EFFICIENCY IN GEOMETRY TEACHING.¹**A Study—An Experiment—A Result.**

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One of the recent questions in education deals with efficiency in the secondary schools. The question is of vital importance to everyone, but to no class is it more pertinent than to the high school teacher of mathematics. Mathematics has held an important place in the courses of study in the past because of the supposed value it gives in formal discipline. This idea now challenged, together with other causes has brought about the violent agitation of the question of how to present high school mathematics, the arrangement, the amount and the kind for each grade, in order to meet effectively the needs of the citizen of to-day. For this purpose improved texts have been published, new material presented in the elementary school and in the high school, mathematical associations organized and committees appointed. Yet in spite of these forces and many others, the teaching of high school mathematics has not yet reached its maximum efficiency. In order to reach a higher degree of efficiency in the teaching of geometry the following study was made and a few conclusions drawn.

The underlying principles of any subject must be understood. Geometry, a study founded upon definitions, axioms, postulates, problems, or theorems has too often been presented to the pupil as a medley of unreasonable, trying situations that have nothing to do with life. The assumptions laid down in geometry may mean and quite often do mean anything that the user sees fit to decide upon. This gives wide latitude to the teacher in the selection and arrangement of working material, but too many have failed to see their opportunity. The tendency of the past few years seems to have been to get away from the pure Euclidean idea and back to the fundamental beginnings. This tendency to change has been attended, as all changes are, by much waste. The text-book writers who have favored a change have held back because they knew that a book that was too radical would not be used, and teachers who were anxious for a change were held in abeyance to custom by fear. Some of the writers in their eagerness for a change have confused themselves by attempting correlation

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with physics², astronomy and other subjects. Others have tried the heuristic³ plan, and others the plan of induction,⁴ while others, each in his own way, have advocated a geometry that was more concrete, practical, inventional,⁵ and empirical.⁶

My thesis is that geometry teaching in order to reach its highest efficiency is neither to make a geometry of "real" or concrete problems nor one of the purely demonstrational type; but rather a presentation of geometrical principles in a concrete way at first followed by the logical, demonstrational type. There is no need of purely utilitarian problems nor purely cultural problems. Whatever is utilitarian is cultural and whatever is cultural is utilitarian. An education that is complete is one that educates vocationally and liberally. The only reason for the present division existing is that our ideas are so faultily worked out that we are not able to see the coördination of the two. At an early age a child is able to build block houses and to work with material that has surface, length, breadth, and depth; he sees triangles, squares, and circles. In the third and fourth grades the home geography with its directions and location employs many of the elementary principles of geometry. In his map making he uses cross-section paper, with the cross roads as his coördinate axes; or if in the city he uses the main streets. The work is continued in geography with respect to latitude and longitude where the equator and prime meridian are the axes. In many of the rural localities the polar coördinates are the basis of location work, especially in directing from one place to another. Nor is geography the only field; the work in agriculture calls for platting of ground either for decorative or practical purposes, and also for architectual forms that are so in evidence about any farm.

The problem of the township with its townships and range lines, together with the section gives a better idea of surface in square units than can ever be obtained from a study of ordinary geometry in that the area of a rectangle is the product of the length by the altitude in terms of a unit square. Again in games and play, as baseball with its diamond, tennis with its court, football with its field, and many others that call for angles, straight lines, area and other geometrical forms. Nor is this all since the child must learn in farm accounts or commercial activities that

²Hiles, *My Class in Geometry*.

³Hopkins, *Plane Geometry*.

⁴Sanders, *Plane Geometry*. Shutts, *Plane Geometry*.

⁵Spencer, *Inventional Geometry*.

⁶Warren, *Experimental and Theoretical Geometry*.

he must make computations based upon data that has been secured in topographical leveling, soil surveying, excavations, drainage ditching, grades for streets and subsequent platting.

A great many of the elementary principles of constructional geometry can easily be given in mechanical drawing in the eighth grade. And there is no reason why the child should not be taught to draw to scale some measurements of his own made in the school yard or surrounding country. Practical problems that one can use in geometry are continually coming up. The engineer has to deal with questions of force and motion, strength of materials, the architecture of houses and bridges, and similar problems. The farmer, the carpenter, the mason, and all other artisans, are confronted with problems each in his own field. And these activities are so interwoven that it is impossible to understand clearly one unless something is known about several.

