XLIX. Experiments on the propagation of longitudinal waves of magnetic flux along iron wires and rods

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XLIX. *Experiments on the Propagation of Longitudinal Waves of Magnetic Flux along Iron Wires and Rods.* By THOMAS R. LYLE, M.A., Sc.D., *Professor of Natural Philosophy in the University of Melbourne*, and J. M. BALDWIN, M.A., B.Sc.*

1. **T**HE following experiments were undertaken in order to investigate the changes in the Fourier characteristics of waves of magnetic flux as they travel along iron wires and rods. In all cases these flux-waves were started at the middle points of the rods or wires by sending alternating currents through short coaxial solenoids placed there.

The subject has already been attacked in different ways by several experimenters, and from the difference in phase of the flux oscillations at two points a so-called "velocity of magnetization" has been obtained from the formula,

$$v = \frac{2\pi l}{\phi T},$$

where v = the "velocity of magnetization,"

T = the period of the oscillations,

ϕ = the difference in phase at two points l centimetres apart.

Oberbeck † worked with rods and bundles of wires, in some cases 40 cm., in others 1 metre long, and by means of a

* Communicated by the Physical Society: read January 26, 1906.

† A. Oberbeck, *Wied. Ann.* vol. xxii. p. 73 (1884).

dynamometer measured the phase-difference of the currents induced in two short solenoids placed at various distances apart on the rod. This was equal to the phase-difference of the two resultant fluxes at the points, provided the time-constants of the circuits of the two solenoids were negligible.

The "velocity" deduced varied from 232,300 cm./sec. for 64 steel wires each 1.8 mm. diam., to 4410 cm./sec. for a steel rod 12 mm. diam., a result which indicates the great influence of eddy currents. He found that the "velocity" was not directly influenced by the character of the iron of the rod

The amplitude F_x of the resultant flux at different points he found to be given by the formula

$$F_x = F_0 e^{-\lambda x},$$

where λ is to a first approximation a constant depending on the material of the bar, but independent of the area of its cross-section.

The values he found for λ when x , the distance from the centre of the magnetizing solenoid, is in centimetres, were

$$\begin{array}{ll} \cdot 1027, \cdot 1017, \cdot 1007 & \text{for soft iron,} \\ \cdot 148 & \text{for hard iron,} \\ \cdot 1451, \cdot 1616, \cdot 1637 & \text{for steel.} \end{array}$$

One frequency only was used.

Trouton* attempted to obtain stationary flux-waves by the interference of two trains of waves travelling in opposite directions round a ring of iron wires, the two trains being produced by an alternating current of known frequency circulating in a short solenoid looped on the ring. He concluded, however, that the effects which he observed were not due to interference but to some peculiarity of the ring.

Zenneck†, assuming the permeability μ and the leakage coefficient $\lambda = -dF/Fdx$ constants, where F is the flux at x , obtained from theory expressions for the flux-waves at different points along iron cylinders in terms of the initial flux where $x=0$, and to verify his theoretical conclusions used a Braun tube to determine the relative amplitudes and phases of the flux oscillations at any two points.

For thick wires (some millimetres in diameter) he found that the "velocity" and the leakage coefficient λ increased with frequency, while for wires less than 1 mm. in diam.

* F. T. Trouton, 'Nature,' vol. xlv. p. 42 (1891).

† J. Zenneck, *Ann. der Phys.* vol. ix. p. 497 (1902), and vol. x. p. 845 (1903).

they were, to a first approximation, independent of the frequency.

More recently Perkins*, working at a frequency of 60, has measured by means of a quadrant-electrometer the amplitude and phase, at different points along an iron bar, of the flux oscillation which had been started in the usual way at the centre. The bar used was 1 metre long, 2.83 cm.² in cross-section, and observations were made over a range of 25 cm. from its centre.

The phase was found to change less rapidly as the distance from the magnetizing-coil increased, and the "velocity of magnetization" obtained varied from

7500 cm./sec. at the centre

to 24000 cm./sec. at 24 cm. from the centre.

The curve he obtained showing the relation between the lag in phase of the flux at any point and the distance of the point from the magnetizing-coil agrees in general character with the corresponding small portions of the curves obtained by us.

The same problem has been theoretically discussed by Thomson †. He assumes that the permeability is constant for all values of the induction, and takes no account of the lag in phase of the induction behind the magnetizing force. As the flux-density or induction diminishes with great rapidity as we pass down the bar, and as both the permeability and the lag in phase of the induction vary within wide limits for different values of the latter, the theoretical conclusions arrived at on the assumption that one of these two quantities is constant and the other negligible, can scarcely be expected to be realized in practice.

2. In all the experimental investigations hitherto carried out on this subject, the range along the rod over which it was possible to determine even roughly the amplitude and phase of the flux oscillations at different points was limited, and the apparent behaviour over a short range afforded a very doubtful and misleading description of the whole phenomenon. No attempt, moreover, was made to determine the change of form of the flux-waves as they passed along the bar.

For this investigation the wave-tracer ‡ designed by one of us is peculiarly well suited, as it enables us to determine

* H. A. Perkins, *Amer. Jour. Sci.* vol. xviii. p. 165 (1904).

† J. J. Thomson, 'Recent Researches,' p. 302.

‡ T. R. Lyle, "Wave-Tracer and Analyser," *Phil. Mag.* vol. vi. p. 549 (1903).

completely, not only current and E.M.F. waves, but also flux-waves over a wide range of amplitudes.

For instance, by its means can be accurately determined flux-waves whose amplitudes range from very large values down to a fraction of a single magnetic line; and by subjecting its readings to harmonic analysis we have been able to obtain the Fourier characteristics of the resultant flux oscillations at all points along rods of iron or steel, $\frac{1}{4}$ in. in diam. and 10 feet long, which have been magnetized by an alternating current circulating in a short solenoid placed at their middle points, when the amplitude of the flux-density at the centre was as low as 10,000.

3. The experiments, of which the results will be given below, were made on

- A. A long rod of Lowmoor iron $\frac{1}{4}$ in. in diameter.
- B. A long rod of silver steel $\frac{1}{4}$ in. in diameter.
- C. A long iron wire $\frac{1}{8}$ in. in diameter.
- D. A bundle of 185 charcoal iron wires, each .079 cm. in diameter, 370 cm. long, tightly taped together so as to form a circular cylinder.
- d_1 . A bundle of 46 of the wires from D.
- d_2 . A bundle of 12 of the wires from D.

In each case at the middle point of the rod or bundle a short coaxial solenoid was placed, and through it an alternating current obtained from a rotary converter supplied with direct current from storage-cells was sent.

The magnetizing-current wave and the resultant flux-waves, crossing the sections of the bar or bundle at different points along its length, were quantitatively determined by the wave-tracer using the galvanometer method * described in the paper already quoted. The wave forms so obtained were subjected to harmonic analysis, the results of which are given in the Tables below. Some of the more interesting of the results of the investigation are also exhibited by means of curves.

The arrangement of the apparatus, the method of experiment, and the reduction of the observations were the same in all essential features as that fully described in a former paper † by one of us. Instead of the fixed secondary coil of 5 or 10 turns wound on the iron rings used in the experiments just quoted, three small search-coils were used. One of these, wound close to the rod or bundle, was fixed at the

* T. R. Lyle, "Wave-Tracer and Analyser," *Phil. Mag.* vol. vi. p. 549 (1903).

† T. R. Lyle, "Variation of Magnetic Hysteresis with Frequency," *Phil. Mag.* vol. ix. p. 102 (1905).

centre of the magnetizing solenoid ; while the other two, of 166 and 805 turns respectively, could be moved from point to point along the rod, the one with the larger number of turns being used for exploring the rod at points near its ends.

During each series of experiments the magnetizing current and the rate of alternation were kept as constant as possible, and a continuous record was made by a chronograph of the times taken for every 200 periods.

The method of procedure in any one series of experiments was as follows :—The speed of the rotary converter having been adjusted to the desired value by means of resistances in the field circuit, or, if necessary, by changing the number of storage-cells used on its direct-current side, the magnetizing current drawn from its alternating side was adjusted by means of resistances to the desired value, and allowed to run until all conductors had attained a steady temperature.

By means of the wave-tracer equi-spaced ordinates of the magnetizing current wave sufficient in number to determine it were now taken. Then the corresponding ordinates of the flux-waves at the centre and at different points along the rod were obtained, and finally the ordinates of the current wave were redetermined to serve as a check on the constancy of the magnetizing force during the run.

Each set of ordinates was now subjected to harmonic analysis and the results affected by their proper factors* to reduce them either to current or flux, as the case might be, in absolute measure.

