machinery. It has been used by myself and my students with success within the last two years, in connection with problems involving the calculation of leakage coefficients, the regulation of large direct current units under conditions of load, the determination of the inductance of the coils on the armatures of direct and alternating current machines, and the complete regulation of alternators and motors in so far as it is affected by disturbances in the magnetic field due to rotating masses of iron or reactive currents in the coils. To touch upon these points in detail would make my present paper too lengthy for this occasion, but I hope to be able, at a future meeting, to call your attention to some of the more important problems that have been treated.

Purdue University, Lafayette, Ind., June, 1898.

(COMMUNICATED BY THE AUTHOR AFTER ADJOURNMENT.)

The importance has been emphasized of making due allowance for the fact that the M. M. F. between a point (d , Fig. 1) on the armature surface and a point on the polar surface (between c and e) within the field coil spools rapidly diminishes as the latter point is taken nearer and nearer the yoke, it may be well to add the formula that applies especially to the reciprocal summation of the points within the field coils.

If the M. M. F. impressed upon the magnetic circuit of Fig. 1, were produced by an energized coil occupying the space between the points c and e , the potential between polar surface points below c and points on the armature surface would be uniform and equal to the maximum magnetizing power of the coil. The potential between points above e and points on the armature would be zero, and the potential of points between c and e and the armature would gradually diminish from the maximum value to zero as the points were taken more and more remote from c . Under these conditions, the effect of the reciprocal of $(m - d)$ or $(c - d)$ at d is proportional to its full value; the effect, however, of the reciprocal of $(p - d)$ at d is only proportional to half its full value, while the effect of the reciprocal of $(e - d)$ is zero.

Therefore, the effect of the surface $(c - m)$ at the point d is proportional to

$$\sum_{x_m}^{x_c} y = \int_{x_m}^x \frac{dx}{\sqrt{c_o^2 + x^2}} = \log(x + \sqrt{c_o^2 + x^2}) \left[\begin{matrix} x_c \\ x_m \end{matrix} \right]$$

while the effect of the surface $(c - e)$ at d is proportional to

$$\begin{aligned} \sum_{x_c}^{x_e} y &= \int_{x_c}^{x_e} \frac{dx}{\sqrt{c_o^2 + x^2}} = \log(x + \sqrt{c_o^2 + x^2}) \left[\begin{matrix} x_e \\ x_c \end{matrix} \right] \\ &= \frac{1}{x_e - x_c} \left[x_e \log(x + \sqrt{c_o^2 + x^2}) - \sqrt{c_o^2 + x^2} \right] \left[\begin{matrix} x_e \\ x_c \end{matrix} \right] \end{aligned}$$

where x_c denotes the distance $(o - c)$ and x_e the distance $(o - e)$. And the total effect of the surface $(m - e)$ at d is proportional to—

$$\sum_{x_m}^{x_c} y + \sum_{x_c}^{x_e} y$$

evaluated as above.