The child as well as the teacher should know the objective point, and the material with which he is to work to arrive at that point. The pupil should always understand why he selects a certain process. In material construction the boy is taught why he uses the try-square at one time, and the T-square at another time. In the construction of a house the boy can see the advancement made in the building, and why at one time certain scaffolding is needed and at another time something else; and in the same way he should see how he builds his geometry. If he knows the purpose he stands a chance of becoming a discoverer. "The pupil must know from the beginning what is aimed at, if he is to display and employ his own strength in the effort of learning; and he will employ it provided he knows definitely what is to be reached. To lead him unconsciously by question and task whose purpose he does not clearly see, has the disadvantage that neither a free rising mental activity nor a clear internally connected insight takes place."⁷ This means that the child should see some reason for the assigned task.

Neither should beginners be limited by restrictions. Their life and experiences have convinced them that certain statements are or are not valid. It is bewildering to them to be required to prove a statement by means of other statements that they have not experienced, and to be careful not to let intuition and former experiences enter into the proof in the slightest degree.

⁷A. L. Baker, *Purposive Geometry*, SCHOOL SCIENCE AND MATHEMATICS, 1906, p. 511.

The teacher must ever bear in mind that not only is the subject new, but that the activity is new. The proof that right angles are equal is foolish to the wideawake boy of fourteen who has worked with a carpenter's square. Again, the teacher should make use of the biological experiences that are worth while, and the high school age is the age at which every boy and girl likes to be doing something.

The work should be grouped about types. This is easily done throughout the grades. A text is usually an economical tool for the assignment of certain problem material. The mathematical facts should be exemplified in the practical world. Model making, another form of paper folding, is worth consideration, if we actually construct instead of merely supposing. "In^s mensurational geometry the balance may be used to advantage if one is careful to use homogeneous material, and squared paper finds a use in determining laws of area. Drawing to scale, a practical problem that confronts the contractor and builder, measurement of inaccessible distances, a study of parallelogram of forces, inclined plane, or images in a plane mirror, together with the elements of surveying, are some of the features of the new geometry." Actual constructions in the manual training department will mean more than any drawing that is copied from a picture. A visit to a building to study architectural form not only adds an interest but "develops the sense of the beautiful in geometrical construction and a love for symmetry." Girls find equally as much interest in arrangement of house furnishings, designs of wall paper, window and mural decorations. All of this stirs up an interest and a love for the subject, and the scope of geometry is widened. If from time to time brief historical data is given it adds human interest.

"For⁹ a class just entering geometry a small equipment of ruler, compasses, scissors, drawing board, protractor, parallel ruler, squared paper, T-square and a triangle of 60 and of 30 degrees with a right angle should be provided for each pupil. In addition there should be accessible to the pupil at all times prisms, pyramids, cylinders, cones, drawing instruments, paste-board for construction, sand for testing capacity, a trip balance, and anything else that would be of value locally, or that might be a help to the child to associate the new idea with his former experiences."

87. C. Packard, Co-ordination of Mathematics and Physics, *School Review*, 1903, p. 791.

88. C. Packard, Co-ordination of Mathematics and Physics, *School Review*, 1903, p. 798.

Coördination has a place if rightly presented. The work in crystallography, optics, and botany have elements in common with geometry. And other branches of geometry, as perspective drawing, may furnish material for the introduction to solid geometry. The work to most pupils can be made real and human interest aroused with problems from the activities that they are acquainted with and are interested in. Types of local influence are Pleasant Hill, Mo., with its vast greenhouse interests, and Deepwater, Mo., with its great tile industry.

Believing thoroughly that the plan discussed above had virtue in it and that it would develop accuracy in conception, accuracy in expression, accuracy in manipulation, economy in thought development, and an easy means of fixing the idea in the child's mind, some of the things desired in geometry teaching, an actual trial of the plan was carried out.

The work began with the simplest fact that had already been made the property of each child. No time was wasted in attempting to define terms the meaning of which was already more definite than any definition could make it. As the definition of a straight line in the books is confusing and the child's idea is clear, at least to him, there is no need of confusing the child by destroying his conception and implanting another. Nor did we think it worth while to accept such definitions as "An axiom is a truth assumed as self-evident," as the same truth is not self-evident to all minds, and again some of these "self-evident" truths are not necessarily true at all times. Nor did we prohibit a child from making use of things that he already knew. Folding paper gave an idea of a straight line. A walk gave opportunity for the observance of straight lines; the boundary of the school yard, right angles; the walks, of parallel lines; the angles of the yard, perpendicular lines and various other ideas. The measurements made were drawn to scale on coördinate paper. Ratio of lengths were determined by the diagonal scale and dividers; this gave the right conception, taught observation, developed an interest, and applied truth in an usable way. In addition, the child was not discouraged by rigidity or a confusion of unintelligible words. The balance and coördinate paper were used for determining laws of area. The problems selected were practical for the particular neighborhood in which the work was offered. No problem was admitted simply because it might meet the needs of some technical artisan or college entrance requirements, but simple problems in sidewalk construction, making of