Three frequencies were used with specimens A and C, and two with specimen D, the central or initial flux being approximately the same for the different frequencies in any one specimen. With A and D additional series were obtained at one of the frequencies already used, but with different initial fluxes. For each of the other specimens B, d_1 , and d_2 , the results of one series of experiments only are given below.

For all the specimens but d_1 and d_2 , by means of a ballistic galvanometer, using the method of reversals, the fluxes at different points along their lengths, due to a continuous magnetizing current in the central solenoid, were determined when the continuous current was such as produced at the centre a flux approximately equal to the amplitude of the central alternating flux previously used.

In addition the statical permeability of the material of each of the specimens for different values of the flux density was obtained by the method of reversals, a long solenoid being used to carry the magnetizing current.

* T. R. Lyle, "Wave-Tracer and Analyser," *Phil. Mag.* vol. vi. p. 549 (1903).

The specific resistance of each specimen was also measured.

In the case of specimen A, after all the observations that were deemed necessary for its full length had been taken, it was shortened by successive steps by cutting off each time equal lengths from both ends; each reduced length was then investigated for one frequency and one central flux.

4. We now proceed to give the results of the different experiments.

For every series the Fourier characteristics of the magnetizing current wave and of the resultant flux wave at different distances from the centrally placed magnetizing solenoid will be given in tabular form, and curves will be given showing how the phase, and the natural logarithm of the amplitude, of the first harmonic of the resultant flux varies as we pass along the specimen from the centre to the end. In the tables will also be given for different points the values of the leakage coefficient λ_1 of the amplitude f_1 of the first harmonic of the flux, where

$$\begin{aligned}\lambda_1 &= -\frac{1}{f_1} \frac{df_1}{dx} = -\frac{d}{dx} (\log_e f_1) \\ &= \frac{(\log_e f_1)_x - (\log_e f_1)_{x'}}{x' - x} \text{ (q.p.)}\end{aligned}$$

at the point $\frac{1}{2}(x+x')$, where x and x' are two adjacent points at which f_1 has been determined. By means of the last expression the values of λ_1 have been calculated, and they are placed in the tables between the two adjacent rows of characteristics from which they have been obtained.

In the tables T will represent the period of the alternating current and flux in any one experiment, x the distance from the centre of the specimen (and of the magnetizing solenoid) of the section across which the resultant flux F (in absolute measure) passes, and C the magnetizing current used during one series (also in absolute measure).

No harmonics higher than the 5th will be given, though they were determined in all cases, unless their amplitudes were below one per cent. of the amplitude of the corresponding first harmonic.

The galvanometer circuit of the wave-tracer when search-coil No. 3 was used had a very small time constant which was allowed for.

[*Note.*—The figures given in the tables are the results from the individual experiments. No smoothing of values has been resorted to, though a small error in the setting of the search-coil would make a large error in the result.]

5. Details of Specimen A.

A straight cylindrical rod of Lowmoor iron.

Length = 267 cms.

Diameter = .644 cm. Section = .3260 sq. cm.

Length of magnetizing solenoid 2.54 cm.

Radial depth of " " 2.40 cm.

Mean radius of " " 1.70 cm.

No. of turns on " " 240.

Specific resistance of the iron = 1.140×10^4 .

Statcal Permeability for different Inductions.															
B ...	25	50	100	250	500	750	1000	2000	3000	5000	7000	9000	11000	13000	15000
μ ...	290	325	370	451	550	650	750	1160	1490	1960	2215	2300	2165	1720	900

The analysed results of the alternating current experiments with this specimen are given in the following tables (I. to IV.), and the values of the flux and leakage coefficients at different points due to a continuous magnetizing force at the centre of the rod are given in Table V.

TABLE I.—Specimen A. T=.053 q.p.

$C = .1371 \sin \omega t + .00642 \sin 3(\omega t - 55.28) + .00057 \sin 5(\omega t - 68.23).$									
$F = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5).$									
$x.$	T.	$f_1.$	$f_3.$	$f_5.$	$\theta.$	$\beta_3.$	$\beta_5.$	$\theta_x - \theta_0.$	$\lambda_1.$
0...	.0537	3798	51.7	23.3	30.64	93.35	31.85	0	.0744
5...	.0536	2618	70.1	14.0	41.42	92.04	32.19	10.78	.0852
10...	.0534	1710	54.8	7.97	53.59	92.35	29.17	22.95	.0826
15...	.0531	1131	37.6	5.54	64.41	94.68	31.68	33.77	.0855
20...	.0532	738	23.4	1.85	74.46	98.09	31.03	43.82	.1009
30...	.0518	269.1	6.83	...	93.21	106.06	...	62.57	.1167
40...	.0528	83.8	1.50	0.49	105.42	109.52	38.67	74.78	.1134
50...	.0530	27.0	0.28	...	107.27	...	...	76.63	.1042
60...	.0526	9.51	...	...	101.97	...	...	71.33	.0775
70...	.0515	4.383	...	...	92.31	...	...	61.67	.0529
80...	.0526	2.583	...	...	84.80	...	...	54.16	.0394
100...	.0527	1.175	...	...	78.00	...	...	47.36	.0333
120...	.0528	0.604	...	...	74.61	...	...	43.97	.0626
130...	.0528	0.323	.0058	.0035	69.64	133.3	...	39.00	

TABLE II.—Specimen A. T=·053 q.p.

$C = .2154 \sin \omega t + .01077 \sin 3(\omega t - 55.82) + .00072 \sin 5(\omega t - 72).$ $F = f_1 \sin (\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5).$									
<i>x.</i>	T.	<i>f</i> ₁ .	<i>f</i> ₃ .	<i>f</i> ₅ .	<i>θ</i> .	<i>β</i> ₃ .	<i>β</i> ₅ .	<i>θ</i> _{<i>x</i>} − <i>θ</i> ₀ .	<i>λ</i> ₁ .
0...	·0530	5009	125.6	43.05	30.72	96.80	37.6	0	·0809
10...	·0530	2232	109.3	23.7	54.80	88.69	25.2	24.08	·0818
20...	·0530	985.1	40.5	4.38	76.18	89.84	28.0	45.46	·0918
30...	·0532	393.4	9.46	...	94.87	95.80	...	64.15	·1091
40...	·0532	132.1	1.64	...	109.27	106.73	...	78.55	·1146
50...	·0532	42.0	...	...	113.64	...	...	82.92	·1076
60...	·0532	14.32	0.15	...	109.13	158.4	...	78.41	·0843
70...	·0531	6.16	0.10	...	98.23	177.2	...	67.51	·0571
80...	·0527	3.48	0.06	...	89.37	186.6	...	58.65	·0397
100...	·0525	1.572	0.027	...	80.84	195.9	...	50.12	·0340
120...	·0526	0.797	0.017	...	75.94	196.8	...	45.22	·0622
130...	·0526	0.428	0.009	...	71.04	203.0	...	40.32	

TABLE III.—Specimen A. T=·032 q.p.

$C = .1649 \sin \omega t + .00655 \sin 3(\omega t - 58.61) + .00096 \sin 5(\omega t - 36.1).$ $F = f_1 \sin (\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5).$									
<i>x.</i>	T.	<i>f</i> ₁ .	<i>f</i> ₃ .	<i>f</i> ₅ .	<i>θ</i> .	<i>β</i> ₃ .	<i>β</i> ₅ .	<i>θ</i> _{<i>x</i>} − <i>θ</i> ₀ .	<i>λ</i> ₁ .
0...	·0320	3730	78.14	26.8	33.04	89.39	32.02	0	·0891
10...	·0320	1530	66.2	14.2	59.70	86.88	24.30	26.66	·0942
20...	·0319	596.6	17.1	...	84.89	91.19	...	51.85	·1134
30...	·0320	192.0	3.6	...	107.21	104.68	...	74.17	·1289
40...	·0320	52.9	0.83	...	122.74	110.30	...	89.70	·1251
50...	·0321	15.14	0.12	...	121.88	136.54	...	88.84	·1027
60...	·0321	5.42	0.056	...	111.21	161.53	...	78.17	·0667
70...	·0321	2.783	0.029	...	99.47	178.23	...	66.43	·0470
80...	·0322	1.740	0.019	...	93.06	182.94	...	60.02	·0351
100...	·0322	0.862	...	...	88.20	...	...	55.16	·0325
120...	·0323	0.450	...	...	84.40	...	...	51.36	·0641
130...	·0322	0.237	...	...	77.3	...	...	44.3	

TABLE IV.—Specimen A. T=·0216 q.p.

