NOTE ON CENTRIFUGAL FORCE IN TURBINES.

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A note by Mr. J. P. Frizell (JOURNAL OF THE FRANKLIN INSTITUTE for June) especially interests the writer, as he has, recently, also noted the same defect in the accepted theory. It is so obvious as to require no proof that centrifugal force, even where it is necessarily considered at all, depends for its measure upon the actual rotation of the mass of fluid, and not at all upon the movement of the wheel, except so far as the latter controls the former. In water-wheels, the two rotations are very different in their angular measures.

Plotting the path of the water in space, it will often be found that the line is so nearly rectilinear that the action of this force may be neglected as unimportant in practice. This is probably the case, for example, in the Vallet wheels. In the Whitelaw turbine, or "Scotch reaction wheel," as it is sometimes called, the spiral form given the arms may often be given such proportions that the path of the water may be nearly a straight line to the exit, and centrifugal force thus rendered so small that it may be neglected in the theory of that wheel. The writer has been interested in noting how much this method of treatment simplifies the theory of turbines, and how much more nearly, in some cases, the results accord with experiment. In the last form of wheel, for example, the expression for maximum efficiency becomes

$$E = \frac{1}{\sec^2 \theta},$$

and the speed of maximum efficiency becomes such that the orifice moves with the velocity due the head. The maximum effort is exerted, as it should be, when the wheel is held fast, and the effort becomes zero, when the speed of orifice becomes twice that velocity.

For the common case, in which the velocity of rotation of the water is a maximum at its entrance into the wheel from the guides, and becomes zero at exit from the wheel, varying in the interval by a uniform retardation, we have, when v_1 is the entering "whirl," r_1 and r_2 are the two radii of the wheel, and $r_2 = n r_1$;

$$v = \frac{r_2 - r}{r_2 - r_1} v_1 = \frac{n - \frac{r}{r_1}}{n - 1} v_1;$$

the work done by or against centrifugal force is

$$\begin{aligned} W &= \frac{D Q}{g} \int_{r_1}^{n r_1} \frac{v^2}{r} dr; \\ &= \frac{D Q}{g} \cdot \frac{v_1^2}{(n-1)^2} \int_{r_1}^{n r_1} \left(n - \frac{r}{r_2}\right)^2 \frac{dr}{r}; \\ &= D Q \frac{v_1^2}{2g} \cdot \frac{2 n^2 \log_e n - 4 n (n-1) + n^2 - 1}{(n-1)^2} \end{aligned}$$

Applying this to the outward flow wheel of the Fourneyron type, having uniform radial flow, it is found that the effort due centrifugal force is about one-quarter the head due the velocity of entering whirl; but this velocity is that due to about one-third the total head, in this particular example, and it follows that the accelerated flow is not far from one-half that predicted by the usual theory, and is not very far from what Mr. Frizell gives as the results of Mr. Francis' experiments, and if the energy is given a uniform decrement, *i. e.*, $dv^2 = \text{constant}$ and $dr \propto \frac{1}{2v}$, for which case, $f(n) = \frac{2n \log_e n - 2(n-1)}{n-1}$, the corres-

pondence is almost exact, a coincidence, possibly, however, as the writer has not investigated that particular case.

It thus becomes evident that, in the general equation of efficiency for wheels of uniform radial flow, for which the writer gets the expression below, the functions representing this force may take the place in the formula there given them, and that the theory of the turbine, thus modified, is a much more manageable one in application than that previously held.

The general equation is, as derived by the writer,

$$E = \frac{(n^2 + 1) (2 a r_1 v_1 - a^2 r_1^2) - f(n) v_1^2}{n^2 v^2 \sec^2 \theta + 2 a r_1 v_1 - a^2 r_1^2 - f(n) v_1^2 + f(f) v_1^2}$$

when the angular velocity is denoted by a and the coefficients for centrifugal force and for friction are taken constant, as they may be for any one class of wheel, as $f(n)$ and $f(f)$, the delivery angle of the wheel being θ .

At the best speed, $v_1 = a r_1$; and

$$E = \frac{(n^2 + 1) - f(n)}{n^2 \sec^2 \theta + 1 - f(n) + f(f)};$$

which becomes unity when $f(f) = 0$ and $\theta = 1$. It is seen that the value $f(n)$ appearing in both numerator and denominator affects efficiency but little, since the expression is, for good wheels, nearly unity.

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