road drags, cutting of handkerchief squares, and so on. The problems were short and definite. Problem material was easily found in mechanical drawing, in elementary physics, in construction of buildings, graphs, and machinery in motion. The motor belt easily shows the common tangents, either internal or external. Graphical representation of statistics, location problems, common forms of symmetry with simple objects as the capital Latin letters, problems in crystallography, as the hexagonal snowflake, botany with the radial arrangement, optics with the equal angles of incidence and reflection, or the angles of refraction, surveying, and common problems of domestic science, furnish a suggestion as to the source of the problem material. In addition, problems were found in carpentry, manual training, laboratory, masonry, blacksmithing, transportation, newspaper offices, and the other activities of the town. We had to make the problems short, simple, and definite, because the average high school pupil is not willing to spend several hours upon a problem that is long, indefinite or tedious in detail. He is not sufficiently developed to analyze a problem that is indefinite or meager in hypothesis.

All problems that presented themselves were not adapted to our particular high school, as some were too complex, some too indefinite and some called for material that was not easily associated with the child's experience. These problems were omitted.

The collection of problem material was carried on by teacher and pupil with as little reference to text-book as possible. In fact, problems from the text were discouraged. There was a tendency at first for the pupil to model his problem after some problem that he had found in a book, but this tendency soon disappeared when he found that such problems would not be accepted. The material was collected in as natural manner as possible, and in the ordinary affairs of life. The pupil in a short time became actively energetic in the seeking of problem material and found problems where he had no thought that a problem could exist. He soon came to use his tools of geometric principles, which were presented as the need demanded, as readily as one of the German pupils used his German vocabulary and more naturally. After a problem had been discovered, studied in the light of previous experience, and geometric principles applied, the pupil was urged to formulate the problem in as general manner as possible.

The pupil was encouraged to select his material from actual life, and from his own life, not someone else's. A book problem

was used only in a suggestive way, or to supplement some other problem material that was inaccessible to the pupil. Part of the book material as well as part of the other material gathered was unsuited to the idea in mind. In order for the boy or girl to appreciate the fact that there is a problem, something must be known about the situation that makes up the problem. If the pupil could not appreciate a problem suggested by another, and there seemed to be a need for that particular situation, an attempt was made to interest him in the material leaving him to invent the problem, rather than attempting an explanation of the "why" or the "wherefore" of the question.

As an example of the above difficulty a construction problem in engineering, while appealing to the ordinary boy sometimes lacked interest to the girl, because she knew nothing about the problem or its conditions, or had no interest in knowing. It was possible to interest her by a field or factory excursion or by experimentation, or by showing her that this at least added to her cultural knowledge, if she had no practical need. Or again she might have been interested by showing her the relation of the problem to wall paper designing, or the symmetrical idea of interior decoration or arrangement of a home. Even if this did not arouse an interest and an activity the engineering material of the boy was more real and held out to her greater possibility in conception than the abstract enigma found in the text, for she had a chance to try and find out, dealing with something that she could measure and verify.

In the work no attempt was made to make the theoretical subservient to the practical, but rather that the pupil could be led to see an association between his acquired knowledge and what he was gaining. This kept ever before the pupil the "worth while" of the subject. After a thorough leading up to the subject the theoretical problems with all the original exercises that time permitted were solved with all the rigor that Halsted would insist upon. This last feature of the work was very limited on account of time.

SOME DISADVANTAGES OF THE METHOD.

1. The problems overlap each other in material, thus destroying an economic principle unless extreme care is used. This objection would in the course of a few years be removed as the plan developed.
2. Differences in need and experience of individuals make mass teaching difficult.

3. There is great difficulty in creating a reconstructed environment in the school.

4. The teachers that are employed at the present are not efficient; are not acquainted with the subject-matter; do not know how to formulate problems and arrange them according to the ability or development of the pupil.

5. Lack of:

a. Strong personality of the teacher—enthusiasm.

b. Familiarity on the part of the teacher of the previous training of the pupil in method and subject-matter. This objection could be removed by a reorganization of the course of study of the elementary school.

c. Familiarity of the question of setting problems.

d. Interest on the part of the teacher for the subject.

e. Superintendent may object if the text is not completed.

f. Meeting college entrance requirements.