C=·1776 sin ωt + ·00728 sin 3(ωt - 58·3) + ·00087 sin 5(ωt - 30·55). F= f_1 sin (ωt - θ) + f_3 sin 3(ωt - θ - β_3) + f_5 sin 5(ωt - θ - β_5).									
x .	T.	f_1 .	f_3 .	f_5 .	θ .	β_3 .	β_5 .	$\theta_x - \theta_0$.	λ_1 .
0...	·0216	3537	90·6	37·7	33·75	88·57	30·68	0	·0932
5...	·0216	2220	96·3	25·6	48·04	85·30	25·96	14·29	·1014
15...	·0216	805·5	29·3	3·4	77·30	85·64	22·53	43·55	·1162
25...	·0216	251·9	4·35		104·37	99·48		70·62	·1401
35...	·0217	62·05	0·67		125·69	111·39		91·94	·1426
45...	·0217	14·90	0·20		132·64	124·01		98·89	·1088
55...	·0217	5·02	0·06		117·00	158·35		83·25	·0723
65...	·0217	2·437	0·038		103·53	169·67		69·78	·0453
80...	·0217	1·236	0·015		94·73	178·70		60·98	·0364
100...	·0217	0·597	0·009		90·59	180·57		56·84	·0287
120...	·0216	0·336	0·006		87·13	183·43		53·38	·0547
130...	·0217	0·194	0·005		79·47	179·73		45·72	

TABLE V.—Specimen A. T = ∞ .
(Static Leak.)

x .	C.	F.	λ .	x .	C.	F.	λ .
0 ...	·0751	3863		40...	·0749	261·3	
5 ...	·0750	2992	·0511	50...	·0752	100·8	·0953
10 ...	·0752	2254	·0566	60...	·0748	38·7	·0955
20 ...	·0751	1237	·0600	80...	·0750	7·44	·0825
30 ...	·0751	609·4	·0708	100...	·0750	2·60	·0525
40 ...	·0749	261·3	·0847	110...	·0750	1·49	·0557

6. In order to exhibit graphically some of the more striking results contained in the preceding tables, the curves in figs. 1 and 2 have been plotted. In fig. 1 the abscissæ are distances x from the origin or magnetizing solenoid and the ordinates of one set of curves are the natural logarithms of the amplitudes of the first harmonic of the flux at different distances, while the ordinates of the other set of curves are the retardations in phase of the first harmonic of the flux at different distances behind that of the initial flux where $x=0$.

It will be seen that the $\log f_1, x$, curves are far from being straight lines as Zenneck assumed for the basis of his theoretical work. This is still more clearly shown in fig. 2,

Fig. 1.

Fig. 2.

where the leakage coefficients $\lambda_1 = -d(\log f_1)/dx$ are plotted against the distances from the origin at which they occur, as well as against $\log f_1$, the logarithm of the corresponding flux. Thus in all cases, including the statical, as we move away from the magnetizing coil, λ_1 first increases, attains a maximum, and then diminishes until near the end of the rod when it again increases. For the same initial flux both the points at which max. λ_1 occurs and the corresponding fluxes are different for different frequencies.

Figs. 1 and 2 show the effects of the end of the rod on the characteristics of the fluxes in its neighbourhood. Thus in fig. 1 the downward turn of the flux and phase curves due to the end effect can be recognized 30 cm. from the end of the rod, and the corresponding rise in the λ curves in fig. 2 is very marked. The end effect always manifests itself in this way, and in very long rods this change in the direction of curvature does not appear.

From the $\lambda_1, \log f_1$ curves in fig. 2 we see that for the same value of f_1 , λ_1 increases with the frequency. In the same set it will be noticed that one curve can be drawn (q.p.) through the two series of points for λ_1 when $T = .053$ arising from two different initial fluxes. This would suggest that λ_1 is independent of x and is a function of f_1 only. In the sequel it will be seen that λ_1 does depend on x as well as on f_1 , and the coincidence of the above two curves is due to the fact that with the initial values used (Tables I. and II.) there is never much difference between the abscissæ at which equal fluxes occur in the two series.

Returning to fig. 1 we see how, for different frequencies, the phase of the first harmonic of the flux at any point x lags behind that of the initial central flux at $x=0$.

Thus as the flux moves away from the magnetizing coil its phase is at first retarded at a fairly regular but diminishing rate for any one frequency, while for the higher frequencies the rate of retardation $d\theta/dx$ is higher. To this part of the phenomenon the observations of Oberbeck, Zenneck, and Perkins have been limited, and following them we would deduce (see § 1) from our results the following mean values for the "velocity of magnetization" over the first 30 cms. of the rod in question :—

3380 cm./sec.	when	$T = .053$ sec.
4790 cm./sec.	„	$T = .032$ „
6230 cm./sec.	„	$T = .0216$ „

Continuing along the rod, however, we see that the space-rate of retardation ($d\theta/dx$) of the phase becomes zero, and

the retardation itself a maximum at distances between 40 and 50 cms. from the starting-point, depending upon frequency and initial flux. After this the retardation diminishes, that is, the phase of the flux advances for the remainder of the length of the rod, and near the end the rate of advance ($-d\theta/dx$) is greatly increased, this latter being due to the end effect.

From this it would seem that the deduction by Oberbeck and others of a "velocity of magnetization" from the rate of retardation ($d\theta/dx$) of the phase of the flux close to the source by means of the formula

$$v = \frac{2\pi}{T \frac{d\theta}{dx}}$$

can hardly be legitimate.

For if so, we should have to allow that the "velocity" is infinite where the phase is stationary and $d\theta/dx=0$, and negative beyond this point, where the phase advances and $d\theta/dx$ is negative.

It is interesting to note that the points at which maximum phase retardation and maximum leakage coefficient occur are always situated very near each other.

7. It was thought that perhaps for the same frequency the phenomena at points in the rod beyond x might depend in great part only on the flux at x ; and if that were so, by making the initial flux as small as that at which maximum retardation occurred in any one of the preceding series, we should get an advance of phase right from the start at the magnetizing coil. To test this a rod of $\frac{1}{4}$ in. Lowmoor iron 12 ft. long, taken from the same stock as specimen A, was used*.

The phase of the flux-wave at any point was taken as given directly by the divided-circle readings of the wave-tracer when the ordinate was zero, and the amplitude was obtained by applying the proper factor to the maximum ordinate of the same flux-wave. This was sufficiently accurate for the purpose in hand. The results obtained are exhibited graphically in figs. 3, 4, and 5. The initial values of the fluxes used were 69, 163, 288, 687, 1732, 2994, and 4100, while the period was .052 sec. (q.p.) in all cases. In fig. 3 the logarithms (nat.) of the fluxes are plotted as ordinates against the corresponding distances from the origin as abscissæ, the points pertaining to any one initial flux being

* When we thought of this, test specimen A had been cut up for the purpose of investigating change of length.

Fig. 3.

Fig. 4.

joined (as also in figs. 4 and 5) by a continuous curve. In fig. 4 against the same abscissæ are plotted the retardation in phase of the flux at any point behind that of the initial central flux; while in fig. 5 the leakage coefficients at different points in the several series are plotted as ordinates against the logarithms of their corresponding fluxes.

Fig. 5.

It will be seen that in all cases over the wide range of initial fluxes used, the general nature of the phenomena is the same as that exhibited by figs. 1 and 2 and detailed in Tables I.-IV. Always as the flux moves away from the origin its phase is at first retarded, the retardation attains a maximum, and then the phase advances and keeps advancing (as the rod is long) until the flux is completely dissipated. The leakage coefficient also always begins by increasing, attains a maximum and then diminishes; and we found in this case as in all others that the points on the rod at which maximum retardation and maximum leakage coefficient occurred were practically coincident. A glance at figs. 4 and 5 will show the effect of change of initial flux in the same specimen at the same frequency on the retardations of phase and the leakage coefficients. Thus increase of initial flux increases the maximum retardation, diminishes the

maximum leakage coefficient, and increases the distance from the origin at which these two maxima concur.

As a result of this series of experiments it is plain that what may happen at any one point of the rod, both as regards rate of retardation (or of advance as the case may be) of phase and leakage depends both on the position of the point and on the value of the resultant flux at the point, and if we followed previous investigators in computing a "velocity of magnetization" we should obtain different velocities for the same flux and frequency, provided the points for which the velocity is computed were at different distances from the magnetizing coil.

Fig. 6.

8. Specimen A was now reduced in length in successive steps by cutting each time equal portions off its two ends. For each length one series of observations was taken in which the initial flux and the frequency were approximately the same as in the series already given for the full length in Table III. The results obtained are given in detail in Tables VI. to XI., and the general effect of reduction in length on the lag in phase and on the logarithm of the amplitude of the first harmonic of the flux is exhibited in fig. 6.

TABLE VI.—Specimen A reduced to $l=225$ cms. $T=.0316$.