(Personally I do not consider the last two objections as I do not see as it matters whether or no the superintendent approves; and there is no reason why a high school should articulate with any particular school.)

SOME ADVANTAGES OF THE METHOD.

1. The pupil works with a known purpose. The geometrical problems growing out of the child's experience founded as they are on their own data, are worked by the pupils with an amount of interest not to say eagerness which artificial problems can never inspire.

2. The pupil works because he needs the geometrical principles in his problems.

3. The method teaches by three effective ways:

a. The child seeks and finds truth largely by his own efforts.

b. By illustrating truth by picture, drawing, and concrete object.

c. By teaching him to express himself in his own way.

4. It trains in organizing ability—facts must be organized in sequence.

5. It develops originality and independence.¹⁰ "It helps to weigh evidence, draw accurate inferences, make fair comparisons, invent solutions, and form judgments."

6. It brings the home, school, mart, and farm into closer relation and gives a bond of common sympathy.

¹⁰Elliott, *Efficiency in Education*, p. 18.

7. Gives life and vital work—as practical utility is a good basis for school work.

8. The material and method follow the laws of mental condition as a result of growth.

9. A definite aim or problem causes the pupil to center his attention on some main idea, and thus furnishes him a proper motive for active thinking. It makes him conscious of the course he is to pursue. He knows definitely what he is to accomplish. "A¹¹ definite aim furnishes a motive for effort."

10. The pupil, not the subject-matter, is the center of the educational process. Instead of giving the pupil some logical truths for future use, or future forgetfulness, it makes geometry an instrument in the process of living in the present, thus giving a more useful instrument or tool for future use.

11. A good aim becomes a standard both for the teacher and the pupil, for judging the value of facts. The pupil is in a situation where he looks for the significance of facts. The systematic application of the psychological factor "suggestion" strengthens the interest and the desire to know, thus making the work easier for teacher and pupil.

12. This method helps to solve the question of relative values in school principles. The pupil learns to use his mathematics. He correlates arithmetic, algebra, geometry, physics, mechanics, and empirical knowledge in a way that convinces him that there may be a relation between school life and life outside of school. The rigidity of the solution is in no way lessened for a pupil has not proved a problem nor understood it until he has assimilated the facts concerning it.

SOME FACTS THAT CANNOT BE DETERMINED IN ONE SCHOOL YEAR IN ONE SCHOOL.

1. Determining the field in which the problems are best suited to the interests and experiences of the child.

2. Whether interest and knowledge of the boys and girls in respect to problem material are different, and if so, why.

3. How much supplementary explanation the teacher must give.

Many questions arise in any study and many cannot be answered until more data is gathered. One asked as often as any other, "Can the pupil stand the examination?" is best answered by the results obtained. The sophomore class in the high school

¹¹McMurry, *The Method of the Recitation*, p. 113.

was divided according to their seats in a study hall, one section following the text in the usual manner, the other section the plan discussed. In this method of division there could be no evident difference in the equipment of the pupils. The section that followed the text was under a different teacher, but one who was well prepared and experienced. Both sections belonged in the same building. The average standing of the two classes in a formal geometrical examination did not differ materially, while on an examination of interpretation, organization, and arrangement the class following the plan discussed averaged nearly twelve per cent above the other section, and stated their ideas in a more definite and economical manner. Enough time has not elapsed to determine whether or no the matter is better retained.

"We¹² should bear in mind that real progress should not be measured by the ground apparently covered, but rather by what the child actually gets in such a way as to make it his own." Probably ninety per cent of the work learned in school is soon forgotten because the child has never learned how to make use of the material gathered. Would it not be better to learn well the part that is covered? The facts learned in school are not the most important feature of the work but it is the attitude of the mind, the habits of study that the student forms that is important. He is prepared then to go out into the world and live the life he has formed the habit of living. Any serious objection to the plan that may be considered, must first be preceded by a consistent thorough trial, unbiased and unprejudiced.

In closing it is not out of place to state that in a brief discussion of any new thing many loose ends appear. They can only be adjusted as the new activity is developed. A recast of the present teacher and curricula, a not impossible thing, to the suggested plan will bring about an education that is utilitarian and cultural, vocational and liberal, and that is the meaning of education for efficiency.

¹²McMurry, *Method of the Recitation*, p. 160.