$C=1595 \sin \omega t + .0054 \sin 3(\omega t - 62.4) + .0007 \sin 5(\omega t - 32.4).$ $F=f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5).$								
$x.$	$f_1.$	$f_3.$	$f_5.$	$\theta.$	$\beta_3.$	$\beta_5.$	$\theta_x - \theta_0.$	$\lambda_1.$
0.....	3605	99.7	26.0	32.40	89.83	37.18	0	.0913
10.....	1446	60.0	9.0	59.98	86.18	28.68	27.58	.0961
20.....	553.3	15.8	1.5	85.28	91.70	35.00	52.88	.1078
30.....	188.3	3.0	...	106.80	100.78	...	74.40	.1355
40.....	48.56	0.42	...	123.28	118.29	...	90.88	.1206
50.....	14.53	0.13	...	121.93	137.48	...	89.53	.1027
60.....	5.201	.062	...	111.64	171.46	...	79.24	.0701
70.....	2.580	.034	...	99.18	173.02	...	66.78	.0474
80.....	1.607	.017	...	92.01	180.20	...	59.61	.0430
90.....	1.045	...	...	88.19	...	...	55.79	.0388
100.....	.709	...	...	84.81	...	...	52.41	.0451
105.....	.566	...	...	81.20	...	...	48.80	.0795
110.....	.380	...	...	74.94	...	...	42.54	

TABLE VII.—Specimen A reduced to $l=165$ cms. $T=.0309$.

$C=.1683 \sin \omega t + .0063 \sin 3(\omega t - 60) + .0011 \sin 5(\omega t - 33).$								
$x.$	$f_1.$	$f_3.$	$f_5.$	$\theta.$	$\beta_3.$	$\beta_5.$	$\theta_x - \theta_0.$	$\lambda_1.$
0.....	3744	85.5	18.1	31.23	87.9	31.37	0	.0908
5.....	2378	95.9	17.7	44.70	88.08	28.94	13.47	.0922
15.....	945.7	37.6	4.84	71.78	89.15	33.77	40.55	.1087
30.....	185.0	1.9	...	106.68	102.12	...	75.45	.1332
40.....	48.84	0.57	...	123.06	115.74	...	91.83	.1224
50.....	14.37	0.15	...	122.20	141.90	...	90.97	.1025
60.....	5.153	.076	...	111.41	165.4	...	80.18	.0719
70.....	2.510	.028	...	97.95	173.9	...	66.72	.0676
75.....	1.790	.021	...	91.85	181.0	...	60.62	.1145
80.....	1.010	.011	...	84.91	183.0	...	53.68	

TABLE VIII.—Specimen A reduced to $l=125$ cms.
 $T=0.0305$.

$$C=1.722 \sin \omega t + 0.066 \sin 3(\omega t - 57.5) + 0.008 \sin 5(\omega t - 27).$$

x .	f_1 .	f_3 .	f_5 .	θ .	β_3 .	β_5 .	$\theta_x - \theta_0$.	λ_1 .
0.....	3787	89.7	20.7	32.86	90.88	29.23	0	.0917
10.....	1514	61.4	11.6	60.17	87.42	23.57	27.31	.0969
20.....	574.2	17.3	...	85.23	91.36	...	52.37	.1156
30.....	180.8	2.9	...	108.09	103.31	...	75.23	.1360
40.....	46.42	0.61	...	124.62	110.18	...	91.76	.1238
50.....	13.46	0.13	...	122.71	140.8	...	89.85	.1212
55.....	7.34	0.07	...	116.67	152.6	...	83.81	.1596
60.....	3.31	0.04	...	105.61	163.5	...	72.75	

TABLE IX.—Specimen A reduced to $l=85$ cms.
 $T=0.0294$.

$$C=1.711 \sin \omega t + 0.065 \sin 3(\omega t - 59.8) + 0.007 \sin 5(\omega t - 31.7).$$

x .	f_1 .	f_3 .	f_5 .	θ .	β_3 .	β_5 .	$\theta_x - \theta_0$.	λ_1 .
0.....	3684	91.5	26.8	32.60	91.4	30.14	0	.0918
10.....	1471	66.6	13.6	59.91	87.31	25.79	27.31	.0974
20.....	555.5	15.9	...	85.32	92.21	...	52.72	.1212
30.....	165.3	2.76	...	107.53	99.57	...	74.93	.1436
35.....	80.62	1.00	...	115.06	100.70	...	82.46	.2114
40.....	28.01	0.26	...	117.12	107.58	...	84.52	

TABLE X.—Specimen A reduced to $l=65$ cms.
 $T=0.0311$.

$$C=1.718 \sin \omega t + 0.067 \sin 3(\omega t - 58.1) + 0.008 \sin 5(\omega t - 26).$$

x .	f_1 .	f_3 .	f_5 .	θ .	β_3 .	β_5 .	$\theta_x - \theta_0$.	λ_1 .
0.....	3794	87.6	26.0	32.66	89.0	31.67	0	.0903
10.....	1539	65.6	12.1	59.72	85.97	27.87	27.06	.0922
15.....	970.4	35.9	5.3	71.41	88.43	25.40	38.75	.1046
20.....	575.1	19.42	1.85	82.82	92.76	...	50.16	.1309
25.....	298.9	7.71	...	91.79	96.36	...	59.13	.2087
30.....	105.3	2.38	...	96.67	96.33	...	64.01	

TABLE XI.—Specimen A reduced to $l=45$ cms.
 $T=0.0311$.

$C=1734 \sin \omega t + 0.0077 \sin 3(\omega t - 58.3) + 0.0003 \sin 5(\omega t - 27)$.								
x .	f_1 .	f_3 .	f_5 .	θ .	β_3 .	β_5 .	$\theta_x - \theta_0$.	λ_1 .
0.....	3820	94.9	29.1	32.01	88.37	27.95	0	.0862
5.....	2480	106.5	24.9	43.93	86.38	28.88	11.92	.1032
10.....	1502	75.3	15.4	55.44	86.16	28.13	23.43	.1139
15.....	850	41.1	7.2	64.26	89.41	31.94	32.25	.2048
20.....	305.3	13.9	2.5	70.11	91.65	30.36	38.10	

From the tables and from fig. 6 it will be seen that shortening the rod makes little perceptible difference either to the leakage of the flux or to its phase-lag until a point is reached at which the end effect begins to assert itself. Thus for the different lengths, flux-waves that are equal at the magnetizing coil are practically equal in all their characteristics after travelling equal distances along the rod, provided they have not reached to within 30 cms. or so from the ends.

9. The next specimen (B) tested was a rod of silver-steel of low permeability of the same section as specimen A. One frequency with one initial flux only was used, these being (q. p.) the same as in one of the series (Table III.) of experiments with specimen A, so that a direct comparison of the behaviour of the two materials could be readily made.

Details of Specimen B.

A straight cylindrical rod of silver-steel.
Length=286.6 cms. Diameter=.653 cm.
Section=.3351 cm².
Magnetizing solenoid and search-coils the same as were used with specimen A.
Specific resistance= 1.859×10^4 .

Static Permeability for different inductions.										
B	50	500	1000	2000	4000	6000	8000	10000	12500	15000
μ	73	96	122	170	248	298	316	290	222	140

TABLE XII.—Specimen B. $T = .0305$.

C=.224 sin ωt; F=f ₁ sin (ωt-θ).															
x	0	5	10	15	20	30	40	60	80	100	110	120	130	140	145
f ₁	3735	1855	700	236	80.3	16.6	5.35	1.34	0.53	.265	.190	.142	.106	.071	.053
θ	23.4	35.7	47.5	53.7	54.7	47.7	42.8	39.0	38.6	38.5	38.5	38.2	39.5	37.4	37.6
θ _x -θ ₀ ...	0	12.3	24.1	30.3	31.3	24.3	19.4	15.6	15.2	15.1	15.1	14.8	16.1	14.0	14.2
λ ₁	.140	.195	.217	.216	.157	.114	.069	.047	.035	.033	.030	.029	.040	.058	

TABLE XIII.—Static Leak. Specimen B. $T = \infty$.

Magnetizing Current $C = .2027$ (abs.).													
x	0	5	10	15	20	25	30	35	40	45	50	55	60
F	3592	1777	695	235	81.5	37.5	17.6	9.35	5.63	3.72	2.57	1.91	1.34
λ	.141	.188	.217	.211	.155	.151	.127	.101	.083	.074	.060	.067	

Tables XII. and XIII. give the results of the experiments with this specimen, the characteristics of the first harmonic of the flux only being given.

By comparing Tables III. and XII. one can see the difference in the behaviour of similar rods of iron and steel for the same initial flux and the same frequency. Fig. 7 illustrates the difference graphically. Thus, though near the origin the rate of retardation of phase ($d\theta/dx$) is nearly the same for both specimens (hence giving nearly equal fictitious velocities of magnetization), yet in the steel rod the retardation attains its maximum at less than half the distance from the origin at which it attains its maximum in the iron rod, and the maximum value in the latter rod is about three times what it is in the former. Again, for the steel rod the leakage coefficients are at first very much larger than for the iron one (see fig. 8), either at equal distances from the origin, or for equal fluxes; but when very low values of the flux have been arrived at, the leakage coefficients are quite as small in the steel as in the iron.

It will be noticed (see Table XII.) that in this specimen also the positions of max. retardation and max. leakage coefficient very nearly if not quite coincide.

10. The cause of the difference in the leakage-curves for the two rods is naturally to be looked for in their very different permeability-curves. Of the flux F which crosses any section

Fig. 7.

of the rod, $-dF$ leaks out in the next elementary length dx , and $F+dF$ continues in the rod, and the ratio of $-dF$ to $F+dF$ (*i. e.*, $-dF/F$ in the limit) will, when the magnetizing force is constant, be equal to the inverse ratio of the reluctances of the two corresponding paths. When the magnetizing force is periodic, the phenomenon is further complicated by the eddy currents set up in the rod, giving rise to back magnetomotive forces. If, for two different rods, the reluctances of two elementary leakage-paths (air) were equal, then

$-dF/F$ for the corresponding elements (and therefore λ , as $\lambda dx = -dF/F$) would be proportional to the reluctances of the second paths (partly in the materials), and thus would be greater for the less permeable substance. However, as λ depends on the ratio of two reluctances, it is quite conceivable that this ratio might be smaller for points on a rod of low permeability than for the corresponding point on a rod of higher permeability. (See fig. 8.)

Fig. 8.

Again, λ at a point near the end of a finite rod for a given initial flux and frequency must be greater than λ for a point equidistant from the magnetizing solenoid in an infinite rod of the same material and cross-section; for the reluctances of the elementary leakage paths through the air will be equal, while the second path for the infinite rod must have a lower reluctance than that for the finite rod, seeing that iron in the former takes the place of some of the circuit, which is air for the latter. Thus the "end effect" on the leakage coefficient is to make its value larger.

11. C, the next specimen tested, was a long thick iron wire. Three series of experiments were made with it for three different frequencies and approximately the same initial flux.

Details of Specimen C.

A straight iron wire.
Length=324 cms. Diameter=.398 cm.
Section=0.1244 cm².
Specific resistance=1.006 × 10⁴.
Magnetizing solenoid and search-coils the same as were used with specimen A.

Statical Permeability for different inductions.													
B.....	100	500	1000	2000	4000	5000	6000	8000	10000	12500	15000	17500	20000
μ	230	435	625	990	1385	1470	1455	1320	1100	800	540	330	170

In Tables XIV., XV., and XVI. are given the analysed results of the alternating flux experiments with this specimen, and in Table XVII. the results with the continuous flux.

TABLE XIV.—Specimen C. T=.0545.

$C = .1742 \sin \omega t - .0091 \sin 3(\omega t + 1.15) - .0011 \sin 5(\omega t - 26.1).$ $F = f_1 \sin (\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5).$								
$x.$	$f_1.$	$f_3.$	$f_5.$	$\theta.$	$\beta_3.$	$\beta_5.$	$\theta_x - \theta_0.$	$\lambda_1.$
0	2360	189.0	41.8	22.05	3.28	45.08	0	.0898
5	1505	100.9	19.0	34.50	- 6.10	45.07	12.45	.1150
10	847.5	59.8	10.2	47.18	-11.81	40.74	25.13	.1245
20	244	16.5	1.6	69.82	-14.88	48.26	47.77	.1760
30	42.0	1.9	0.3	83.97	- 8.82	63.34	61.92	.1774
35	17.3	0.44	0.12	79.09	- 1.85	68.62	57.04	.1800
40	7.02	0.09	0.05	71.05	- 7.73	79.06	49.00	.1207
50	2.10	0.09	...	53.54	-11.45	...	31.49	.0759
60	0.98	0.06	...	46.23	- 5.57	...	24.17	.0510
70	0.59	0.05	...	39.72	+ 1.90	...	17.67	

TABLE XV.—Specimen C. T=·034.

C=·1737 sin ωt - ·0096 sin 3(ωt - 0·24). F= f_1 sin (ωt - θ) + f_3 sin 3(ωt - θ - β_3) + f_5 sin 5(ωt - θ - β_5).								
x .	f_1 .	f_3 .	f_5 .	θ .	β_3 .	β_5 .	$\theta_x - \theta_0$.	λ_1 .
0	2220	87	34	27·00	- 1·33	42·56	0	·1034
5	1326	54	15	40·55	- 16·36	36·21	13·55	·1272
10	702	36	7	56·07	- 20·51	36·24	29·07	·1351
20	181·7	8·9	...	81·94	- 18·71	...	54·94	·1904
30	27·1	0·6	...	96·95	- 6·62	...	69·95	·1792
35	11·05	0·07	...	88·46	+ 16·98	...	61·46	·1687
40	4·75	0·04	...	77·14	66·04	...	50·14	·1123
50	1·55	0·04	...	56·41	130·2	...	29·41	

TABLE XVI.—Specimen C. T=·0234.

C=·1803 sin ωt - ·0072 sin 3(ωt + 3·32) + ·0016 sin 5(ωt - 6·6). F= f_1 sin (ωt - θ) + f_3 sin 3(ωt - θ - β_3) + f_5 sin 5(ωt - θ - β_5).								
x .	f_1 .	f_3 .	f_5 .	θ .	β_3 .	β_5 .	$\theta_x - \theta_0$.	λ_1 .
0	2115	75	28	30·40	- 15·46	42·70	0	·1148
5	1192	59	16	45·26	- 26·67	35·00	14·86	·1388
10	595·4	29·4	6·4	62·07	- 27·00	30·72	31·67	·1515
20	130·9	4·9	...	91·13	- 22·21	...	60·73	·2053
30	16·80	0·18	...	104·61	+ 5·08	...	74·21	·1763
35	6·96	0·10	...	90·53	49·92	...	60·13	·1494
40	3·30	0·07	...	76·41	66·49	...	46·01	·1057
50	1·14	0·06	...	58·76	94·31	...	28·86	

TABLE XVII.—Static Leak. Specimen C. T= ∞ .

Magnetizing Current, C=·1844 (abs.).												
x	0	5	10	15	20	25	30	35	40	45	50	55
F	2220	1476	919	559	318	162	77·5	31·6	12·6	5·83	3·01	1·98
λ	·082	·095	·099	·123	·135	·147	·179	·183	·154	·132	·084	

In the series of experiments with this specimen the flux density at the centre was much higher (18,000 instead of 10,000) than in the corresponding series with specimen A. The series with rod A, given in Table II., in which the induction at the centre was about 15,000, will, however, afford a comparison under more nearly similar conditions, and we can see that the leakage coefficients for the thinner rod are considerably higher than for the other (see fig. 8). On account of the smaller sectional area and greater leakage, observations were of necessity limited to a shorter range than for the other specimens, but within this range the phenomena observed and recorded in the preceding tables are similar in all respects to those observed with the previous specimens. In particular the leakage coefficient again attains a maximum value at or very near to the position for maximum phase-lag.

It is also worthy of notice that the value of $d\theta/dx$ near the origin is larger than for specimen A, the mean values of the fictitious "velocity of magnetization" for the first 20 cms. (a range within which in this specimen $d\theta/dx$ is fairly uniform) being

$$\begin{array}{ll} 2750 \text{ cms./sec.} & \text{when } T = \cdot 0545, \\ 3850 \text{ cms./sec.} & \text{,, } T = \cdot 034, \\ 5060 \text{ cms./sec.} & \text{,, } T = \cdot 0234, \end{array}$$

values considerably smaller than those for specimen A. This

Fig. 9.

agrees with the results of some experiments that will be described further on, which show that reducing the diameter

of the specimen, other things being equal, increases the space rate of retardation of the flux near the origin.

There might seem to be a discrepancy when the values of f_1 for $x=0$ in Tables XIV., XV., XVI. relative to the first harmonics of their magnetizing currents are compared with the value of F_0 in Table XVII. relative to the continuous current producing F_0 . The iron seems to be quite as permeable under alternating magnetization as under continuous. This apparent discrepancy will disappear (and this remark will apply to the other specimens) when it is remembered (a) that the form of the current wave is peaked and its max. ordinate is greater than the amplitude of its first harmonic, (b) that the form of the flux-wave is flat and its max. ordinate is less than the amplitude of its first harmonic.

Thus the max. values of C and F_0 for the different values of T in the above are as follows :—

T.....	∞	·0545	·034	·0234
C _{max.}	·1844	·1854	·1850	·1866
F _{max.}	2220	2160	2100	2034

12. Eddy currents must play an important part in the phenomena under consideration, and their effects in this connexion may, in a general way, be explained as follows. If the vector OF (fig. 10) represent the resultant flux down

Fig. 10.

any cylindrical portion of a rod, the E.M.F. produced by its variation in a circuit in the rod round the outside of this cylinder, being in quadrature with OF and behind it in phase, may be represented by a vector FE where the angle OFE is a right angle. This E.M.F. generates an eddy

current which generates a flux down the cylinder that lags behind FE in phase, and hence may be represented by some such vector as FM or IF, where IF=FM. Hence the vector OI will represent in amplitude and phase the flux that would have passed down the cylinder if the eddy current had not been present. Thus we see that the effects of eddy currents at any point of one of our specimens are (a) an increase in the phase retardation, *i. e.* in $\theta_x - \theta_0$; (b) a diminution in the amplitude of the flux there, which manifests itself as an increase in λ the leakage coefficient.

A further set of experiments was undertaken in which these eddy-current effects were much reduced by replacing the solid rod by a cylindrical bundle of soft iron wires, called specimen D, of which the details are as follows:—

Details of Specimen D.

A cylindrical bundle of soft iron wires.

Number of wires = 185.

Length of each wire = 368 cms.

Diameter „ = .0791 cm.

Area of iron section of specimen = .909 cm.²

Diameter of bundle = 1.5 cm.

Specific resistance of iron = 1.366×10^4 .

No. of turns on magnetizing coil = 366.

„ „ search coil central = 5.

„ „ „ „ *a* = 10.

„ „ „ „ *b* = 50.

„ „ „ „ *c* = 100.

Statical Permeability for different inductions.

B	100	1000	2000	4000	5000	6000	8000	10000	12000
μ	202	475	750	1140	1240	1260	1125	875	575

In Tables XVIII., XIX., and XX. are given the results of three series of experiments with this specimen at one frequency ($T=.0255$), but with different initial fluxes, and in Table XXI. those of one series at the frequency for which $T=.0515$, the initial flux being approximately the same as in one of the preceding series, while in Table XXII. are given the results for continuous magnetizing force also with approximately the same central flux.

TABLE XVIII.—Specimen D. T = .0255.

$C = .1697 \sin \omega t + .0296 \sin 3(\omega t - 58.6) + .0091 \sin 5(\omega t - 65.4).$ $F = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5).$								
$x.$	$f_1.$	$f_3/f_1.$	$f_5/f_1.$	$\theta.$	$\beta_3.$	$\beta_5.$	$\theta_x - \theta_0.$	$\lambda_1.$
0.....	16190	.086	.020	15.77	16.49	29.88	0	.0285
7.3.....	13150	.097	.021	19.52	12.33	26.98	3.75	.0460
10.....	11620	.107	.021	21.19	10.56	24.44	5.42	.0414
15.....	9440	.121	.021	24.27	8.34	20.73	8.50	.0278
20.....	7816	.132	.022	27.38	6.77	17.72	11.61	.0383
25.....	6456	.141	.025	29.97	5.68	15.62	14.20	.0396
30.....	5292	.151	.024	32.50	4.72	11.33	16.73	.0455
40.....	3360	.166	.033	36.30	4.17	9.86	20.62	.0530
50.....	1976	.179	.041	39.23	3.61	8.47	23.46	.0614
60.....	1070	.184	.045	40.97	4.35	8.58	25.20	.0716
80.....	255.4	.182	.046	41.20	5.52	9.46	25.43	.0651
100.....	69.5	.160	.041	37.90	6.98	12.83	22.13	.0501
120.....	25.47	.147	.037	33.38	9.14	15.56	17.61	.0381
140.....	11.91	.133	.031	30.43	9.04	18.42	14.66	.0270
160.....	6.94	.145	.027	28.40	11.32	19.95	12.63	.0487
180.....	2.62	.135	.036	26.83	12.52	19.43	11.06	

TABLE XIX.—Specimen D. T = .0254.

$C = .0846 \sin \omega t + .0078 \sin 3(\omega t - 62.17) + .0013 \sin 5(\omega t - 4.49).$ $F = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5).$								
$x.$	$f_1.$	$f_3/f_1.$	$f_5/f_1.$	$\theta.$	$\beta_3.$	$\beta_5.$	$\theta_x - \theta_0.$	$\lambda_1.$
0.....	10030	.057	.005	18.98	26.27	39.69	0	.0304
7.3.....	8035	.065	.009	22.67	20.12	33.50	3.69	.0450
10.....	7121	.073	.010	25.07	16.81	30.69	6.09	.0433
15.....	5730	.085	.010	29.27	13.05	27.63	10.29	.0432
20.....	4618	.098	.012	32.16	10.86	24.44	13.18	.0437
25.....	3710	.106	.014	34.35	10.07	22.20	15.37	.0545
30.....	2826	.116	.014	37.23	8.71	20.85	18.25	.0545
40.....	1639	.136	.022	41.35	7.60	16.26	22.37	.0652
50.....	854	.142	.025	43.89	7.64	15.63	24.91	.0700
60.....	424	.149	.031	45.05	8.07	15.34	26.07	.0741
80.....	96.4	.136	.030	43.35	10.08	18.86	24.35	.0592
100.....	29.5	.118	.021	39.10	15.05	22.70	20.12	.0469
120.....	11.55	.101	.020	35.73	13.69	20.63	16.75	.0320
140.....	6.09	.092	.020	32.56	16.56	28.71	13.58	

TABLE XX.—Specimen D. $T = .0255$.

$C = .0353 \sin \omega t + .0012 \sin 3(\omega t - 58.29).$ $F = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5).$								
$x.$	$f_1.$	$f_3/f_1.$	$f_5/f_1.$	$\theta.$	$\beta_3.$	$\beta_5.$	$\theta_x - \theta_0.$	$\lambda_1.$
0	2958	.049	.006	13.72	27.77	35.36	0	.0469
7.3	2100	.058	.006	17.55	21.02	32.80	3.83	.0761
10.....	1710	.066	.008	19.42	19.47	28.70	5.70	.0751
15.....	1175	.077	.011	22.77	17.29	28.91	9.05	.0745
20.....	809	.084	.014	24.68	15.51	26.75	10.96	.0771
25.....	550	.092	.013	26.03	14.98	25.48	12.31	.0814
30.....	366.4	.096	.016	26.98	14.98	24.07	13.26	.0805
40.....	163.8	.100	.019	27.35	15.60	25.08	13.63	.0824
50.....	71.8	.100	.019	27.39	16.49	25.41	13.67	.0756
60.....	33.7	.103	.018	26.40	17.95	28.16	12.68	.0677
70.....	17.13	.089	.015	25.77	18.58	28.79	12.05	.0591
80.....	9.48	.082	.016	24.73	18.51	26.98	11.01	.0458
90.....	6.00	.079	.010	24.35	17.68	30.36	10.63	.0459
100.....	3.79	.083	.013	23.54	16.81	30.66	9.82	.0345
120.....	1.90	.072	.005	20.19	18.16	30.96	6.47	.0287
140.....	1.07	.067	.015	18.85	19.50	31.50	5.13	

A glance at the columns for $\theta_x - \theta_0$ and λ_1 in Tables XVIII. to XXI. will show that so far as phase retardation and leakage coefficients are concerned, the general character of the results is the same as has been obtained from the other specimens; and, as was expected, not only are these quantities much reduced in magnitude, but also the effect of change of frequency on them is less marked. In addition the apparently coincident point of maximum phase retardation and maximum leakage coefficient is much further removed from the origin than in any of the other specimens.

13. A much more striking result, however, is the way the upper harmonics of the flux, and hence its wave form, vary as the flux-wave passes along the specimen. In these series the ratios f_3/f_1 and f_5/f_1 of the amplitudes of the third and fifth harmonics to that of the first are much larger than in the previous ones, where the flux wave was almost sinusoidal. We were therefore able to determine for this specimen these upper harmonics both in amplitude and phase with certainty for greater distances from the origin than for the other

TABLE XXI.—Specimen D. $T = .0515$.

$C = .0790 \sin \omega t + .0050 \sin 3(\omega t - 58.46) + .0006 \sin 5(\omega t - 64.86).$ $F = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5).$								
$x.$	$f_1.$	$f_3/f_1.$	$f_5/f_1.$	$\theta.$	$\beta_3.$	$\beta_5.$	$\theta_x - \theta_0.$	$\lambda_1.$
0	9472	.065	.012	16.92	19.80	33.23	0	.0304
7.3	7592	.077	.013	19.90	14.58	28.61	2.98	.0454
10.....	6700	.083	.014	21.68	13.83	28.83	4.76	.0510
15.....	5193	.098	.017	25.15	10.83	23.91	8.23	.0441
20.....	4167	.115	.016	27.78	9.34	22.38	10.86	.0455
25.....	3319	.123	.019	30.09	8.81	17.68	13.17	.0519
30.....	2560	.136	.022	32.16	8.00	17.50	15.24	.0594
40.....	1414	.153	.033	36.00	7.75	15.05	19.08	.0653
50.....	736	.159	.032	38.16	6.59	13.65	21.24	.0721
60.....	358	.165	.039	39.00	7.42	14.31	22.08	.0789
70.....	162.9	.155	.036	38.38	8.22	14.35	21.46	.0727
80.....	78.7	.152	.038	36.91	9.33	14.39	19.99	.0600
100.....	23.7	.131	.028	33.41	10.42	18.50	16.49	.0443
120.....	9.76	.121	.026	30.06	11.16	22.35	13.14	.0333
140.....	5.01	.107	.014	28.73	12.78	19.78	11.81	.0269
160.....	2.92	.099	.023	28.26	10.25	22.50	11.34	.0354
170.....	2.20	.099	.023	26.27	14.24	28.24	9.35	.0596
180.....	1.13	.093	.020	25.65	14.62	29.63	8.73	

TABLE XXII.—Specimen D. Static Leak. $T = \infty$.

Magnetizing Current. $C = .0915$ (obs.).															
$x..$	0	7.3	10	15	20	25	30	40	50	60	75	90	100	110	120
F...	9775	7700	6517	5235	4154	3267	2542	1429	740	355	120	45.6	26.9	15.8	11.1
$\lambda...$	.0327	.0617	.0439	.0462	.0481	.0502	.0576	.0658	.0735	.0723	.0644	.0528	.0533		.0531

specimens; and the results given in the tables show that as we move away from the magnetizing coil we find (α) that the ratios f_3/f_1 and f_5/f_1 first increase, then both attain a maximum at approximately the same place, and after this diminish, and this maximum point is the same as that at

which the phase retardation and the leakage coefficient of the first harmonic attain their maxima; (b) that the retardations in phase β_3, β_5 of these harmonics behind the primary first diminish, become minima, and then increase, and that the point of minimum values of β_3 and β_5 is the same apparently as that of maxima f_3/f_1 and f_5/f_1 as well as that of maxima $\theta_x - \theta_0$ and λ_1 . The above may be differently stated as follows:—As the flux-wave moves away from the origin its form gradually changes, becoming flatter and flatter until it reaches the point of maximum lag and maximum leakage coefficient. After this as it continues to move on its flatness steadily diminishes. This change of form is exhibited in fig. 9, in which are plotted the two wave forms for $x=0$ and $x=60$ in the series given in Table XVIII. They are placed inaccurately as regards relative phase, but the ordinates of the one for $x=60$ are increased so that the amplitudes of the first harmonics of the two curves are the same.

Further interesting information as to the behaviour of the upper harmonics of the flux may be obtained by considering their actual amplitudes. Taking the third harmonics for any of the series in this paper, we find that the leakage coefficients are different from what we estimated they would be for a primary or first harmonic of the same amplitude and frequency at the same section of the rod. In some cases, indeed, as will be seen from the tables given, the amplitude of the third harmonic actually increases as the distance from the magnetizing coil increases.

Hence we cannot consider the upper harmonics as being independent of the first harmonic or of each other. Each seems to get continually reinforced by the first harmonic as well as by other harmonics lower in order than itself as it travels along. Such a transference of energy from the first harmonic to the higher ones must be accompanied either by a reduction of the amplitude of the first harmonic, which in these experiments would manifest itself as leakage in λ_1 , or by a retardation of its phase helping to increase $\theta_x - \theta_0$, or by both; so it is probable that this reinforcement of the higher harmonics by the first has an important bearing on the theory of the experiments described in this paper.

14. In order to investigate the effect of change of diameter we made two smaller bundles with wires taken from specimen D. One of these called d_1 contained 46, and the other called d_2 12 wires, and as D originally consisted of 185 wires the sectional areas of D, d_1 , and d_2 were very nearly in the proportions of 16 to 4 to 1 or their diameters as 4 to 2 to 1.

One series of experiments was performed with each of d_1 and d_2 under approximately the same conditions of frequency and initial flux-density as the series already performed on D and recorded in Table XXI.

In Tables XXIII. and XXIV. are given the results of the new series, and it will be seen on comparing them with Table XXI. and with each other, that the effect of change of diameter is if anything more marked than that of permeability which was illustrated by the comparative series on specimens A and B.

Thus we find that the values of $d\theta/dx$ near the origin for the diameters 4, 2, and 1, are 4.75, 9.9, and 15.3 respectively; so that if we deduced a "velocity" from these figures we should get very different values for it, in the same material, with the same frequency and flux-density.

The change of diameter seems to have little or no effect on the max. value of the retardation, though it has a marked effect on the distance from the origin at which this maximum occurs. Thus the coincident points of max. retardation and max. leakage coefficient are at distances from the magnetizing solenoid approximately proportional to the diameters of the specimens in which they occur.

Again we find that reduction of diameter increases the leakage coefficients (for the same conditions of frequency and flux-density) approximately in the same ratio as the diameter is reduced.

TABLE XXIII.—46 wires from Specimen D.

$$T = 0.050.$$

$C = 0.0742[\sin \omega t - 0.042 \sin 3(\omega t + 0.28) + 0.035 \sin 5(\omega t - 66)].$ $T = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5).$								
$x.$	$f_1.$	$f_3/f_1.$	$f_5/f_1.$	$\theta.$	$\beta_3.$	$\beta_5.$	$\theta_x - \theta_0.$	$\lambda_1.$
0.....	2583	0.0603	0.0095	14.32	17.09	25.24	0	0.0857
10.....	1096	0.1113	0.0158	24.23	7.07	14.75	9.91	0.1181
20.....	336.6	0.1530	0.0302	31.41	5.59	9.76	17.09	0.1508
25.....	158.3	0.1630	0.0345	33.45	4.83	8.93	19.13	0.1596
30.....	71.3	0.1583	0.0374	32.86	5.69	9.35	18.54	0.1559
40.....	15.0	0.1400	0.0321	31.18	8.44	13.64	16.86	0.1147
50.....	4.76	0.1214	0.0225	28.14	10.37	18.80	13.82	0.0742
60.....	2.32	0.1033	0.0205	25.92	10.45	19.94	11.60	

TABLE XXIV.—12 wires from Specimen D.

T=·050.

$C = \cdot0463[\sin \omega t - \cdot05 \sin 3(\omega t + 1\cdot63)].$ $F = f_1 \sin(\omega t - \theta) + f_3 \sin 3(\omega t - \theta - \beta_3) + f_5 \sin 5(\omega t - \theta - \beta_5).$								
$x.$	$f_1.$	$f_3/f_1.$	$f_5/f_1.$	$\theta.$	$\beta_3.$	$\beta_5.$	$\theta_x - \theta_0.$	$\lambda_1.$
0.....	681	·080	·0164	15·60	11·35	16·24	0	·1690
10.....	125·6	·1534	·0312	30·88	4·88	6·61	15·28	·2820
15.....	30·7	·1637	·0396	34·72	4·89	6·92	19·12	·3142
20.....	6·37	·1485	·0312	32·10	7·61	11·04	16·50	·2230
25.....	2·09	·1210	·0259	25·51	10·73	16·73	9·91	·1641
30.....	0·92	·0776	·0172	25·26	11·71	25·18	9·66	

In particular it is most striking that the max. values of λ_1 , viz.:—

·3142, ·1596, and ·0789,

should be so nearly inversely proportional to the corresponding diameters, 1, 2, and 4 respectively, the corresponding products being

·3142, ·3192, and ·3156 respectively.

In addition we find with the smaller bundles d_1 and d_2 the same change of form of the flux-wave as it passes along them as in the case of specimen D, and also the same peculiarity in the behaviour of the upper harmonics both as regards their amplitudes and phases as has been drawn attention to in the preceding paragraph in relation to the thicker bundle.

15. The fact that so many of the characteristics of the alternating flux attain either their maximum or minimum values at practically the same point in the specimen (particularly in D, d_1 , and d_2) is undoubtedly the most striking result of this investigation.

This point might be called the critical point for the specimen under the conditions of initial flux and frequency used, its distance from the magnetizing solenoid the critical distance, and the values of the different characteristics of the flux there the critical values. For the existence of such a critical point we are unable to offer any explanation.

A more fundamental knowledge than any we at present possess of the nature of magnetization and permeability is required. Our want of any exact knowledge of the amplitude and phase relations between induction and magnetizing force and of the relations between the different harmonics of an

induction wave precludes the possibility of a satisfactory solution on the basis of our present knowledge.

16. It was thought, however, that a partial explanation of some of the phenomena observed might be obtained if the initial flux was made up of elementary magnetic lines very heterogeneous as regards their phases. As these passed along the specimen there would be a retardation of the phase of each line due to "velocity" as well as a change of the phase of the resultant flux at any section due to groups of lines in special phases leaking out at special distances from the origin. If this were so, the phases of the lines at different distances from the axis of the rod or bundle should be different, as at any section those lines about to leak out would be near its circumference. To test this the wires of specimen D were divided into three groups. The first group of 28 wires was taped to form a circular cylinder, and round it was wound a search-coil of 50 turns insulated. Over this central cylindrical bundle the second group of 45 wires was uniformly distributed and taped so that the whole formed a circular cylinder coaxial with the first. An insulated search-coil of 50 turns was wound round the middle of the compound bundle formed. The remaining group of 110 wires was now added and taped so as to be coaxial with the others, and a third search-coil of 50 turns was wound round the middle of the whole.

The bulging caused by the search-coils was not very great. The magnetizing coil was wound on a bobbin, and could be placed at any distance on the specimen from the search-coils. By means of a switch the search-coils could be connected in different ways, so that when continued to the wave-tracer the latter determined for us the flux-waves through the central bundle or through either of the surrounding tubular bundles of wires for different distances from the magnetizing coil.

The result of this investigation was to show that while at the centre of the magnetizing coil the flux-density and phase varied over the section—the phase of the first harmonic for the outer layer being 4·56 degrees ahead of that for the central portion and the amplitude varying from 67 lines per wire for the outside portion to 58·5 lines per wire for the central,—yet for distances greater than 15 cms. from the magnetizing coil there was no appreciable difference in the phases of the first harmonics, wave-forms, or densities of the fluxes through the three different portions of the compound bundle. Hence no such explanation as that imagined at the beginning of this paragraph can be tenable.

17. It was also thought that if we determined in amplitude and phase the magnetizing force required to produce the

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different resultant fluxes at different sections of the specimen in any one series of the transmission experiments, we might obtain some information that would help to explain the phenomena.

This was done for specimen A and for a cylindrical bundle made up of 46 of the iron wires from specimen D.

The specimen was inserted in a long solenoid and by means of the wave-tracer, using it as described in a former paper * by one of us, the amplitudes and phases of the magnetizing forces required to produce uniform oscillating fluxes of different amplitudes and phases were obtained and the results plotted. From these curves and the results given in this paper on specimen A, and similar results obtained from the narrow bundle, we were able to obtain the amplitude and phase of the *resultant* magnetizing force that acted at each section of the specimens in the transmission experiments. When these characteristics of the magnetizing force were plotted against corresponding distances along the specimen from the magnetizing solenoid, the curves obtained were less instructive than those for the characteristics of the flux at different distances which have been given in this paper.

18. To sum up :—When waves of magnetic flux that have been started by alternating currents in a short solenoid placed at the centre of a long iron rod, or bundle of iron wires, are transmitted along the rod or bundle, we find that—

(1) The retardation of phase of the first harmonic of the flux at any point distant x from the centre, behind the first harmonic of the initial flux, first increases with x , attains a maximum and then diminishes, and keeps diminishing until the flux is dissipated if the specimen is sufficiently long for this to be effected.

(2) The leakage coefficient λ_1 of the amp. f_1 of the first harmonic, which we define by the equation $\lambda_1 = -\frac{1}{f_1} \frac{df_1}{dx}$, first increases with x , attains a maximum, and then diminishes and keeps diminishing until the flux is dissipated.

(3) The distances from the magnetizing solenoid at which the phase retardation and the leakage coefficient become maxima are equal (or very nearly so), and the point at which these maxima occur we call the *critical point* of the specimen, and its distance from the magnetizing solenoid the *critical distance*, for the particular initial flux and frequency used.

(4) Previous investigators of this subject, using less sensitive methods, were only able to obtain observations within the

* T. R. Lyle, "Variation of Magnetic Hysteresis with Frequency," Phil. Mag. vol. ix. p. 102 (1905).

critical distance, and from their observations concluded (1) that the leakage coefficients were practically constant at all distances x , thus arriving at a logarithmic decrease of flux amplitude; (2) that the fairly regular space-rate of phase retardation $d\theta/dx$ observed would be completely accounted for by a velocity of magnetization v , deduced by the formula

$$v = \frac{2\pi}{T d\theta/dx}.$$

We find that neither of these conclusions is correct, as (a) λ , for the same rod and conditions of experiment, varies within wide limits along the rod, and (b) the retardation, after increasing as they observed within the critical distance, becomes stationary at the critical point, beyond which the phase of the flux advances: so that if their conclusion (2) were correct, we should have to admit an infinite velocity at the critical point and a negative velocity beyond it.

(5) As the flux passes along the specimens made of wires, the amplitudes f_3, f_5 of its upper harmonics get continually reinforced by a transference of energy from the first harmonic (see § 13, Tables XVIII.-XXI.). The ratios $f_3/f_1, f_5/f_1$ first increase with x , attain maxima, and then diminish; at the same time the differences β_3, β_5 between the phases of these harmonics and the phase of the first diminish first with x , attain minima, and then increase; and the positions of maxima of $f_3/f_1, f_5/f_1$ and minima of β_3, β_5 are at (or near) the critical point. From this it results that as the flux-wave moves away from the origin its form changes, becoming flatter as x increases (see fig. 9) until the critical point is reached, when its flatness is a maximum; beyond this as x increases its flatness diminishes.

(6) The effects of an end on a flux-wave approaching it are to increase the leakage coefficient and to cause an advance in the phase. (See fig. 6.)

(7) When a rod whose behaviour has been determined is shortened and subjected to the same magnetization as before, no difference is observed in the flux-waves until (for a $\frac{1}{4}$ in. rod) within 30 cms. from the new end. (See § 8 and fig. 6.)

(8) For the same frequency, initial flux, and cross-section, increase of permeability increases the critical distance, increases the critical value of the retardation (see fig. 7), and diminishes the critical value of the leakage coefficient. (See § 9 and fig. 8.)

(9) Eddy currents diminish the critical distance, increase the critical value of the retardation, and increase the leakage coefficients. (See § 12.)

(10) For the same frequency, initial flux-density, and material, increase of diameter of the specimen increases the critical distance, has little or no effect on the critical retardation (hence making $d\theta/dx$ near the origin less), and diminishes the leakage coefficients: in fact, we find that the critical distance is approximately proportional to the diameter, and the critical value of the leakage coefficient inversely proportional to the diameter. (See § 14 and Tables XXI., XXIII., XXIV.)

(11) For the same specimen and initial flux, increase of frequency increases the initial value of $d\theta/dx$, increases the critical values of the retardation and the leakage coefficient, and slightly diminishes the critical distance. (See figs. 1, 2, and the Tables.)

(12) For the same specimen and frequency, increase of initial flux increases the critical distance (see fig. 4), increases the critical value of the retardation (fig. 4), and reduces the critical value of the leakage coefficient (see fig. 5): also for low values of the initial flux F_0 , $d\theta/dx$ near the origin increases with F_1 , but seems to approach an upper limit for high values of F_0 . (See fig. 4.)

L. On the Use of Chilled Cast Iron for Permanent Magnets.
By ALBERT CAMPBELL, B.A. (From the National Physical Laboratory.)*

EARLY in the past year an interesting paper† was published by Mr. B. O. Peirce of Harvard University, drawing attention to the fact that chilled cast-iron is in many instances a suitable material for permanent magnets. As the subject is of interest to scientific experimenters and of considerable importance to instrument makers, I undertook some time ago a short research upon it, with a view, firstly of obtaining some measurements by standard methods, and secondly of finding, if possible, an easy method of chilling the material so as to give good results.

Form of Test Pieces.—The cast iron tested was of ordinary commercial quality and was obtained in the form of rods and rings. The rods, which were of rectangular section, were shaped to dimensions usual in such tests, viz., 10 cm. × 1 cm. × 1 cm.

* Communicated by the Physical Society: read January 26, 1906.

† Amer. Acad. Proc. xl. 22. pp. 701-715, April 1905. Dr. Watson has kindly drawn my attention to Mr. J. R. Ashworth's experiments on chilled cast-iron rods (Proc. Roy. Soc. vol. lxiii. p. 210, Dec. 9, 1897); he found that the magnetic quality of these was comparable with that of tungsten steel.